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CONSIDERATIONS REGARDING THE STRUCTURAL AND KINEMATIC ANALYSIS OF A CONICOGRAPH MECHANISM

Abstract: *A conicographic mechanism of I. I. Artobolevski is studied structurally and kinematically (positions). The mechanism is R-RPP-RPR-PRP type. The positions of the mechanism are calculated by the closed-loop method. It is found that the mechanism can draw ellipses, parabolas and hyperbolas. Considerations are made regarding the geometric-kinematic synthesis of the mechanism and the type of the generated curve. Some dimensions of the mechanism were then modified to obtain other types of curves, not conical, curves with several branches, some with closed loops.*

Key words: *conicograph, mechanism, curves.*

1. INTRODUCTION

In mathematics, a conic is a curve obtained as the intersection of the surface of a cone with a plane [1]. The three types of conic section are the hyperbola, the parabola, and the ellipse; the circle is a special case of the ellipse. Alternatively, a conic section, in terms of plane geometry, it is defined as the locus of all points P whose distance to a fixed point F (called the focus) is a constant multiple (called the eccentricity e) of the distance from P to a fixed line L (called the directrix) [2]. For 0 < e < 1 we obtain an ellipse, for e = 1 a parabola, and for e > 1 a hyperbola (Figure 1).

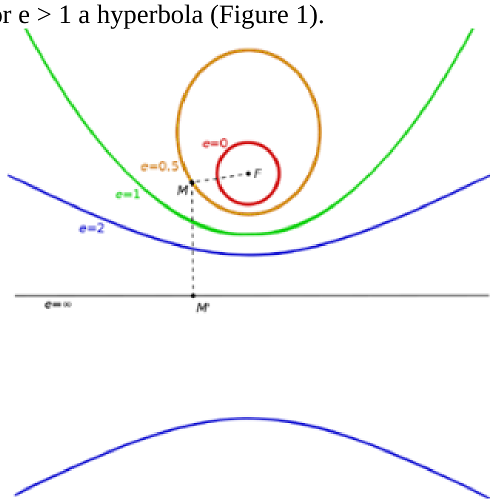

Figure 1 Conic generation with focus and directrix L (e = ∞).

Based on the presented geometric considerations for generating conical curves, mechanisms capable of tracing them have been discovered. Numerous methods of analysis and synthesis of these mechanisms can be found in the history of the Theory of Mechanisms. Acad. Artobolevski created many such curve-generating mechanisms, which he could generate by synthesizing mechanisms based on geometric properties [3], [4]. In [6], [7], [8] some new mechanisms for generating conics are given. Examples of the generation of these curves are presented in [5], [11]. Although the analysis and synthesis of the motion of plane links is a classic problem, which has been studied extensively in the literature, new research shows the concern for the discovery of modern methods, more accurate for the generation of plane and space curves [9], [10]. Next, the curves generated by an Artobolevski mechanism are studied.

2. INITIAL DATA

We study the kinematic possibilities of a conicograph created by Acad. I. I. Artobolevski (Figure 2). By rotating the driving element 1, the point E will draw a conical curve, as follows: ellipse if e = FB / GA <1; hyperbola if e> 1 and parabola if e = 1.

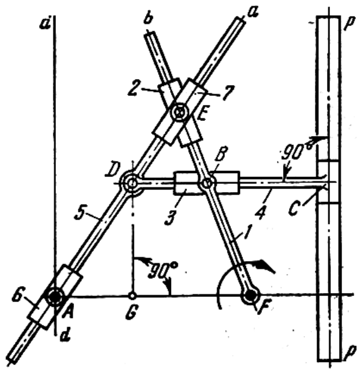

Figure 2 Artobolevski conicograf.

3. THE STRUCTURE OF THE MECHANISM

The kinematic diagram of the mechanism is given in Figure 3.

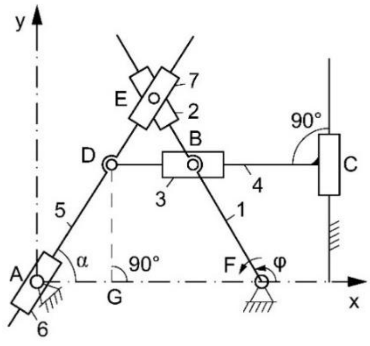

Figure 3 Kinematic diagram of the mechanism.

In Figure 4 the structural diagram and the component parts of the mechanism are presented. The mechanism consists of a driving element with rotational motion, an RPP type dyad, an RPR type dyad and a PRP type dyad.

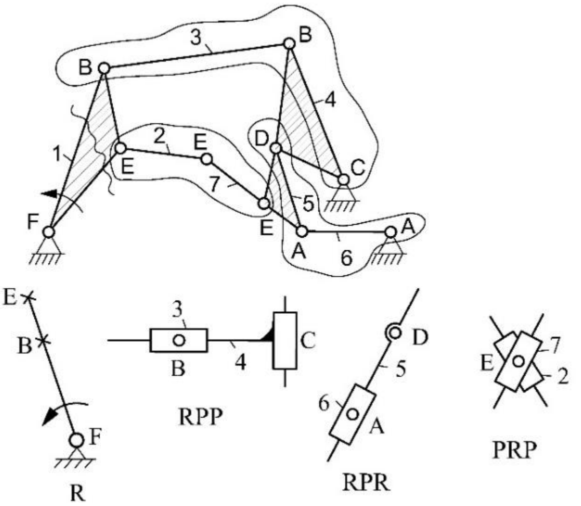

Figure 4 Structural analysis of the mechanism.

4. CALCULATION OF THE MECHANISM POSITIONS

Based on Figure 3, with the closed-loop method, the relationships are written:

$$GF/GA = FB/GA = e \quad (1)$$
$$X_B = X_F + FB.cos\varphi \quad (2)$$
$$Y_B = Y_F + FB.sin\varphi \quad (3)$$
$$X_D = GA \quad (4)$$
$$Y_B = Y_B = Y_C \quad (5)$$
$$X_C = const. \quad (6)$$
$$AD^2 = GA^2 + Y_D^2 \quad (7)$$
$$\cos\alpha = GA/AD \quad (8)$$
$$sin\,\alpha = Y_D\ /\ AD \quad (9)$$
$$X_E = AE.cos\alpha = X_F + FE.cos\varphi \quad (10)$$
$$Y_E = AE.sin\alpha = Y_F + FE.sin\varphi \quad (11)$$
$$FE = (Y_F - X_F.tg\alpha)/(cos\varphi.tg\alpha - sin\varphi) \quad (12)$$
$$AE = (X_F + FE.cos\varphi)/(cos\alpha) \quad (13)$$

4. OBTAINED RESULTS

The following initial values for the dimensions of the mechanism (in millimeters) were considered:
GA = 20; XF = 56; FB = 37; XC = 76.

For e = 1.85 > 1 results the hyperbola in Figure 5.

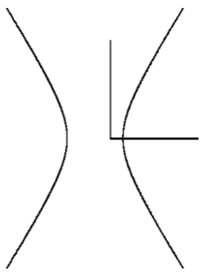

Figure 5. Hyperbola for e=1.85.

The diagram in Figure 6 shows that in the working cycle of the mechanism there are jumps to infinity (the maximum value of XE and YE, respectively at 300 millimeters, was blocked). These jumps are specific to hyperbolas.

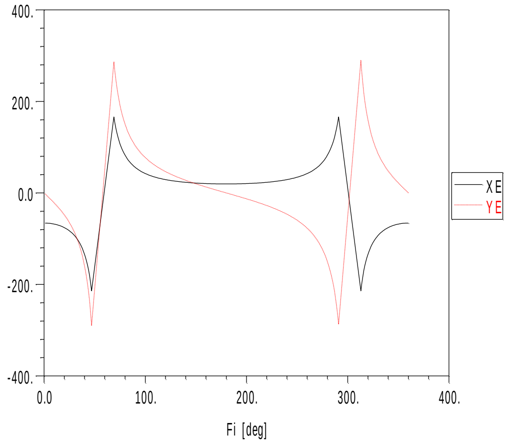

Figure 6 Jumps in the working cycle.

If e = 0.75 <1 results the ellipse in Figure 7.

Figure 7 Ellipse for FB = 15, GA = 20.

In this case the mechanism works throughout the whole cycle, according to the diagram in Figure 8.

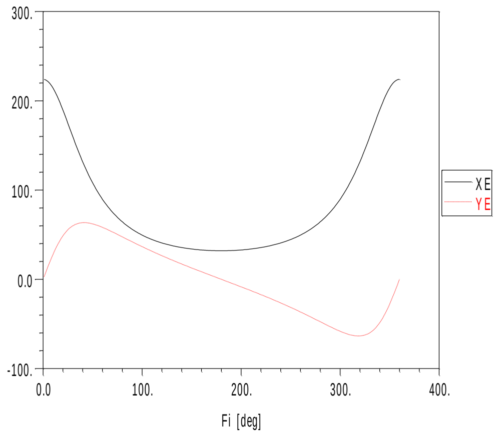

Figure 8 The cycle for the ellipse.

In the case of the parabola e = 1, the Figure 9 results.

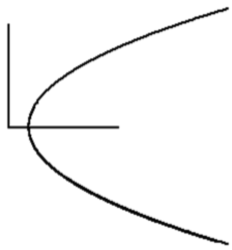

Figure 9 The parabola for FB = 37, GA = 37, e = 1.

The diagram in Figure 10 shows that even in this case there are no jumps with interruption in the working cycle of the mechanism.

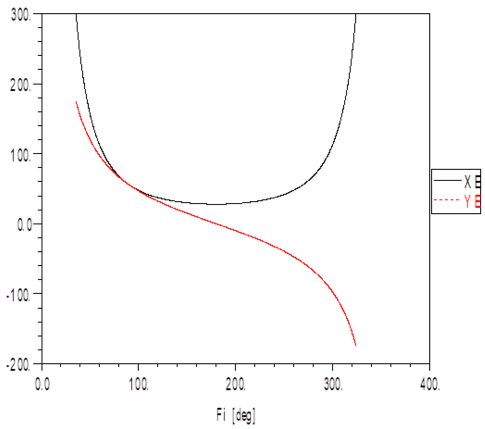

Figure 10 The cycle for the parabola.

All the above proves that the point E can generate conical curves.

5. OTHER KINEMATIC POSSIBILITIES OF THE MECHANISM

In the above relationships, the presence of YF is observed, although in Figure 3, YF = 0. Some curves were determined considering non-zero YF. Subsequently, other dimensions of the mechanism were modified, highlighting the shape of the trajectory of the E point (Figure 11 … 21).

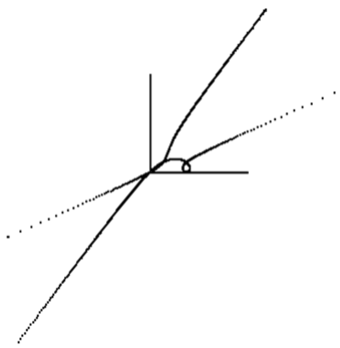

Figure 11 The trajectory of E for: FB=15; GA=20; YF=15; e=0.75.

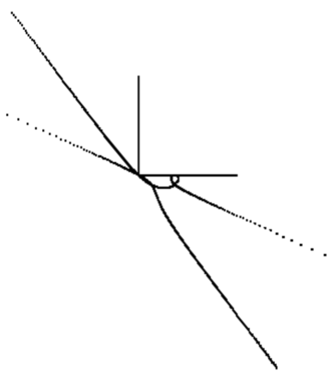

Figure 12 The trajectory of E for: FB=15; GA=20; YF=– 15; e=0.75.

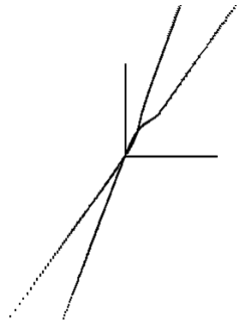

Figure 13 The trajectory of E for: FB=15; GA=20; YF=40; e=0.75.

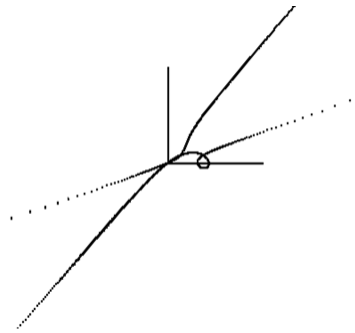

Figure 14 The trajectory of E for: FB=15; GA=20; YF=12; e=0.75.

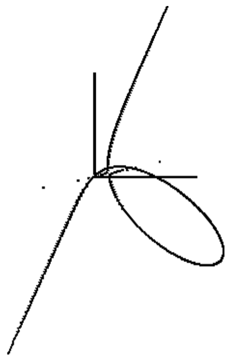

Figure 15 The trajectory of E for: FB=35; GA=20; YF=12; e=1.75.

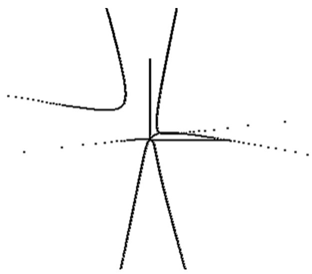

Figure 16 The trajectory of E for: FB=85; GA=20; YF=12; e=4.25.

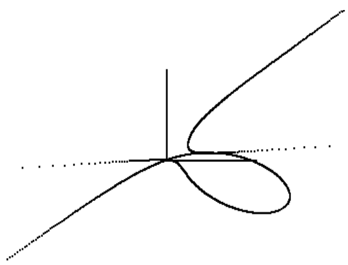

Figure 17 The trajectory of E for: FB=85; GA=85; YF=12; e=1.

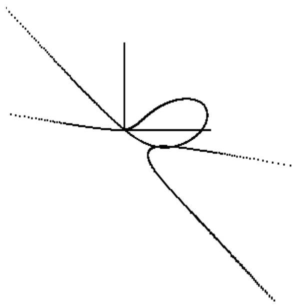

Figure 18 The trajectory of E for: FB=85; GA=85; YF= –30; e=1.

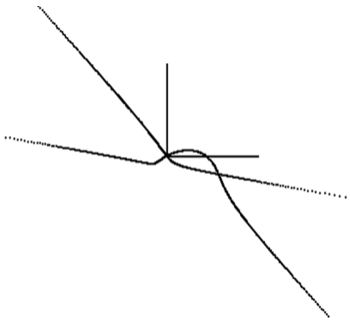

Figure 19 The trajectory of E for: FB=85; GA=85; YF= –30; XF=– 15; e=1.

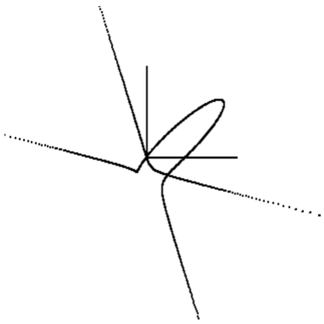

Figure 20 The trajectory of E for: FB=85; GA=35; YF=–30; XF= –15; e=2.428572.

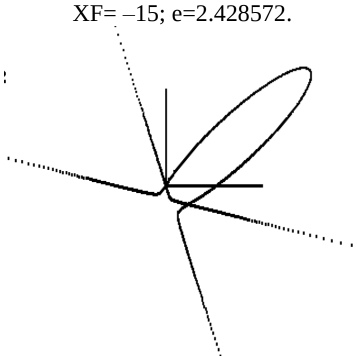

Figure 21 The trajectory of E for: FB=85; GA=35; YF= –30; XF=0; e=2.428572.

From the figures above it is clear that other curves have resulted, with several branches, some with loops, totally different from the conical curves.

6. CONCLUSIONS

A conicograph mechanism designed on the basis of geometry by Acad. Artobolevskii was studied. We made the structural and kinematic analysis based on the closed loop method, strictly necessary to calculate the mechanism, in order to confirm it's capability of tracing the conical curves. A computer program was developed so that the desired curves, the successive positions and also the kinematic diagrams were obtained, for the studied mechanism. It turned out that the mechanism generates ellipses, hyperbolas and parabolas. Then some geometric dimensions of the mechanism were modified, resulting in other different conical curves, with several branches, some even with closed loops. The equations given for the points that draw the mathematical curves, can be used to create programs for the CNC machine tools that can generate these curves.

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