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MODELING THE DOUBLE WORM-FACE GEARS

Abstract: *The worm-face gear family, invented 60 years ago, contains in its structure several variants which have the following defining elements: tapered worm, reverse tapered worm and cylindrical worm. This type of gear can be realized with a single wheel and also in engagement with the second embodiment of the front worm wheels. This paper presents the matrix - vectorial mathematical model of the double worm-face gear with cylindrical worm and a graphical modeling which is based on the specific geometrical characteristics accomplished by means of the Autodesk Inventor 3D modeling program. The applicability of the study, considering the solutions which it suggests, aims to create opportunities for the use of modern rapid prototyping and analysis of stress FEM technique.*

Key words: *double worm-face gear, mathematical model, 3D modelling worm, worm-face wheels*

INTRODUCTION

The worm-face gears is a skew axis gearing consisting of a tapered worm engaging with a tapered worm gear, patented in 1954 [9]. A further embodiment is derived from the gear consisting of a worm cylinder and a worm gear meshing with a flat surface. A constructive solution was patented in 1960, [10] in which a tapered worm engages simultaneously with two front wheels, followed by the variant of the worm gear cylindrical worm wheel engaging with two frontal plane (Fig 1).

The specific geometric and functional characteristics of these gears with crossed axes are: high reduction ratios, high torque capability, the possibility of using the torque hardened steel on hardened steel materials, the possibility of adjusting the clearance between the sides. As practical applications we mention speed reducer construction and kinematic chains of various machinery and equipment.

The particular situation of the double worm face gear implies a great number of teeth in simultaneous contact which, for the same input torque, ensures the diminution of the contact pressure on the flanks of the teeth. Consequently, the gear can be additionally charged compared to the gear which drives the single worm wheel. It should however be noted that the experimental studies which were performed [6] show that an additional amount of heat is produced, which requires additional cooling measures for the gear reducer.

Fig. 1. Double worm-face gear with cylindrical worm

The constructive variant which uses a cylindrical worm as gear actuator can be fitted with a double worm wheel gear monoblok and a worm gear made up of two separate, but simultaneously processed parts. The toothing of the double worm-face wheels is made by hobbing or, in the case of single objects or small series, by means of the fly cutter [1,3].

The object of this paper is the modeling of the double worm-face gear which uses a cylindrical worm. The definition of the worm flank elements of the double worm wheel will be a vectorial / matrix definition, which then allows determining the coordinates of the points that materialize teeth flanks, this process being a mathematical modeling and a graphical modeling using the Autodesk Inventor environment.

2. MATHEMATICAL MODELING OF DOUBLE WORM-FACE GEARS

The specific cylindrical worm for the double worm-face gear can be achieved in various ways: archimedean worm, involute worm or convoluted worm. In all cases, the helical flanks are characterized by the strong asymmetry of the flanks while maintaining constant pitch on both flanks.

Starting from the analysis presented in [2],[5],[7],[8], using the reference systems O_1 X_1 Y_1 Z_1, connected to the worm, O_0, X_0 Y_0 Z_0 reference system linked to the generating curve of the worm, the surfaces of the flanks may be formed by helical movement, defined by the parameter h M_{10} matrix, rectilinear G_0 generating profiles of the sections (one for the left flank and another one for the right flank) defined by expression (1).

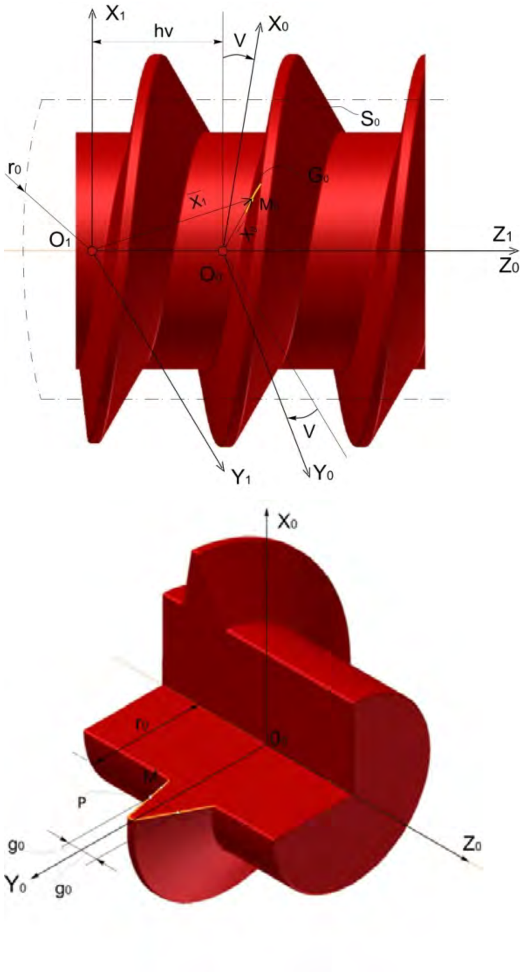

Fig. 2. Generation cylindrical worm

$$M_{10} = \begin{Vmatrix} \cos v & -\sin v & 0 & 0 \\ \sin v & \cos v & 0 & 0 \\ 0 & 0 & 1 & hv \\ 0 & 0 & 0 & 1 \end{Vmatrix} \tag{1}$$

$$\bar{x}_0 = \bar{x}_0 \cdot (p) = \begin{Vmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{Vmatrix} \tag{2}$$

where: p is an independent parameter .

An arbitrary point M_0 of the helical surface S_0 is defined by the relation vector (3)

$$\bar{x}_1 = \begin{Vmatrix} x_0 \cos v - y_0 \sin v \\ x_0 \sin v + y_0 \cos v \\ x_0 + hv \\ 1 \end{Vmatrix} = \begin{Vmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{Vmatrix} = \bar{x}_1(p, v) \tag{3}$$

In order to distinguish the points on the left flank and the ones on the right flank, using the principle in [8], the index "k" is used as notation. It will have the value 1 for the left flank and value 2 for the right flank.

In the case of the double worm-face gear, in order to distinguish between the points that will generate the flanks of the lower and respectively upper ring gear it is necessary to use the additional index j which will have the value 1 for the lower ring and 2 for the upper ring.

Considering the above mentioned issues, the axial profile of the ZA archimedean-type worm will be defined by the expression (4):

$$\bar{X}_{okj} = \begin{Vmatrix} 0 \\ r_0 + p_{kj} \cos a_k \\ (g_0 + p_{kj} \sin a_k)(2k-3) \\ 1 \end{Vmatrix} = \begin{Vmatrix} x_{okj} \\ y_{okj} \\ z_{okj} \\ 1 \end{Vmatrix} = \bar{X}_{okj}(p_{kj}) \tag{4}$$

The expressions of the worm flanks in the system O_1 X_1 Y_1 Z_1 will be:

$$\bar{X}_{1kj} = \begin{Vmatrix} -(r_0 + p_{kj} \cos a_k) \sin v \\ (r_0 + p_{kj} \cos a_k) \cos v \\ (g_0 + p_{kj} \sin a_k)(2k-3) + hv \\ 1 \end{Vmatrix} = \begin{Vmatrix} x_{1kj} \\ y_{1kj} \\ z_{1kj} \\ 1 \end{Vmatrix} = \bar{X}_{1kj}(p_{kj} v) \tag{5}$$

The normal in helical flanks of the same reference systems will be defined by the equation (6):

$$\bar{N}_{1kj} = \begin{Vmatrix} -(2k-3) \sin a_k \sin v - H_{kj} \cos a_k \cos v \\ (2k-3) \sin a_k \cos v - H_{kj} \cos a_k \sin v \\ -\cos a_k \\ 0 \end{Vmatrix} = \begin{Vmatrix} n_{1xkj} \\ n_{1ykj} \\ n_{1zkj} \\ 0 \end{Vmatrix} = \bar{N}_{1kj}(p_{kj} v) \tag{6}$$

The value H_{kj} (7) from the normal equation is illustrated by the ratio

$$H_{kj} = \frac{h}{r_0 + p_{kj} \cos a_k} \tag{7}$$

The vector relations (5) and (6) define the points which form the helical flanks of the cylindrical worm for the double worm-face gear.

Similarly with the kinematic generation modality of the simple worm face-gear, the functional double worm-face gear can be replaced , in terms of kinematics, with a gear consisting of a rack and a specific worm gear for the lower ring and another rack in a symmetrical position for the upper ring gear.

Figure 3 shows schematically the relative position of the worm-face gear components for the proposed kinematic generation system.

The meanings of the notations used are: O_F X_F Y_F Z_F, fixed reference system; O_1 X_1 Y_1 Z_1 worm-linked system, O_2 X_2 Y_2 Z_2 system linked to the double worm wheel; z_1, z_2 the number of beginnings for the worm and the number of teeth on the two ring gears; W_1, W_2 angular speeds of the worm and wheel, indices k and j for the lower and upper rings.

The movements of the elements are: the translation movement of the worm with speed hW_1, the rotation movement of the wheel with angular speed W_2.

The relative position of the schemes related to the worm and the double worm wheel is determined by the parameters u_1 and u_2

$$i_{12} = \frac{W_1}{W_2} = \frac{Z_2}{Z_1} = \frac{u_1}{u_2}$$

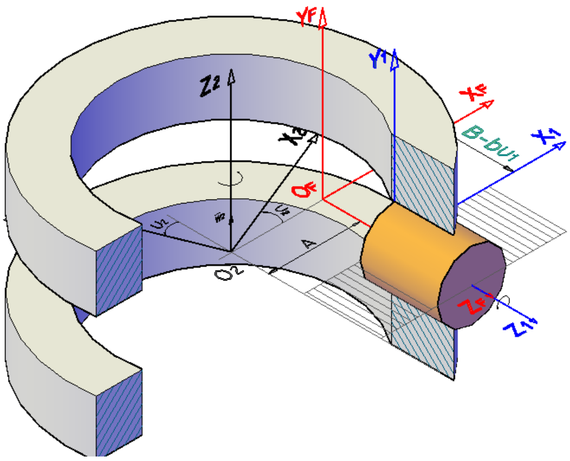

Fig. 3. The relative position of the worm and double worm wheel

The tooth flanks of the two worm gear rings will be expressed by means of the radius vector $\overline{X}_{2k}$ (8)

$$\overline{X}_{2kj} = M_{21} \cdot \overline{X}_{1kj} \qquad (8)$$

$$\overline{X}_{2kj} = \left\| \begin{matrix} cosu_2 \cdot x_{1kj} - sinu_2 \cdot z_{1kj} - (B - hu_1) sinu_2 + Acosu_2 \\ -sinu_2 \cdot x_{1kj} - cosu_2 \cdot z_{1kj} - (B - hu_1) cosu_2 + Asinu_2 \\ y_{1kj} \\ 1 \end{matrix} \right\| = \qquad (9)$$

$$= \left\| \begin{matrix} x_{2kj} \\ y_{2kj} \\ z_{2kj} \\ 1 \end{matrix} \right\| = \overline{X}_{2kj}(p,v,u_2)$$

The expression of the normal vector to the tooth flank is

$$\overline{N}_{2kj} = \left\| \begin{matrix} n_{1xkj}\, cosu_2 - n_{1zkj}\, sinu_2 \\ -n_{1xkj}\, sinu_2 - n_{1zkj}\, cosu_2 \\ n_{1ykj} \\ 0 \end{matrix} \right\| = \left\| \begin{matrix} n_{2xkj} \\ n_{2ykj} \\ n_{2zkj} \\ 0 \end{matrix} \right\| = \overline{N}_{2kj}(p,v) \quad (10)$$

The vector relations (9) and (10) define the points which form the double worm wheel tooth flanks.

3. A GRAPHICAL MODELING OF THE DOUBLE WORM FACE GEAR

To model a double worm face gear with a cylindrical worm, we considered the concrete case having the following values as geometrical characteristics: axial module m_a = 2.5, number of worm beginning z_1 = 1, z_2 =47 number of teeth wheel, the axial distance A = 56mm worm reference radius R_0=18,24 mm, flank angles a_1 and a_2 = 10^0 and 30^0. (1) (2.9)

The modeling of the worm gear which is shown below is based on the drawings of the two components of the gear , the worm and the double worm wheel [4]. Due to its specific features, the Autodesk Inventor 3D modeling software enables modeling the gear and its components.

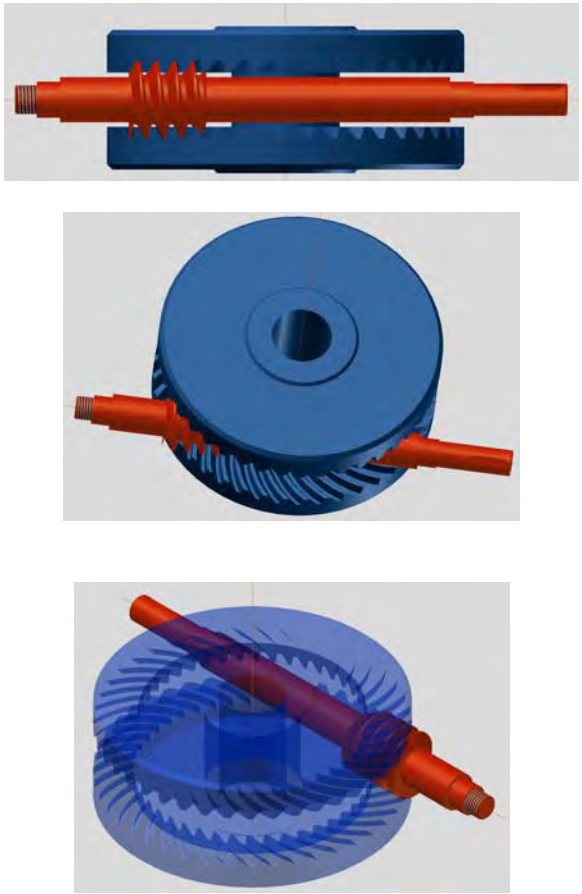

Fig. 4. Double worm-face gears 3D graphic model

The first element which is generated is the worm, which is the defining element of the gear. To generate worm teeth with a predefined generating curve (profile of the generator worm flank) and guiding curve (cylindrical propeller pitch) modeling is done in the " Part " module using the command **Sweep**, which automatically generates the worm profile surface. Once the worm teeth portion generated, the other parts of the worm are constructed.

The worm wheel is modeled as a semi-finished product up to toothing, in the module "Part". To achieve the profile of the double worm wheel tooth flank, the guiding curve of the worm propeller is printed both on the lower part of the teeth and on the upper part of the wheel. Once the guiding curve defined, it creates a profile almost identical to the profile of the worm flanks, while observing the geometry of the worm. Using the SWEEP command, a "gap" between the two consecutive flanks is obtained. This "gap" is multiplied by the given number of teeth, having the geometrical axis of the blank as an axis of rotation, thus resulting in lower teeth on the worm wheel. For the upper part, the procedure is similar. It can be seen that the upper teeth is not a picture in the "mirror" compared to the lower teeth, being slightly rotated by one half of the worm pitch.

The 3D model of the worm-double worm wheel assembly is done in module "Assembly", observing the proper positioning of the worm.

4. CONCLUSIONS

The functional double worm face gear, which is formed between the worm and the double - matrix worm wheel can be mathematically modeled as matrix / vector. This model can provide the coordinates of the points which form the flanks of the worm and of the double worm wheel.

The resulted matrix / vectorial mathematical model developed matrix - vector can be used to obtain worms and double worm wheels by using modeling and prototyping 3D technologies. It can also be used in FEM analysis.

The 3D Autodesk Inventor modeling program allows the adequate modeling of the double worm face gear based on the drawings of the worm and double worm wheel.

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