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GEOMETRICAL STRUCTURES AND SYMMETRY LANGUAGE IN GEORGE DAVID BIRKHOFF'S STUDIES

Abstract: *Our study is a theoretical approach to David Birkhoff's theory on aesthetical measure and other theories of measure, as a potential path to improve aesthetic engineering theory. Golden ratio, fractal dimension, a few principles of design, self-organizing maps, evolutionary systems and the concept of complexity measures are the steps we considered. Our conclusion is that an objective measure of the aesthetic concept is far from being put to life.*

Key words: *Symmetry, aesthetic engineering, measurement, beauty, fractals*

1. INTRODUCTION

Our study aims to approach G.D. Birkhoff's theory on aesthetical measure from a slightly different angle, taking into account not only more recent studies on the matter, but also the advantages or disadvantages of applying his theory to the modern day aesthetic engineering which is also a relatively new concept.[1]

In his book, *Sapiens: A Brief History of Mankind,* Yuval Noah Harari [2] develops an exciting theory on the history of civilization, destroying several myths in the process and covering almost all areas of culture with new ideas. He begins his journey by stating that our history as we know it started quite late on the scale of evolution, some 70,000 years ago, long after the first humans populated the world (some 300,000 years ago people already knew and used fire, but that didn't change significantly their lives, for a very long time). As the author says, if the "Cognitive Revolution" is the foundation of our culture (70,000 years ago, as mentioned before), the "Agricultural Revolution" accelerated history around 12,000 years ago and the "Scientific Revolution", which began 500 years ago somewhat put an end to the history and triggered something completely different.

The key concept of Harari's theory is the creation of imagined systems by our ancestors, systems that shifted the accent from nature to human activity. They lived in small groups, but they needed to cooperate and to communicate, so, the first great achievement was the creation of a new language – different from the sign language. This is the Cognitive Revolution that covered some 40,000 years. This is also the start of that imagined system that completely separated humans ("homo sapiens", in fact) from any other form of life. They were able to communicate more information on the world, on the social relations, on things that are not real, for example "the tribal spirits", as Y. N. Harari considered. Later on, instead of just continuing to live as simple hunters and gatherers, going from one place to another and following the change of seasons and the trails of animals, humans settled down and built shelters that were to become real houses, they worked the land and their work was definitely harder and harder, demanding more and more possessions, more tools and a way to use something more sophisticated than the ten fingers to add numbers. The necessity to have more imagined systems grew even more. To shorten this introduction, suffice to say that all systems operating today, from ancient times or more recent, are created by us, with no similarity to the natural world: a new kind of order was necessary, as opposed to the chaos of everyday life. Language and writing and the need for order made possible the apparition and the implementing of the first code of law, Hammurabi's code, in 1776 BC. Next in line were most branches of science, slowly developed in the years to come.

Starting from some simple questions (for instance, how did people succeed in organizing cooperation systems?) Harari concludes that people created "imagined orders" and "writing systems". These two inventions filled the gaps left by our biological inheritance.

Therefore, philosophical concepts or scientific ones, all such concepts help people to communicate, to create and to enjoy their creations, in an orderly manner. What could be wrong if we have a theory on aesthetical measure? Is there anything wrong with a mathematical approach to beauty, as we are used to perceive it from a totally different angle? Is there an aesthetic angle to engineering that should be considered quite seriously? Or is function still more important than form? We will try to find some answers through our study.

2. APPLICATION FIELD

As mentioned in the previous chapter, we aim to get some new angles in aesthetic engineering, maybe to underline a few more aspects of such a challenging domain.

Aesthetic engineering should answer several questions, and one of them is truly important: Is there a way to blend engineering and scientific methods with aesthetic design and even with the evaluation process? (in a different way than simple intuition of the designer could do it, or the specific knowledge of the analysts concerning the trends). Philosophers and art critics have not reached a consensus concerning the nature of beauty or any other aesthetical concepts. In an academic area or during a lofty speech the term "aesthetics" is used mainly with reference to arts theory (Honderich, 1995). At the same time, the main goal of aesthetic engineering should be the use of engineering, mathematical and other science methods to identify and also to quantify the role of the aesthetic elements in design.

Also, we should note that the positive aesthetic appreciations of the products and systems have 2 specific characteristics: 1) they are multi-modal – it means we may be confronted with more than a single sensorial experience (not only a visual one, but also aural, tactile or olfactory): 2) they tend to be interactive – the client, appreciating a product or a system does not remain a simple observer. He may interact with the object, he tests it. These 2 characteristics provide the distinction between the aesthetical appreciations of a product/system and the appreciations of an object of art, thus determining the specificity of the aesthetical engineering.

It is obvious that aesthetic engineering should develop its own theories and methods in order to answer all challenges.

There are a few theories that try to build a solid foundation for the aesthetic engineering: philosophical, psychophysical, cognitive and social, even ecological. However, our interest now is to find a place here for the mathematical theories.

3. MATHEMATICAL THEORIES APPLIED TO AESTETHIC

George Birkhoff is a mathematician who in the 1930s wrote a book "Aesthetic Measure" where he suggested a formula to measure aesthetics, as a function *f* of the ratio: M = f(O/C) where (M) is measure, O is Order and (C) is the Complexity. (Fig. 1)

Philip Galanter also commented on this formula, as we can see below, in Fig. 1:

"M / C

Where:

M = aesthetic effectiveness
O = degree of order
C = degree of complexity" [3]

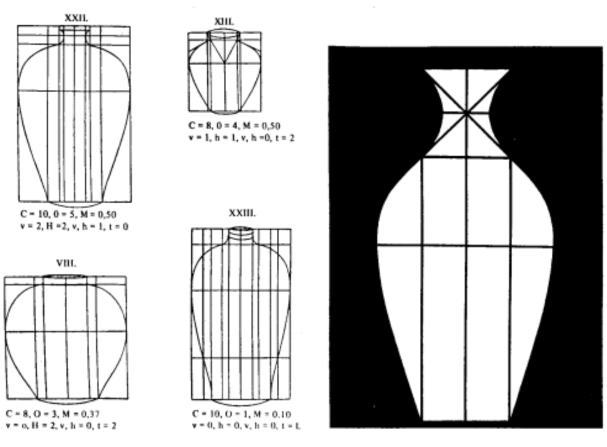


Fig. 1 Birkhoff's Aestethic Measure, as reflected in P. Galanter's work

In the Introduction, BirkoffI states that he started to be interested in the structural aspects of aesthetic perception while listening to music, almost 30 years before. He somewhat became aware of the fact that order, or pattern of the tones has a significant importance in the aesthetic awareness of music, as stated by Derek Thomas in 2018.

Birkoff considers that aesthetic experience consists of 3 primary consecutive stages: "… 1) a preliminary effort of attention, which is necessary for the act of perception, and which increases in proportion of what we shall call the *complexity* of the object (C); 2) the feeling of value or *aesthetic measure* (M) which rewards the effort; 3) a realization that the object is characterized by a certain harmony, symmetry or *order* (O), more or less concealed, which seems necessary to aesthetic effect…" (Birkhoff, pp. 3-4).

What is interesting about this is that the formula did not come from nowhere and it reflects a sort of physiological theory of his: order is a sort of released unconscious tension when the perception is real, therefore complexity describes to what degree unconscious and psychological efforts should work to reach an aesthetic value. It is sort of the effort made divided by the relaxation released once the aesthetic object is perceived. So, in order to actually make this practical he broke the equation out and came up with ways to do the summation and these measures are always in a group of similar objects or a group of paintings or a group of pottery or something like that. Even with some very simple shapes his formula is not satisfying, though he tries hard to exemplify hid theory, as in Fig. 2, also given by Birkhoff:

"M = O / C

C = number of extended lines
O = V + E + R + HV – F
V = vertical symmetry
E = equilibrium
R = rotational symmetry
HV = relation to horizontal/vertical network
F = unsatisfactory form"

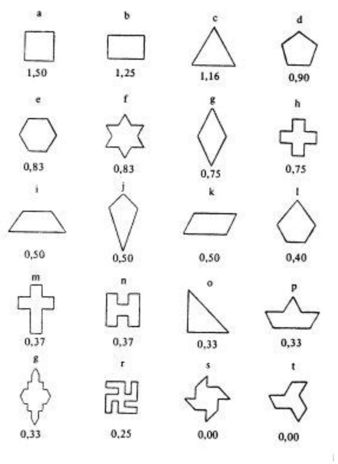


Fig. 2 Birkhoff's Aestethic Measure

Initially, most reactions to Birkhoff's theory came from psychologists. In their research papers they published results of empirical studies and compared them with Birkhoff's theory. They tried to find a correlation between the empirical results and theoretical predictions and the results were usually negative in the sense that the theory of aesthetic measure was not corroborated empirically. However, his work continues to influence various fields of research. Other researchers often develop some of his ideas, but cannot provide some comprehensive critical analysis of his work. Other studies discussed the appropriateness of the mathematical formula itself – more recently such discussions originated from information aesthetics or computational aesthetics. It seems that his formula measures rather the aesthetic efficiency than the level of aesthetic quality, prefers symmetry over beauty and penalizes complexity and views order and complexity as opposing concepts. One of his critics was Douglas Wilson (1939).

Another theory to be taken into consideration is **the Golden Ratio** – also one of the oldest theories. "Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid, through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler, to present-day scientific figures such as Oxford physicist Roger Penrose, have spent endless hours over this simple ratio and its properties. But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics" (Livio Mario, 2003) [4]. The author considers that the Golden Ratio is specific not only to human calculations and therefore solely determining human activity. The same ratio has been observed in many patterns abundant in nature: the spiral arrangements of leaves, flower petals, seed heads, shells, hurricanes and so on.

From the mathematical point of view, the Golden ratio, also known as the golden section or the golden mean or the divine proportion ("sectio aurea" in Latin), is the first irrational number discovered and also defined in history roughly equal to 1.618033, and the symbol Φ (the Greek letter) is used for it. To calculate it we use the equation:

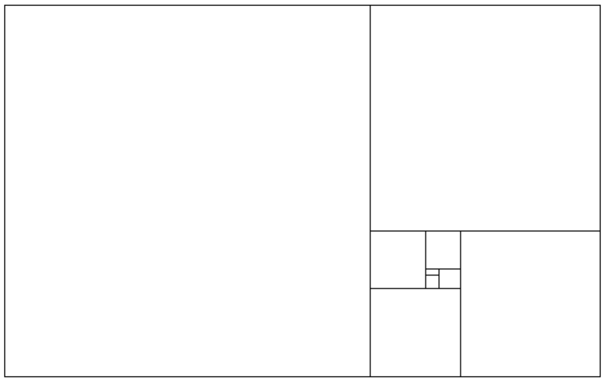
$$(a + b)/a = a/b = \Phi,$$

which leads to:

$$\Phi^2 - \Phi - 1 = 0.$$

It is always a positive number.

Considering Fibonacci's series, we have an interesting relation quite nicely described: any odd number of rectangles with their sides equal to Fibonacci's successive numbers will fit perfectly in a squared number. Thus we obtain a rectangular shape, similarly appearing any time we cut off a square. (Fig. 3a, 3b).



Fibonacci sequence
1,1,2,3,5,8,13,21,34

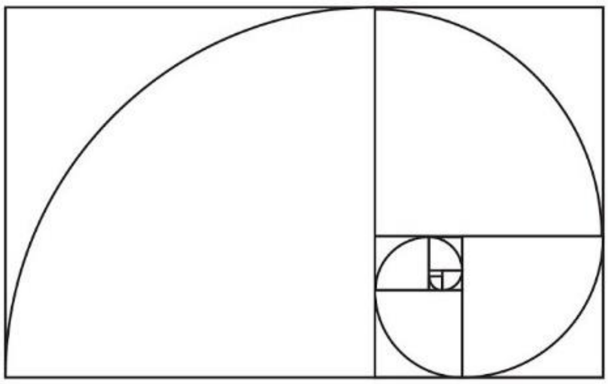


Fig. 3a Golden Section/Ratio

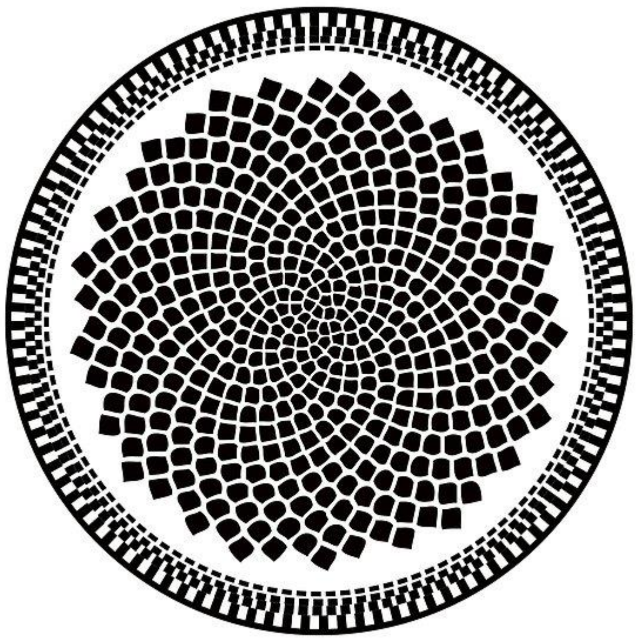


Fig. 3b Sunflower Fibornacci fractal spiral

Approaching modern time, in the 19^{th} century, Gustav Fechner – psychologist, medic, physicist, the founder of experimental psychology – tried to understand the human response to aesthetics, conducting some empirical studies on the matter. The results he obtained emphasized the idea that "the golden rectangles had the greatest appeal" (Philip Galanter, 2012). However, later studies denied Fechner's results, considering that this golden ratio cannot explain or describe beauty inherent to any natural or artificial creation.

Another measure took into consideration is represented by **the fractal dimensions,** as Philip Galanter explained in his works. In 2019, Felix Hausdorff introduced this concept. Fractal dimensions represent a set of small geometrical forms that are the exact replica of larger geometrical forms they are part of – like fern, cauliflower, broccoli, snow-flakes and so on. They also represent a real number found between the topological size of an object and the dimension of the space that defines that object. However, there is still no exact definition of the fractal dimension and no one found a general formula to calculate it.

As P. Galanter mentioned, there are people who try to find a mathematical dimension of aesthetics, applying mathematical calculations to paintings, for instance. It is the case of Taylor (2006) who "did a study of Jackson Pollock's late period 'drip' paintings" which seems to be "fractal-like", as P. Galanter says (Fig 4). And the interesting fact is that if you measure the fractal dimension over time it increases. So, in some sense, the fractal dimension can be related to the aesthetic quality of his paintings. But this is rather a niche use and we cannot derive any generalization here.

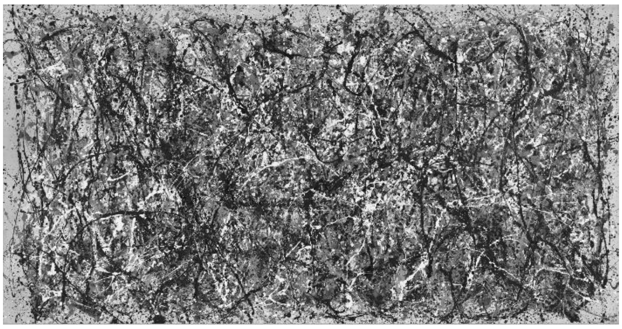


Fig. 4 Jackson Pollock painting fractal-like (Autumn Rhythm)

Speaking of **the principles of design**, by no means to be overlooked when considering the aesthetic aspect of an object, one of the oldest and most famous is **the multitude in unity** ("il piu nell'uno"), unity in variety, variety in unity, as described by the Roman school of painting. Then we reach the notion of **balance,** which comes from our awareness of the surroundings, where we need stability and when we see abstract objects our brain is constantly evaluating the stability, so if we have a couple objects, imagined the on a fulcrum, if they are balanced on the fulcrum then they tend to be balanced in the canvas (Fig. 5).

Another important principle of design derives from **the scale and proportion** of the objects we consider: proportion relates to "the relative size within the image and scale relates to absolute size relative to the body" (P. Galanter, 2012). **Color** has its own importance, as color gives harmony, it contributes to weight. There is also **the rule of thirds:** if we divide a painting into thirds, the intersection points tend to be areas of significant activity, which creates a sort of emotion and also gives balance to the entire painting.

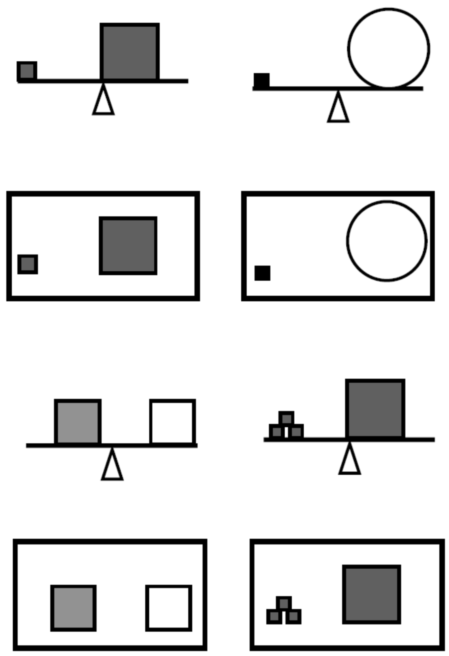


Fig. 5 Balance and weight in the canvas

Sometimes **self-organizing maps** are used, in order to select a certain structure from any piece of music and then develop a critical view over "an evolutionary composition system" (Phon-Amnuaisuk, 2007) [5]. Thus, for a time based work they found that for a very short period of time you get some sort of a coherent measurement or coherent generation, but the global coherence is not here. They tried to create some hierarchical models to show some promise but no good results so far. [4]

Odd enough, there was an evolutionary model, using **evolutionary systems** to reach aesthetic measurement. In short, there is genotype and phenotype: genotype is like the DNA and the phenotype is like the body (Fig. 6)**.**

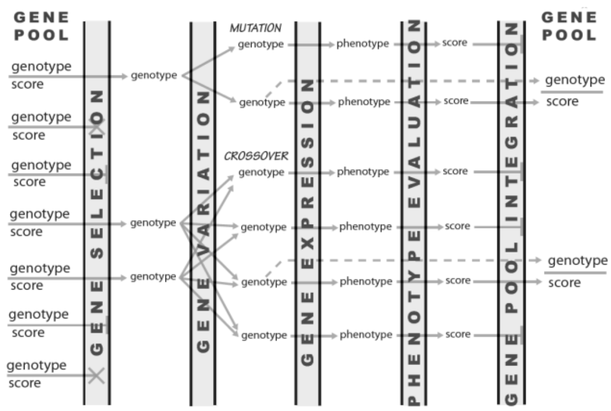


Fig. 6 Genotype Pool

In the art system the genotype would be some kind of data structure, it will be input in an algorithm and then the output will be the piece of art. The idea is that you evolve the genotype and then at some point you have the genotype expressed as phenotype and you look at the work of art and say "good", "bad" or "indifferent" getting a score that gets back to the gene pool that could get better and better art. Of course, there can be all kinds of mutations, crossover and other measures. The problem is that for most industrial application of evolutionary systems of algorithms there is an objective dysfunction: at some point you have the phenotype and you have to make the judgment and have a sub-measure, to maximize the result – but we do not know how to get that number for art. There is still the problem of "evaluating the phenotype to assign a fitness score – what kind of fitness function can measure aesthetic fitness?" (P. Galanter, 2012). The best answer is still to be found, as several people tried the same old approach: getting a bunch of people to look and then to appreciate the aesthetic appeal of the evolutionary system. As P. Galanter mentiones, Scott Draves (2005) [6] "created the Electric Sheep system that allows his genetic screen saver users around the world to approve or disapprove of phenotypes via the internet and this is the crowd source evaluation" (Fig. 7).

Fig. 7 Scott Draves – Electric sheep (2005)

It is again P. Galanter who speaks about "the concept of **complexity measures**, where you use Machado and Cardoso's NEvAR system (2002, 2003) as computational aesthetic evaluation methods with Sim-like evolving expressions. Their fitness function is related to Birkhoff's aesthetic measure". There, as Machado and Cardoso claim, "… the aesthetic value (AV) is, to some extent, linked with the complexity of the image and with the mental work necessary for its perception" [7].

In a way it is an update of the original theory of Birkhoff and they extended it in some interesting ways. To measure the variety of visual stimulus they used jpeg compression and the idea is that the more resistant the image is to jpeg compression the more complex the visual stimulus is. Equally, the more resistant to fractal compression the image is, the more complexity of the perception we get. They developed a formula where "the left side of the formula rewards those images, which have high CV and low CP estimates at the same time, while the right side rewards those images with a stable CP across time." [8]

$$AV = \frac{CV^a}{(CP(t_1) \text{ x } (CP(t_0))^b} \text{ x } \frac{1}{\left(\frac{CP(t_1) - CP(t_0)}{CP(t_1)}\right)^c} \quad (1)$$

Therefore, there is a protection from a bad estimate of a fractal compression. However, their influence was limited to this kind of images and there is no indication of a possible generalization.

To conclude, as P. Galanter said, "aesthetic judgments are typically multidimensional. Evaluation of a traditional painting could generate a set of scores on color, balance, value, etc. A typically multi-objective fitness (F) function could however involve a weighted sum of factors."

$$F = (w_0 \text{ x } color) + (w_1 \text{ x } balance) + (w_2 \text{ x } value) \quad (2)$$

But, assuming we can individually measure these things, they are not correlated, there is a question of where these weights come from and we can assume there is no relation between them so how can we preserve in the gene pool a particular strength in one aspect?

4. FURTHER RESEARCHES

Geometry is an exact science, operating with exact measurements, defining even the most complex forms by decomposing them into the basic ones: a rectangle, a triangle, a circle, lines and points. Things get complicated when works of art combine geometry with the artistic view of the creator, in an amalgamated and new form, entering the aesthetic domain.

However, most important to any domain of activity, as we strongly consider, is creativity. There is no novelty, no innovation, in the absence of creativity. There is no proper design without creativity.

That is why our further researches should aim to explore new ways to develop creativity, to give the next generation a powerful tool for improvement in any given area. Aesthetic measurements are not an answer, in our opinion, it should be far better to hunt for creativity.

Psychology might be the right answer to such a quest, helping both professors and students to understand the full meaning of a creative personality and how to point it in the right direction.

Therefore, our further researches might seem very far from the present study, but we consider the study of creativity a correct answer to most problems related to exact sciences and aesthetics at the same time. We should study the three factors specific to the creative potential: the cognitive/intellectual factors, special abilities and personality factors (motivation, attitudes, character). Also, generally speaking, any pursuit of personal, cognitive, emotional, volitional, psycho-motor development is beneficial for the development of creativity.

That is why our studies in the years to come will be directed to the understanding of creativity and the possibilities to develop it rather than trying to apply mathematics to aesthetics.

5. CONCLUSION

It is our belief that so far no efficient measurement of the aesthetic aspect has been successfully created. Our response to the aesthetic concept is highly subjective and is governed by other laws than the objective world. Aesthetics is one of the imagined systems we talked about at the beginning of our paper and it is not a concrete object but rather its reflection in our mind. For several millennia humans created objects to serve many purposes, first having in mind the sole function of the object and little by little they began to embellish those objects, adding another dimension: beauty. Of course, the standards of beauty varied a lot in time, according to various factors: the raw material used for the making of the object, the ability of the one shaping the object, the tools used, the disposition towards creating something beautiful (not merely useful), the continuous change in taste and trends, the models available and how they were perceived and so on.

Today we are more and more specialized, therefore aesthetics does not answer to arts only, but to several other domains of human activity. Since we created more and more products and we work in such different conditions, the role of engineering is ever growing and is demanding improvement, by taking into consideration the link to aesthetics. However, improving the aesthetic aspect of engineering does not need – in our opinion – measurement, but a better understanding of the clients' desires and beliefs, as well as an open mind towards our culture, that is changing so fast and in so many ways.

On the other hand, critical opinions and novelty have always lead to evolution so there might be some good in trying to measure aesthetics – if we can discover or create the appropriate method of measurement.

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