

STUDENTS' APPROACHES ON WARPED SURFACE

Abstract: The geometric challenges in the architectural design of warped surface come mainly from the physical materialization. This paper presents structural forms based on warped surface and shows the types of structures made during the 20th and 21st century, the geometric scheme, the structural behaviour and a projection of their potential trends. The analysis creates the possibility of comparisons between original and current design. Also, this work analyses the ways in which descriptive geometry, by warped surface allows the students architects the realization of volumes and architectural spaces, especially the role it plays in the development of spatial thinking.

Keywords: education, architectural design, hyperboloid, hyperbolic paraboloid, conoid.

1. INTRODUCTION

This analysis presents a geometric study of the architectural curvilinear forms that are based on warped surface. For the acquisition of professional attributes, architect students must be able to perform a comparative analysis of the structure, shape and functionality of a building. This synthesis begins in the first year of study where various forms and surfaces underlying the complex structures of the 20th and the 21st century are studied at the Descriptive Geometry discipline. This knowledge is applied in the architectural design workshop of the year 1, projects that have to meet requirements related to architectural design, functionality, structural geometry and technology.

The structure clearly plays a fundamental part in this equation, and the stiffness and strength it shows when responding to various loads depends in turn on the used materials, the basic details of the structure, the technology of the epoch and last but not least the scale of the architectural project. Nevertheless, the geometric profile of a structure is of crucial importance.

Mathematician Carl Friedrich Gauss (1777-1855) grouped the infinity of curved surfaces into three main categories: spheres, cylinders, and saddle-shaped surfaces. Emphasis will be laid on the last category in this study, saddle-shaped surfaces or more precisely ruled minimal surfaces. One of the advantages of these types of structures in architecture is that they owe their resistance strictly to their form, hence their being called self-bearing structures.[1] As these architectural forms evolved, the structural analysis had a long and difficult history. As they were developed and perfected sometime between 1950 and 1960, at a time when architects were using them as a means of artistic expression, long before the computer ever entered the architectural scene, a considerable amount of effort was required to check the designs.

2. RULED SURFACES

A ruled surface is a surface formed by a motion of a straight line through a space, which is moving according to a certain rule. For instance, a line, D, lying on three

arbitrary curves Γ_1 , Γ_2 and Γ_3 in the space, called directrices, may intersect three surfaces. The D line is then called generator of the ruled surface. This study is focused on the non-developable ruled surfaces or skew surfaces, in other words those surfaces cannot be unrolled onto a plane. They are characterized by the variation of the tangent plane to the ruled surface as the point of tangency changes on the generator. Thus there is a new tangent plane to the surface corresponding to each position of the point of tangency on the generator.[2]

The non-developable ruled surfaces are generated by a line lying on: Three curved directrices Fig. 1; Two directrices and a core surface; Two directrices and it is parallel to the generators of a directrix cone; Two curves and a directrix line – cylindroid Fig. 2;

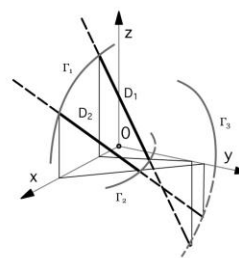


Fig. 1 Three curved directrices.

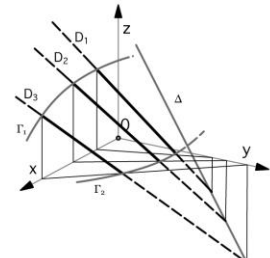


Fig. 2 Cylindroid.

One curved directrix and two line directrices – conoid Fig. 3; Three line directrices, separated by a finite distance – hyperboloid Fig. 4;

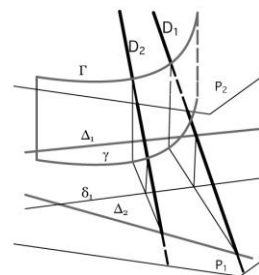


Fig. 3 Conoid.

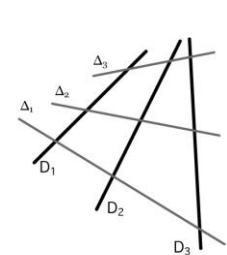


Fig. 4 Hyperboloid.

A line directrix to the infinite – *hyperbolic paraboloid* Fig. 5.

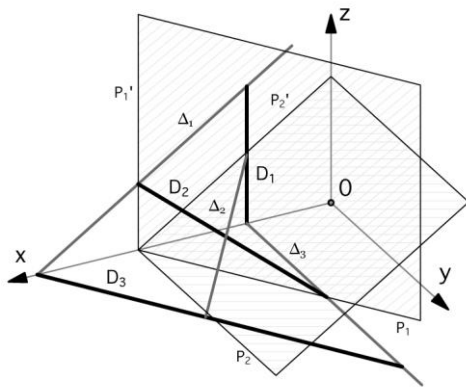


Fig. 5 Hyperbolic paraboloid.

The surfaces that will be further presented in this report are: the hyperboloid, the hyperbolic paraboloid and the conoid.

3. THE HYPERBOLOID OF REVOLUTION OF ONE SHEET

The one-sheet hyperboloid of revolution is a doubly ruled surface of the second order, generated by a straight line D , revolving around another straight line (ω, ω') , vertical axis, not situated in the same plane as the straight line D Fig. 6. The one-sheet hyperboloid of revolution can be also obtained by revolving a hyperbola around its axis (ω, ω') not transversal Fig. 7. Every point of the generator describes by its revolution a parallel circle of the surface.

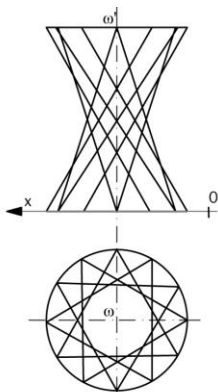


Fig. 6 Hyperboloid.

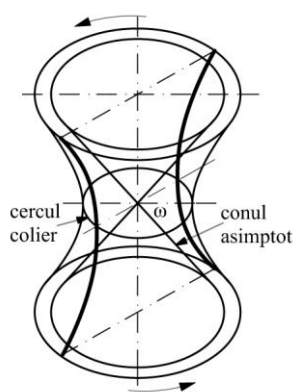


Fig. 7 Hyperboloid obtained by revolving a hyperbola.

As there are two lines that can generate the same surface, the one-sheet hyperboloid of revolution is a doubly ruled surface. The two lines are part of two systems of generators Fig. 8. The front generators are called main generators. The vertical projections of these main generators can be taken two by two, making up the apparent contour of the asymptotic cone of the surface. The asymptotic cone of the surface is the cone whose vertex is in the center of the contour circle and its

generators can be obtained by directing all the parallels from the center of the contour circle to the hyperboloid's generators. Every hyperboloid generator has a corresponding parallel generator, on the asymptotic cone and every generator on the asymptotic cone has two corresponding parallel generators, of different systems on the surface of the hyperboloid.

The general hyperboloid or scalene is a second order surface and it is generated by a straight line lying at all times on three directrices not parallel to the same plane Fig. 9.

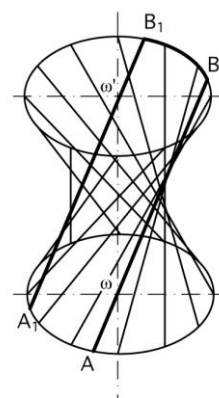


Fig. 8 Hyperboloid is a doubly ruled surface

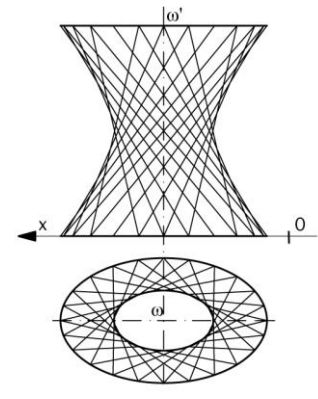


Fig. 9 General hyperboloid.

3.1 Fields of use

The hyperboloids of revolution are extremely important for the development of architectural projects as their doubly curved surface may be executed from straight fragments. The pre-stressing strands may be efficiently arranged in the direction of the generators; and the concrete forming of the doubly curved surfaces, when concrete is used, can be achieved extremely easy, by using straight wooden segments.



Fig. 10 Saint Louis Science Center, United States, 1963 [3].

The hyperboloid has many applications in constructions and architecture. Thus, for executing various roof systems, joining together a number of hyperboloid sections would be an excellent option. The simple surface hyperboloid of revolution can be obtained by revolving a hyperbola around its main axis, frequently employed when building cooling towers which can be made either from concrete cast on site or by using pre-cast elements. The St. Louis Science Center

Planetarium's hyperboloid of one sheet exterior curved surface was conceived by Gyo Obata in 1963. The thin shell concrete structure rests on 12 pillars around the building Fig. 10. The generators stay visible when there are other types of bearing structures, rhombic entanglements of straight generators, materialized in 2008 as the Tornado Tower, a 200 m-tall skyscraper in Doha, Qatar, Fig. 11.



Fig. 11 Tornado Tower, Doha, Qatar, 2008 [4].

The general hyperboloid can be seen in the architecture of two iconic buildings: The Canton Tower building in Guangdong, China, 2010, Fig. 12.



Fig. 12 The Canton Tower, Guangdong, China, 2010 [5].

The former is a 600 m tall building presenting an open network structure, allowing the hyperboloid's generators to stay visible. Museo Soumaya in Mexico City, 2011, also built on a steel framework, is hidden underneath an opaque facade featuring 16,000 hexagonal aluminum tiles, Fig. 13.

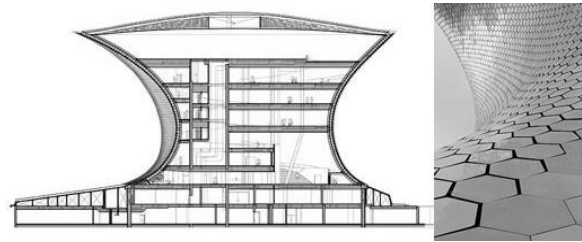


Fig. 13 Museo Soumaya in Mexico City, 2011 [6].

3.2 Answers given by students

By courses, works and workshop exercises the students can follow historical, theoretical fundamentals and the technical bases of projective geometry which will offer the bases of development of a specific vocabulary. That is why, the theoretical courses take the shape of debates and the practical workshop mean effective construction of models and representation of technical drawings, the very confrontation with built reality. We want to present below the use of hyperboloid and their implementation in models which surprise both the volume of architecture object and the indoor space, they were made by the first-year students of the Faculty of Architecture. The tridimensional model of concept drawings, offers a very good visual support, being the most suitable communication pathway between the designer and beneficiary. According to the design theme of the first year – "Temporary Pavilion", students have a great liberty in choosing the architectural forms.

The use of primary forms, an example is the circle, as generator of architectural forms is one of the most encountered methods in design. [7] Thus, some of them chose hyperboloid as a process of creation of space of architectural object, materialized by a straight line, revolving around a vertical axis, not situated in the same plane as the straight line. The student has chosen to materialize this surface by several straight lines as can be seen in Fig. 14. This generates a circular space that offers a 360 degree panorama over the city. As we can see the circular movement, will emphasize rounded spaces. At certain times, this rounded movement will be used, in order to emphasize the center of interest or to reveal more of the spatial surroundings.

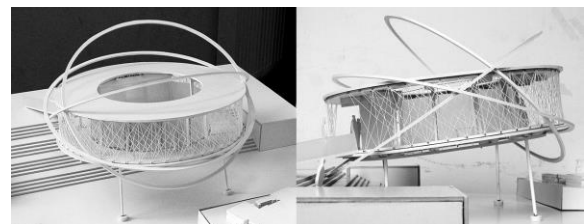


Fig. 14 Model made by student using hyperboloid.

4. THE HYPERBOLIC PARABOLOID

The hyperbolic paraboloid is a quadric ruled surface generated by a straight line that lies on two straight directrices and is at all times parallel to a director plane. It is built by tracing one generator at a time, as a distinct variation of the general hyperboloid. The third straight directrix opens upward to the infinite and is replaced by a director plane parallel to the surface's generators.

The hyperbolic paraboloid is a doubly ruled surface. It contains two families of mutually skew lines that can generate the same hyperbolic paraboloid. The first generator family is made of generators parallel to the first director plane P Fig. 15. The second family is made of generators parallel to the second director plane.

The second director plane is parallel to the two straight directrices $\Gamma_1 \Gamma_2$, which support the generators in the first family Fig. 16. Thus the generators in the first family may become directrices for those in the second family and the other way round. The hyperbolic paraboloid is the only ruled surface with two director planes.

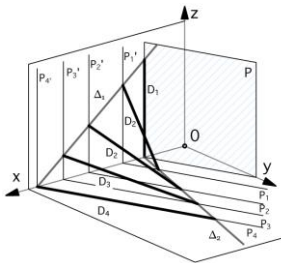


Fig. 15 Hyperbolic paraboloid.

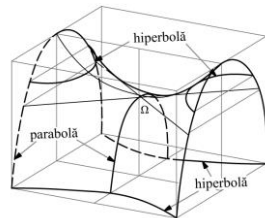


Fig. 16 Hyperbolic paraboloid.

A hyperbolic paraboloid can be also defined by means of a skew quadrilateral ABCD Fig. 17. A skew quadrilateral determines one hyperbolic paraboloid and only one. The axis of the hyperbolic paraboloid is the straight line parallel to the intersection line of the two director planes; it may be determined by joining the middle sections of the skew quadrilateral's diagonals ABCD. The tip of the hyperbolic paraboloid is the point on its surface where the tangent plane in that particular point is perpendicular on the axis of the hyperbolic paraboloid.

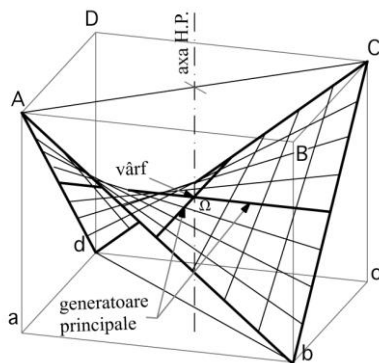


Fig. 17 Skew quadrilateral forms hyperbolic paraboloid.

The two generators that pass through the tip of the hyperbolic paraboloid are called main generators. The main generators are the diagonals of a parallelogram that can be obtained by joining the middle of the skew quadrilateral's sides $AbCd$. The surface of the hyperbolic paraboloid contains two series of straight generators Fig.18.

They allow the delineation of skew quadrilateral sections of equal measure. Or, in other words: any skew quadrilateral may be adjacent to a section of a surface of a hyperbolic paraboloid. If two straight opposite sides are equally subdivided and the subdivision points are united by straight lines, they become generators of the doubly curved surface by the hyperbolic paraboloid. [8]

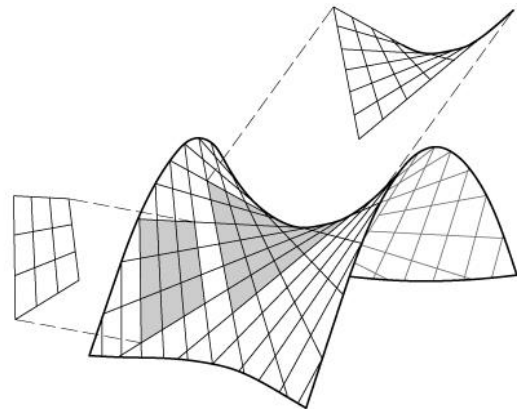


Fig. 18 The hyperbolic paraboloid contains two series of straight generators.

The paraboloid, as a translation surface, may be also generated by a parabola which is moving parallel to its axis, along with another parabola, having parallel axes and pointed in different directions.

4.1 Fields of use

There are many examples of hyperbolic paraboloid in constructions and architecture where it can be encountered in the manufacturing of roof systems or in other projects which require a large number of surfaces, Kuala Lumpur International Airport in Malaysia as an example, completed in 1998, Fig. 19.



Fig. 19 Kuala Lumpur International Airport, Malaysia, 1998 [9].

An illustrative example in this respect is the Oceanographic in Valencia, Spain, a work of the architect Félix Candela, completed in 1999. Eight intersected hyperbolic paraboloids form a radial shell. The lines of striction confer stiffness to the structure and the edges of the paraboloids remain free, Fig. 20.

In 1966, Marcel Breuer designed the church Saint Francis de Sales Parish in Muskegon, Michigan, known for his curving Brutalist form, hyperbolic paraboloid concrete wall, Fig. 21.



Fig. 20 Oceanographic in Valencia, Spain, 1999 [10].

From a structural point of view, the double curvature of opposite direction deals very well with the changing game of the internal forces of tension and compression, maintaining balance under any strain, if there is a minimum manifestation of forces. The surface may bear in any point or given direction the compression or applied stress, tangential to its curvature. It may be rest on the two lower points, taking Kuala Lumpur International Airport, Fig. 22.



Fig. 21 Saint Francis de Sales Parish in Muskegon, Michigan 1966.

Fig. 22 Kuala Lumpur International Airport [11].

For transmitting the self-load which is the most important load of a roof, the suspended parabolas with the curvature downwards are preferred for the tension efforts, and the parabolas with the curvature upwards are preferred for the compression forces fig. 23.[12]

It's a positive thing that the parabolas coincide perfectly with the pressure lines, and that they are capable of supporting their own weight. The deviation tendencies of the forces from the parabolic curvature of

the thin surface are therefore very reduced from the very beginning. [13]

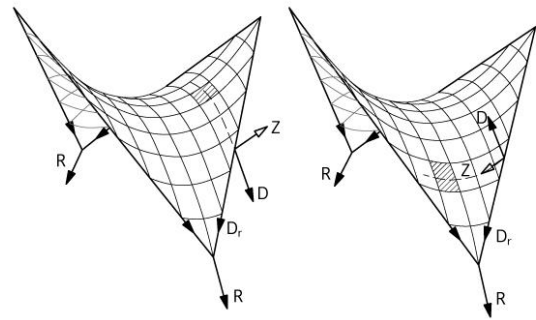


Fig. 23 The tensions involved in the paraboloid.

4.2 Answers given by students

The hyperbolic paraboloid generates as we have seen a beautiful cover, so students have approached this type of surface as an entry signal in the example of the first pavilion, Fig. 24. This surface in relation to the other proposed spatial forms generates an animated external space.

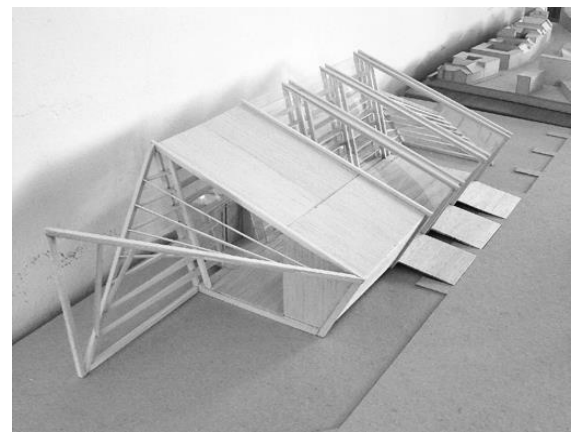


Fig. 24 Model made by student using hyperbolic paraboloid.

In the following example we can see a space that is generated by a band that surrounds the interior space, and the links between them are made with the hyperbolic paraboloid surfaces, Fig. 25. By using the surface of the hyperbolic paraboloid a single inconvenience appears, in this example the indoor space resulted is longitudinal, thus the students are forced to add other volumes in composition.



Fig. 25 Model made by student using hyperbolic paraboloid.

5. THE CONOID

The conoid surfaces are ruled surfaces generated by a straight line that lies on a straight directrix D and a curve directrix Γ , staying parallel at all times to a plane called director plane Fig. 26.

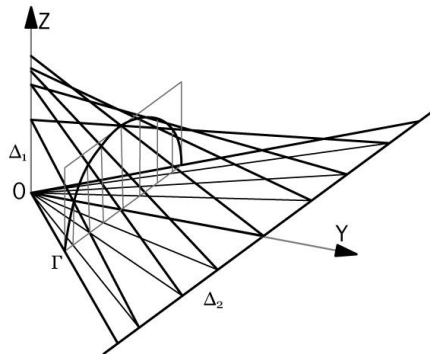


Fig. 26 The conoid.

The conoid surfaces are a particular case of ruled surfaces, generated by a straight line that lies on three given arbitrary directrices, but where one of the directrices opens onward, creating the conoid with a director plane: it may be right or skewed. A right conoid with a director plane that has a circle as a curve directrix, as a directrix line a line parallel to the plane of the circle and as a director plane a plane perpendicular on the directrix line Fig.27. In Fig. 28 we can see the double orthogonal projection of a right conoid. The two sheets of the conoid intersect after the straight-line directrix that is called the surface's line of striction. This straight line represents the shortest distance between any two generators of the surface. Sometimes, the curve directrix Γ may be replaced with a surface S , tangential to all the conoid's generators. In this case the surface S is called a nucleus or a core. [14]

Thus we may define a right or skew conoid circumscribed to a given sphere. If the curve directrix Γ is a helical line then the conoid becomes a helical surface with a director plane. And if the curved directrix Γ is reduced to a straight line then the conoid may be confused with the hyperbolic paraboloid.

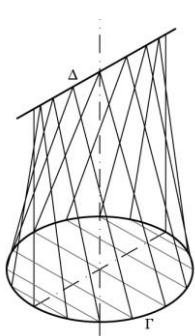


Fig. 27 The right conoid.

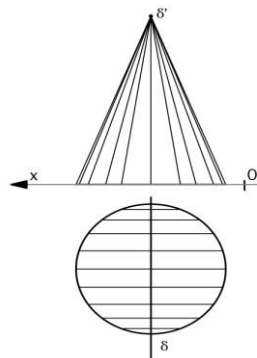


Fig. 28 The double orthogonal projection of a right conoid.

The conoid surfaces are parabolic-like surfaces, meaning that in any of the surface's points one of the main curvature axes is infinite.

5.1 Fields of use

In architecture these surfaces have been extensively used in the manufacturing of roof systems and coverings. The shape's plastic simplicity is remarkable. The fact that conoids are doubly curved and can still be made of straight lines is of great importance when it comes to building thin surfaces.

Successively arranged conoids may be used in the construction of sheds, such as the Oxford Road Station, Manchester, Great Britain (1958-60), Fig 29. The image represented shows the structure of the roof where we can notice the straight-line directrix that intersects a curve and another straight line directrix.



Fig. 29 Oxford Road Station, Manchester, Great Britain (1958-60) [15].

In Gossau, Switzerland, 1954-1955 architects Heinrich Danzeisen, Hans Voser and engineer Heinz Hossdorf work on the development of an industrial building using conoids, Fig. 30. The construction of shed roofs using thin conoid surfaces allows a straight roof edge. The surface is supported by the superior footing of a section of the roof and the inferior footing of the next. The curve of the superior footing may be a construction that follows the pressure line or another bending-resistant form, which stiffens the thin surface just like a buffer. [16]



Fig. 30 Factory Gossau, St. Gallen, Switzerland; 1954-55 [17].

One might even include the cover of the entrance of the UNESCO building in Paris amidst the conoid use examples, a project developed by architects Marcel Breuer, Bernard Zehruss and engineer Pier Luigi Nervi. Two such conoid surfaces emerge in a console from a parabolic arch. The broader surface which emerges from the front side is delineated by two parabolas in opposite directions, of which the upper one is slightly curved. The surface is obtained by translating a straight-line generator on these two parabolas Fig.31.



Fig. 31 the entrance canopy at the UNESCO building in Paris, France, 1958 [18].

5.2 Answers given by students

Students can be attracted to circular or curvilinear forms, and the conoid can satisfy this requirement. An example is given in the Fig. 31 where the student used the conoid to generate a longitudinal space through the transversal frames that materialize the directrix plane. It will have wooden frames that through their succession will form the image of a curved structure. Inside the pavilion there is a space that provides flexibility of exposure, while keeping the visual connection to the exterior through the transparency of the closure.



Fig. 31 Model made by student using conoid

Concepts are ideas which integrate various elements in a whole and become an important part of architectural design.

We can thus notice the concepts proposed by students in these models and the way in which thinking and geometric knowledge made possible the materialization of architectural shapes.

6. CONCLUSION

Whenever we approach an architecture project, we must take into account several aspects:

- Defining an architectural program, understanding a socio-urban context, understanding the site - typology, climate, orientation, views, access, context, history, boundaries, shape.
- Defining a conceptual design strategy.
- Defining a architectural constructive system.
- Establishing a geometry capable of satisfying the conceptual, functional and building structure of the building.
- Establishing a relationship between the interior spaces and the exterior image of the building.
- The premeditation of some aspects of time (its action on the building as a qualitative parameter).

The message of architecture needs a visual language to communicate our thoughts and choices about how we imagine an architectural object or spatial organization. The drawing facilitates the thinking process and allows for interdisciplinary communication with all those involved in the constructive process of architecture.

In this paper we have analyzed several architectural volumes and we have seen how easily these complex surfaces materialize in the descriptive geometry if they are known in the analytical aspect.

Thus, in order to involve the students' critical capacity, it is necessary to blend the real/practical experiences with geometric analysis and with descriptive and constructive methods.

This procedure generates a greater independence in making decisions, an overview and adoption of the most suitable strategies for solving various design and structural problems. [19]

As we could see in the study, an infinite variety of forms and structures may be produced by using non-developable ruled surfaces.

The imagination of the architects and their desire to produce unique works is today possible as high-performance materials and equipment are used in constructions and as computer aided modeling is a support instrument.

Thus, using portions of such surfaces, by reassembling them, we can obtain new forms, varied in terms of functionality and aesthetic purpose.

As we are in possession of an optimum structural solution, we can obtain in the same time specific plastic forms for directing the stress and bearing capacity to the most important parts, for effectively creating reinforcements and ribs, for the expressive form of the footings and the visible transfer of the load.

Students' Approaches on Warped Surface

Students find inspiration in the great achievements of the last century, knowing that they already have the answers to a large set of limitations.

This work provides a basis for future exploration of new concepts in architecture and construction.

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