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INNOVATIVE REPRESENTATION OF TRIPLE ORTHOGONAL PROJECTION USING 3D PRINTING TECHNOLOGY

Abstract: *In this paper the authors present the use of 3D printing to create a physical model that clarifies for students the triple orthogonal projection of a point, an essential concept from descriptive geometry. The physical model features a sphere representing a point in 3D space, along with three plates hinged together and mounted at right angles to represent the projection planes. The 3D printing offered the required precision for component fabrication while a sphere imagining a light spot facilitated projection interpretation. The possibility to rotate the plates facilitated comprehension of the point's draught, resulting in enhanced comprehension of the geometric relationships between the projections of that point. This physical model proves that 3D printing can be used to modernize teaching materials in descriptive geometry.*

Key words: *Orthogonal Projection, 3D Printing, Descriptive Geometry, Educational Model, Draught, Precision.*

1. INTRODUCTION

Three-dimensional (3D) printing is an additive technique that uses successive layers of material deposited on top of each other to create three-dimensional objects. This technique is also known as Fused Filament Fabrication (FFF) [1]-[3]. Figure 1 illustrates how the Fused Filament Fabrication (FFF) technology works, outlining the fundamental steps of the process of depositing molten thermoplastic material layer by layer to create 3D objects.This computerized process melts and extrudes filaments, which are typically made of plastics such as PLA or other termoplasti material[1], [4]. Due to its flexibility and affordability for customized manufacturing, 3D printing is widely used in various fields, including industry, education, and research [2], [3], [5].

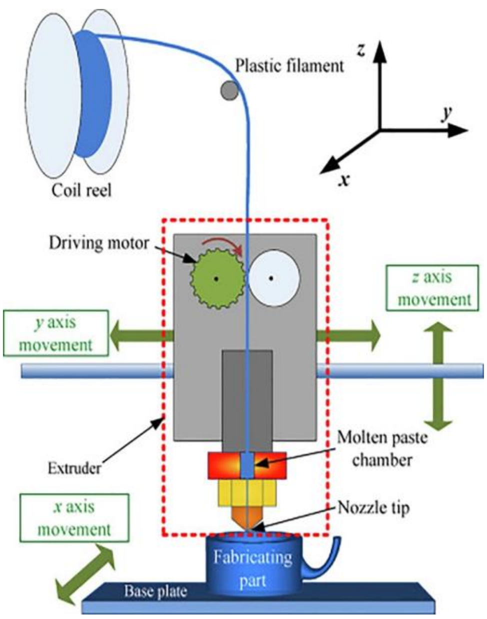

Figure 1 Principle of FFF [6].

In descriptive geometry, the triple orthogonal projection of a point is essential for the orthogonal projection system. This system employs three planes of projection, each orthogonal to the other two. These planes are referred to as the horizontal (H), vertical (V), and lateral (or profile) (L) planes. It is important to note that the projection lines are orthogonal to the projection planes and are also parallel to one another. The planes H, V, and L divide 3D space into eight trihedra.

A point in 3D space is projected onto each of the three planes, resulting in three distinct projections, one for each plane. Each projection serves as an accurate two-dimensional representation used for graphical engineering and analysis. The position of the point's projections in the projection planes is determined by the coordinates of the point in 3D space (Figure 2). Furthermore, the projections that lie in the projection planes correspond precisely to one another [7].

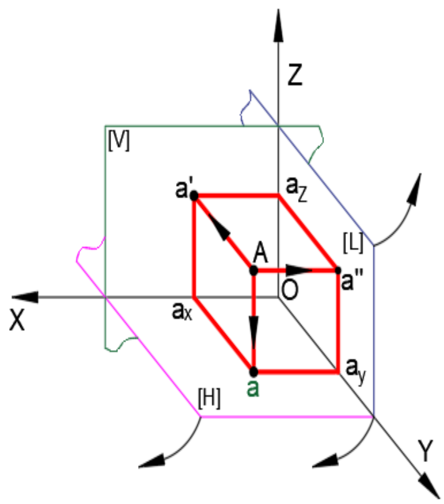

Figure 2 Spatial Representation of a Point Located in the First Trihedron [16].

This technique facilitates the transformation of a point's 3D representation into a 2D representation across at least two projection planes[8]. This process is then employed to depict lines and planes on these projection surfaces. The 2D representation of a point on two or three projection planes results in what is commonly referred to in specialized literature as a draught or projection drawing. From this draught, it is always possible to pinpoint the corresponding point in 3D space. Moreover, the draught serves as a valuable tool for visualizing and solving complex geometric problems, making it indispensable in the fields of technical drawing. [9-11].

The draught is created by rotating the horizontal plane around the intersection of the vertical and horizontal planes until they overlap. Simultaneously, the lateral plane rotates around the line where the vertical and lateral planes intersect, until the lateral plane overlaps the vertical plane (Figure 3)

Representing a point in the triple orthogonal projection involves finding the three projections, each belonging to a two-dimensional system, while maintaining the spatial relationships. First, the projection planes are drawn, then the projections of the point onto these planes are plotted. To ensure consistency of the drought, the projection lines that connect the projections of the point are represented (Figure 3). Generally, it is difficult to understand how the draught looks when a point is placed in a trihedron or another. Starting from these, the authors decided to build a physical model to facilitate the understanding of a point's draught. They chose to use 3D printing to create the physical model, recognising the precision of this technology [12]-[15].

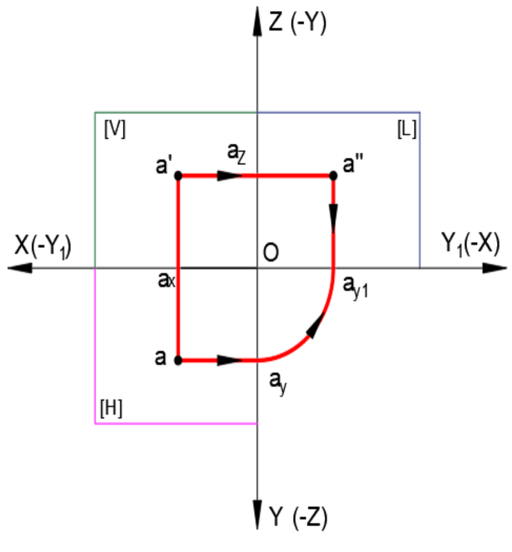

Figure 3 Draught representation of a Point Located in the First Trihedron [16].

2. METHODS

2.1 Materials and equipment used

The Creality Ender-3 V3 SE 3D printer is a state-of-the-art piece of equipment that focuses on rapid prototyping of objects and manufacturing three-dimensional objects. The printer uses Fused Filament Fabrication (FFF) printing technology. By using this technology, precise and detailed models can be obtained by successively depositing layers of material.

The Creality Ender 3D printer uses the aforementioned technology to extrude and deposit thermoplastic filament, such as PLA, layer by layer. First, the filament is heated in an extruder to a temperature between 180 °C and 220 °C. Then, the filament is placed on the printing platform.

The device is distinguished by a large print volume, which can reach dimensions of 220 x 220 x 250 mm. This allows the construction of cube-like objects, making it very suitable for both creating educational pieces and testing highly complex engineering concepts (Figure 4).

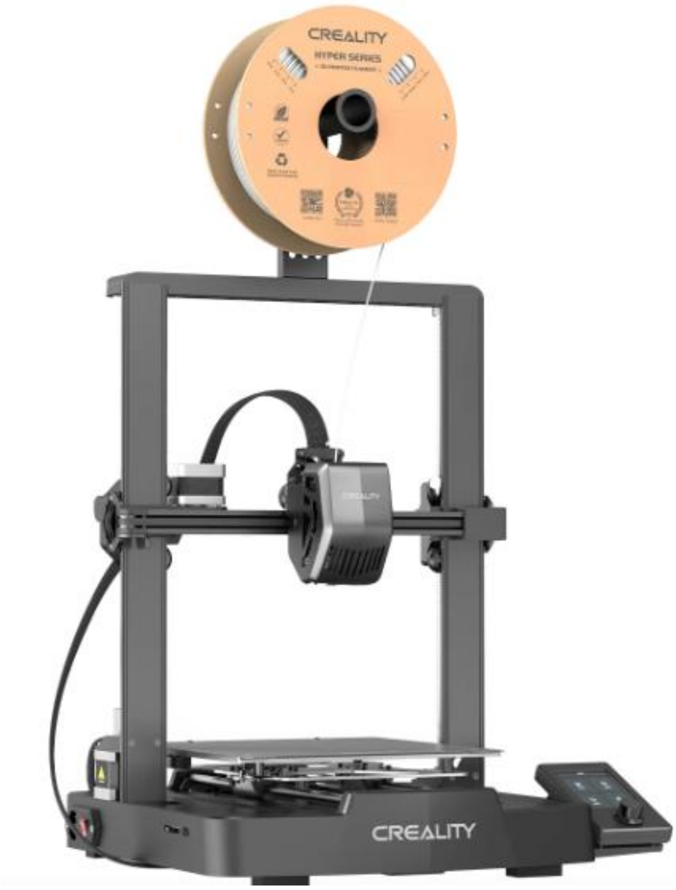

Figure 4 Creality Ender-3 V3 SE 3D printer [20]

This type of printer was chosen for this study because it offers the following advantages:

- It has a high printing speed of up to 250 mm/s while maintaining product quality.
- It supports a wide range of materials such as PLA and PETG, as well as other filament types compatible with fused filament fabrication (FFF), making it extremely versatile.
- The printer ensures continuous and precise feeding during printing, with the filament used having a standardized diameter of 1.75 millimeters.
- This allows the thickness of the layer applied during printing to vary between 0.1 and 0.4 millimeters,

which gives the researcher the opportunity to adjust both the surface quality and the level of detail.

- The printer allows for the faithful reproduction of complex details also because the print head positioning accuracy is very high, 0.01 millimeters on the X and Y axes and 0.002 millimeters on the Z axes. This also contributes to better print quality in general.
- There is no risk of damaging the printed object when it is removed from the printing platform because it is magnetized and flexible.
- The printer features an advanced auto-leveling system that uses an integrated sensor to accurately measure the distance between the print head and the platform surface. This improves the uniformity and quality of each applied layer.
- It prevents slippage or blockages that may occur during manufacturing by incorporating a direct-type extruder for the "Sprite" model. It also ensures a continuous and stable filament supply.
- The printer also features a dual Z-axis that improves the stability and precision of vertical movements, reducing vibrations and guaranteeing superior final print quality.

Therefore, the researchers chose for this research activity a 3D printer that meets the following general requirements:

- it is reliable;
- it is efficient;
- it works well for a variety of applications, such as complex descriptive geometry projects where model quality and accuracy are essential [20].

PLA (polylactic acid) is an easy-to-use, biodegradable material suitable for desktop printers thanks to its smooth finish and dimensional stability. The Ender printer is recognized for its adjustable platform, 0.4 mm layer width and ±0.1 mm accuracy. G-code generated by CAD software controls the printer's movements to ensure layering accuracy. The Ender printer is commonly used for prototyping and educational projects due to its reliability.

2.2 Physical model construction

A demonstration system was designed using SolidEdge software, consisting of three flat plates hinged by pivot points, allowing 90° tilting or extension alignment.. Circular holes were created in each plate, sized so that flexible cylinders could be fixed into them. These cylinders have a sphere at the end, symbolizing the point represented (Figure 5).

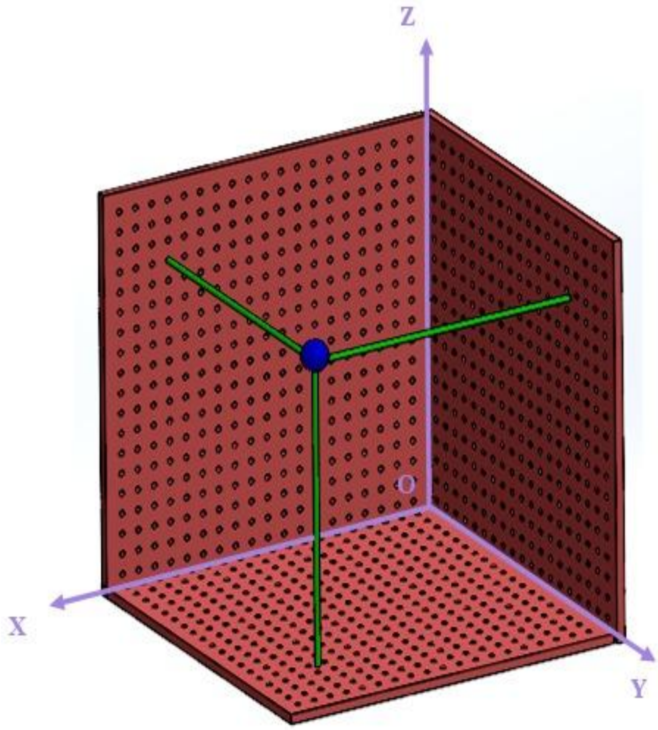

Figure 5 Digital approach to Triple Orthogonal Projection Representation.

During the rotation of the plates, the cylinders adjust their position, visually revealing the projections of the point on the three planes: horizontal, vertical, and lateral (Figure 6).

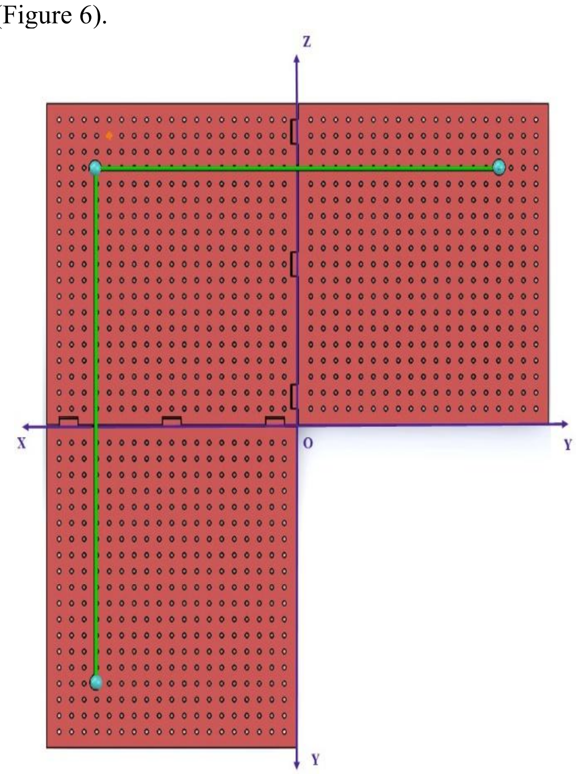

Figure 6 Digital approach to the draught for Triple Orthogonal Projection Representation.

The system includes a directed light source that emphasizes how the coordinates of the point are reflected in the orthogonal triple projection.

This model provides a clear and interactive visual representation, facilitating the understanding the system of the triple orthogonal projection.

When the point is located in the first trihedron, the draught is relatively easy to understand. In the case of the other trihedra, imagining how the draught forms is more difficult, and the visual-spatial intelligence level of each student plays a determining role.

2.3 Integration of 3D printing in the triple orthogonal projection model

3D printing plays a central role in realizing the components of the demonstration system. The articulated plates, flexible cylinders and sphere were designed using CAD software and then printed from PLA. The plates are perforated with perfectly sized holes, guaranteeing precise cylinder fit. The sphere is designed to be mounted at the end of these, completing the point assembly.

Through the integration of 3D printing, the physical models are rapidly created, with precise dimensions, a fact that provides the opportunity to demonstrate theoretical concepts practically.Furthermore, the projections obtained from the triple orthogonal projection are transformed into digital 3D models and printed for deeper understanding and comparison.

This technology offers flexibility and precision in creating educational materials.

Regarding surface roughness, the layer's height is the most important, according to a study by Zhang et al. on soft pneumatic gripper made of TPU [21]. They found the optimal settings, namely: 0.1 mm layer thickness, 0.2 mm wall thickness, 200 °C nozzle temperature and 120 mm/s print speed.

These settings are performed to minimize the roughness Ra to 18.752 μm, using the Taguchi method with L16 matrix [17].

In medical applications, Kwon and collaborators used FDM to optimize the circularity and roughness (Ra) of implantable prototypes. Using the Taguchi method with L9 matrix, they highlighted that layer thickness most affects surface quality (by 54%), build orientation (by 28%), and printing temperature (by 18%). Experimental results revealed a 67% reduction in Ra roughness after adjusting from standard settings to optimal parameters: 0.1 mm layer thickness, planar orientation and 205 °C printing temperature [18].

Smith et al. proposed a post-processing technique called "ironing", which, by applying a pressing beam at temperatures between 190-210 °C and speeds of 50-150 mm/s, reduces the roughness Ra of PLA components from 12.8 μm to 4.2 μm, uniformizing the geometric contour and achieving a slight melting of the layer ridges. Thus, printing parameters, especially layer thickness and post-processing methods, are essential for achieving low-roughness surfaces in 3D printing [19].

To achieve good quality printed parts, the printing parameters must be set appropriately. In this study, the extruder temperature was set to 210°C for PLA material and the build plate temperature was set to 60°C, ensuring optimal adhesion and reducing the risk of delamination or warping.

At the same time, for better adhesion of the material layers to the build plate, the brim function in the printer software was used. This function aims to provide better adhesion through an additional number of material perimeters that are glued around the piece.

The brim function is essential because it provides proper adhesion of the first layer of material, which prevents the part from deforming throughout the printing process. The printing speed was 50 mm/s, which provides a balance between detail and production time. The reduced printing speed played the role of preventing delamination of the deposited material layers, but also better quality of the obtained part. The layer height, 0.1 (mm), determines the fineness of the details for each hole on the board, thus achieving positioning with minimal deviations. The low printing speed, as well as the 0.1 mm height of the material layer, led to the production of a part with good roughness and reduced dimensional deviations.

The Creality Ender printer offers an accuracy of ±0.1 mm, which guarantees accurate dimensions and quality finishes. By respecting these parameters, the parts obtained comply with the requirements of the demonstration system, demonstrating the efficiency of 3D printing in education and technical applications.

3. RESULTS

The use of 3D printing to create a physical model of the triple orthogonal projection has proven that this technology can be used as a foundation in building of the physical models that allow understanding of complex geometric relationships. The horizontal, vertical, and lateral projection planes have passed in this way from the sphere of the abstract to the sphere of the real and are articulatedly joined at precise 90° angles.

The order lines that are represented by the flexible cylinders in the model were easily integrated thanks to the calibrated holes that were strategically distributed on the surface of the plates. This constructive solution allowed for quick and stable positioning of the cylinders. In addition, it allows for precise adjustment of their trajectory according to the needs of the exercise.

The placement of a sphere at the end of each cylinder was intended to mark the precise position that corresponded to a geometric point in 3D space, both visually and palpably. The sphere facilitates quick identification of the point and allows visual appreciation of its relationship to the other components of the construction..

To visualize the model outline, the plates representing the projection planes were rotated. This procedure caused each plate to move to a common plane. This allowed the transposition of the point projections onto each of the three reference planes - frontal, horizontal and lateral. Therefore, it has become much simpler to see and analyze the relationships between point positions on the three main axes (x, y, z). Following the rotation, it was possible to obtain a complete and clear image of the correspondence between the point's location in space and each of its plane projections. This made it easier to understand the essential geometric relationships in descriptive geometry.

The use of the spotlight improved interpretation and eliminated any ambiguity regarding the positioning of the point in space, as it allowed the projections on each plane to be clearly highlighted.

The use of 3D printing technology allowed the components to be made in accordance with all project requirements, ensuring the accuracy of the plates, cylinders and spheres. Each geometric element was created with accuracy in both dimensions and shape through this method, which is essential for the correctness and functionality of the final assembly.

A significant advantage of the suggested solution was the flexibility of the cylinders, which allowed them to adjust perfectly to the moment of plate rotation. Basically, this emphasized how the point and its projections are connected to each other in three-dimensional space.

The cylinders allowed for effortless and error-free positioning due to the material properties and the precision with which they were printed. This supports the visual and didactic clarity of the model.

Therefore, the correct transposition of descriptive geometry concepts into an interactive and easy-to-explore physical model was possible thanks to the efficiency of 3D printing, combined with intelligent constructive solutions.

In addition, the model that was realized significantly helped improve the theoretical understanding of the triple orthogonal projection. This has proven to be useful for both educational and technical purposes. Through the physical representation of points, lines, and projections, the perception of spatial relationships was facilitated. Students were also able to directly observe how orthogonal projections relate to each of the three reference planes.

The main objective, the development of a didactic model that is geometrically accurate and useful for teaching and learning, was successfully achieved. A practical and interactive approach to specific concepts of descriptive geometry was possible thanks to the model, which helped clarify some abstract concepts and consolidate theoretical knowledge.

Thus, the use of this model in pedagogical or technical contexts can help to form a thorough and applied understanding of the triple orthogonal projection. This model serves both as a demonstration support and as a tool for exploration and practical experimentation.

4. DISCUSSIONS AND CONCLUSIONS

Using 3D printing to obtain the physical model of the triple orthogonal projection has brought numerous benefits, but also a number of problems. First, the use of flat plates that are hinged at 90° angles provided an accurate representation of projection planes and provided a view of the geometric relationships between point coordinates (x, y, z), (Figure 7). An interactive demonstration, which improved understanding of how orthogonal projections are formed and interconnected, was made possible by the integration of flexible cylinders and a sphere, which were precisely made through 3D printing.

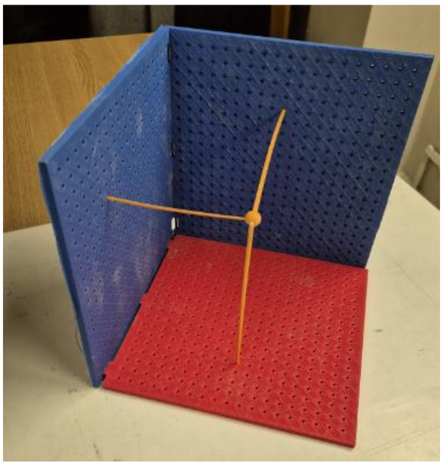

Figure 7 Assembling 3D printed plates, cylinders and spheres on the Creality Ender-3 V3 SE printer

A simple but effective way to emphasize the projections of the point on the three planes was the spotlight. This eliminated any ambiguity in interpreting the position of the point, which made the idea of the draught easier to understand. In addition, when the plates were tilted to illustrate the point draught, it provided a practical picture of how two-dimensional projections are connected in three-dimensional space.

A significant advantage of 3D printing was the high precision in creating system components, which ensured the dimensional compliance necessary to successfully present theoretical ideas. At the same time, the flexibility of the cylinders allowed them to adapt during the rotation of the plates, emphasizing the relationship between projections and spatial coordinates.

Instead, the printing process required fine-tuning of variables such as temperature, speed, and layer height to achieve the best results. In terms of widespread use, the time required to manufacture components through 3D printing is another limitation.

Ultimately, this project successfully proved the effectiveness of 3D printing in creating a model for triple orthogonal point projection that is both functional and educational. The physical model offers a practical and interactive method that facilitates the understanding of complex geometric concepts for students in technical study programs. This study can extend and apply the method to other chapters of descriptive geometry, helping to develop innovative and useful teaching materials. There are also several directions for future research and application that could help both education and the engineering fields:

- implementing the technique in chapters related to descriptive geometry, such as the line or the plane, to create innovative and interactive educational resources;
- examining the impact of different filaments, such as ABS or TPU, on the accuracy and quality of models

created by using other materials for 3D printing instead of the PLA that is currently used;

• promoting 3D printing in education through courses and projects that facilitate the practical understanding of theoretical notions.

• the use of 3D printing in mechanical, civil and architectural engineering for rapid prototyping and practical evaluation of projects.

These directions offer new perspectives on the integration of 3D printing into learning and research processes. They also increase the relevance and effectiveness of the method in a variety of technical and educational contexts.

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