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SPECIFICS OF THE ORTHOGONAL PROJECTION OF INCLINED CONICAL SURFACES AND A STUDY OF THEIR SHAPE AND SIZE

Abstract: *In descriptive geometry classes students are taught the basic rules of the graphic representation of geometric bodies on a plane according to Monge's method, referred to as orthogonal projection. The main geometrical elements of space are projected - points, straight lines, planes and bodies –angular and rotating, as well as their relative position. This knowledge is subsequently applied in engineering graphics. The bodies that are projected are generally in a particular case and regular ones, i.e. their altitudes are perpendicular to their bases.*

This paper focuses on inclined round conical surfaces, also known as rotating bodies. But are all cones rotating ones, although they are presented as such in high school studies?

This paper is a study of the shape of the different types of conical bodies, as particular consideration is given to the inclined ones and the way they are projected according to the Monge's method.

Keywords: *orthogonal projection, descriptive geometry, inclined conical surface.*

INTRODUCTION

In the first year of their studies, along with the other general engineering disciplines, prospective engineers study Applied Geometry and Engineering Graphics. The aim of the training is to learn the principles and methods of orthogonal projection. The first step towards this is applied descriptive geometry. The good knowledge of this kind of geometry enables even individuals with a total lack of visual graphic and spatial thinking and without any past experience to learn to design.

The projection of the main types of bodies, such as cones, cylinders, pyramids and prisms, as well as their intersection is studied. These bodies are united into two main groups-angular and rotating ones and their intersections.

This paper examines the shape of the different types of conical bodies, as particular consideration is given to the inclined ones.

Simulations of programs for three-dimensional object modeling are introduced in order to give a full and complete vision of the size and shape of the projected conical surfaces.

2. THEORETICAL RESEARCH

In the course of their high school studies in geometry and stereometry, pupils solve problems associated with conical and cylindrical surfaces, and the right circular cones and cylinders are the ones referred to most often. In fact, it is these shapes that are the most common in technical practice. They are said to be rotating surfaces, and the bodies they determine ï rotating bodies.

Let us examine the definitions of a conical surface and a cone proposed in the reference sources. The most common ones read as follows:

1. The right circular conical surface is a rotating one, because it may be generated by rotating a straight line around an axis which intersects it. The right circular cone is referred to as a rotating one [1].

2. A body bounded by a conical surface and the plane of its directing circle is referred to as a circular cone. It is referred to as an inclined cone when its surface is inclined and a right one if its surface is right. The right circular cone is also referred to as a rotating one [2].

There are also some more thorough and extensive definitions, such as:

3. A conical surface (a cone) is generated from the straight lines passing through one common point and intersecting a given curve that is not incident with their common point. Their common point is referred to as an apex, the given curve - as a directrix, and the straight lines generating the surface ï generatrices of the conical surface.

The directing curve may be a spatial or a plane one.

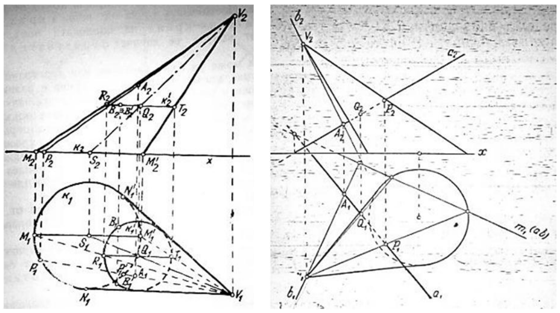
Generally, we limit ourselves to curves of second degree - a circle, an ellipse or generated by the intersection of the conical surface ï a hyperbola or a parabola. In this case, the conical surface is also of second degree.

For simplicity of illustration, it is assumed that the directing curve is a circle. This surface is also referred to as a circular conical surface. A circular conical surface is referred to as a right one if the perpendicular **that is dropped from the apex of the conical surface to the plane of the directing curve passes through the centre of the latter**. Otherwise, it is an inclined one [1], [2].

In this sense, a special case is a directing curve with the shape of a polygon - in such case the obtained shape is a pyramid. Undoubtedly, this body is neither a round one, nor a rotating one.

It is obvious that only the simplest definitions and the ones that are not extensive enough read that cones are rotating bodies. Why, in such case, everybody is so firmly convinced of the view that, as far as cylinders and cones are concerned, the reference is to rotating bodies?

Indeed, when it comes to solving problems about cylindrical and conical surfaces in descriptive geometry classes, mainly right circular cones and cylinders are considered, for which the definition that they are rotating ones is completely applicable. The problems of the inclined conical and cylindrical surfaces are somehow set aside, and the few such mathematical problems in descriptive geometry textbooks are mainly associated with inclined **circular surfaces**. Such examples are illustrated in Figure 1a.



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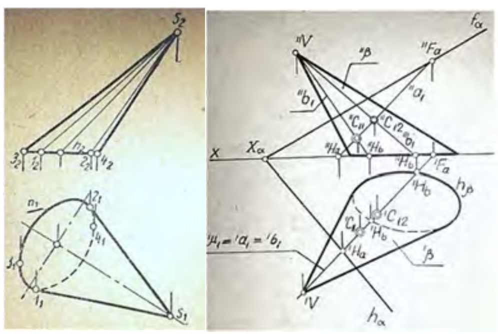


b)

Fig. 1 Inclined cones [1], [2], [3], [4].

An interesting construction is proposed in a small number of descriptive geometry textbooks. Such an example is illustrated in Figure 1b. Unfortunately, there is not any clear indication of the exact type of the cone ï a circular, a rotating one or a conoid. It should be noted that the base is elliptical, but there is not any explanation on how it was chosen and why it has such an orientation.

3. A SolidWorks SIMULATION AND A COMMENTARY

This paper is focused on studying inclined conical surfaces. Is it possible that they are referred to as rotating ones if (for a better convenience) their directing curve is a circle? What types of sections are generated on their intersection with a plane parallel to their base? What types of sections are generated on their intersection with a plane perpendicular to their axis (i.e. what is their cross-section)? What is their size and shape in an orthogonal projection?

Figure 2 illustrates a visualization of a right cone and an inclined cone, with a directing curve that is a circle, as well as their sections with a plane parallel to their base (the latter lies in the horizontal projection plane). It is obvious that the sections obtained are again circles, according to the properties of homothety and stereometry.

There are differences when comparing an inclined

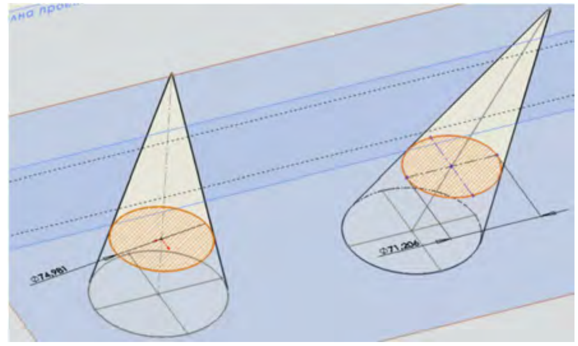


Fig. 2 A simulation of a right cone and an inclined cone with a directing curve - a circle. An intersection with a plane parallel to their base.

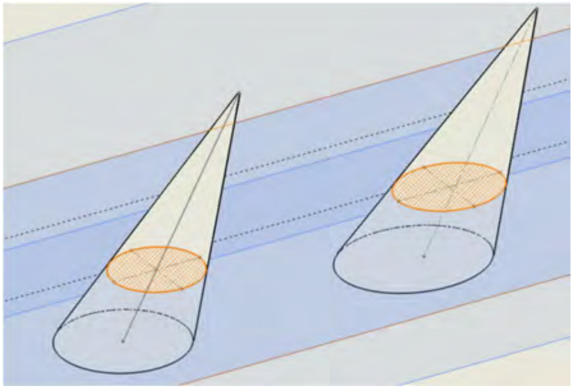
circular and rotating cone. Such visualizations are illustrated in Figure 3. Viewed without their dimensions determined, the surfaces look almost alike (Figure 3Ѳ). Nevertheless, there is some difference and it consist in the fact that the base of the inclined rotating cone lying in the horizontal projection plane is elliptical ï Figure 3b. It stands to reason that this is due to its surface having been obtained by rotating a straight line around a rotation axis, and it is well-known that the section between a right rotating cone and a plane forming an angle different from 0^0 and 90^0 with its axis is an ellipse. Therefore, unlike the section of an inclined circular cone with a plane parallel to its base, where the section is a circle, in the case of the inclined rotating cone, an elliptical section is obtained (Figure 3b). Again, this is in accordance with the properties of homothety and stereometry.

An interesting distinctive feature is also observed after performing the rotation by the three-dimensional modeling programe StslidWorks, in order to obtain an inclined conical surface. The same parameters for a radius and obliquity of the body were used in the rotation, as in the building of the inclined circular cone (this is visible in Figure 3fη). The obtained elliptic directing curve has a minor axis with the size of the diameter of the circular cone and a major axis generated by construction. A distinctive feature is that after the rotation, the centre of the obtained elliptical base does not coincide with the centre of the original directing circle any longer. **The centre of the elliptical base is shifted in the direction of the longest generatrix and, at the same time, in the opposite direction to the obliquity of the body.**

It is obvious that, the larger the angle of the obliquity, the larger will be the shifting of the centre of the elliptic base. **Correspondingly, after shifting the centre at the base, the axis of the whole body shifts**, and this is also visible from the section parallel to the base, again in Figure 3fη

In orthogonal proiection, apart from knowing the true size and shape of the conical surface, it is important to know the cross-section obtained.

Figure 4Ѳ illustrates a visualization of the section of an inclined circular cone with a plane perpendicular to its axis. The obtained section is an elliptic one. The size in



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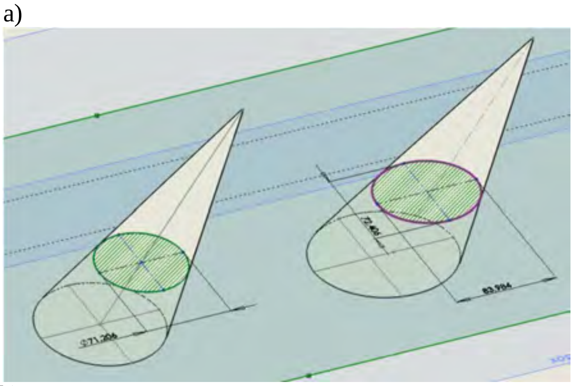

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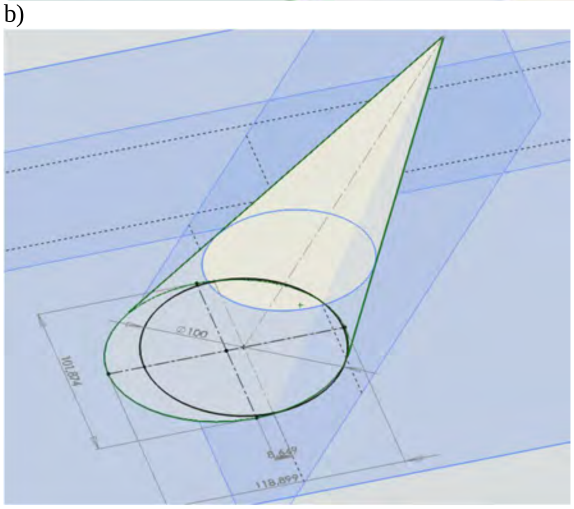


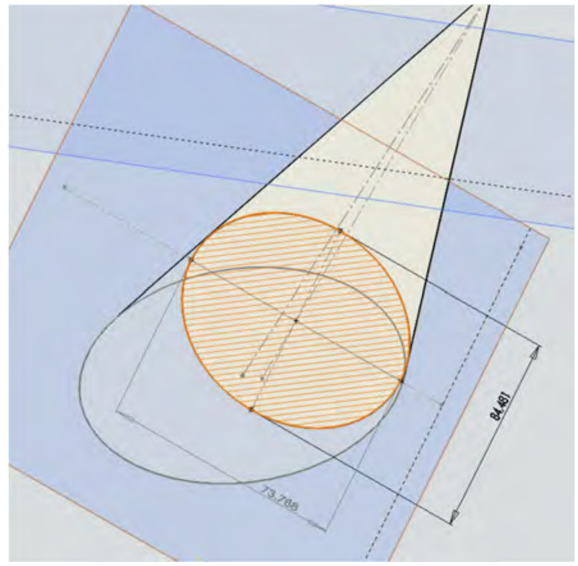
c)

Fig. 3 A SolidWorks simulation of an inclined circular cone and an inclined rotating cone. A section with a plane parallel to their base.

the visualization clearly indicates that the reduction in the size of the minor axis of the elliptic section is in the direction of the obliquity. The larger the angle of the obliquity, the shorter becomes the minor axis of the ellipse.

Therefore, when referring to **an inclined circular cone,** the axis of the cone may not be referred to as an “axis of rotation”.

If we examine a section of an inclined rotating cone with a plane perpendicular to its axis, we shall notice that it is a circle, as it is illustrated in Figure 4b and Figure 5b.



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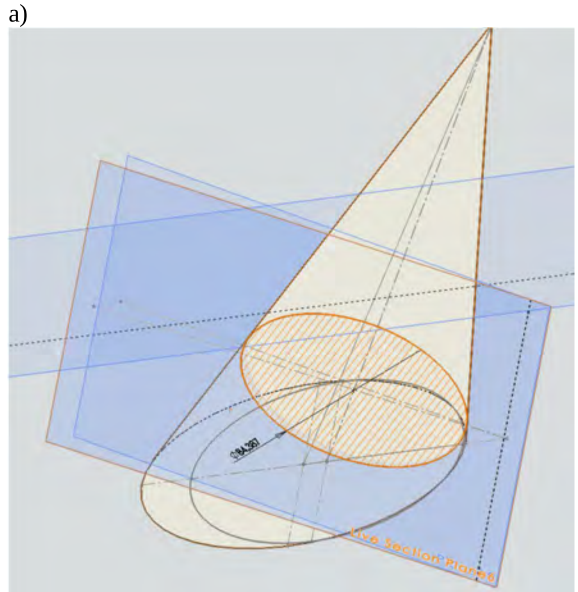
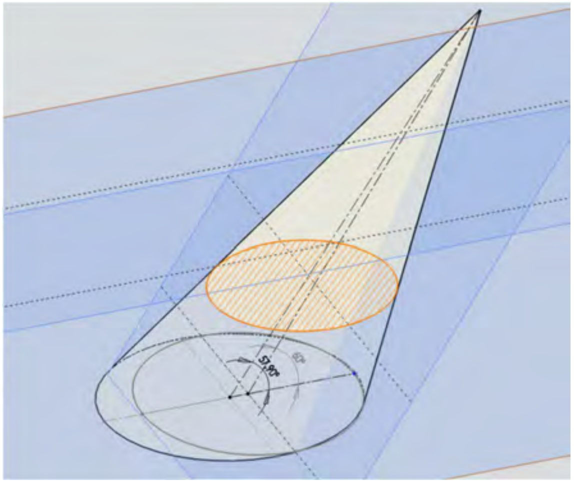


b)

Fig. 4 A section of an inclined circular (a) and rotating (b) cone with a plane perpendicular to their axes.

As result of the axis shifting of the rotating cone, the angle between the axis of the body and the plane in which its base lies changes. In the example simulated in Figure.4b, the angle decreases from 60^0 to 57^0. This is also illustrated in Figure 5a. It should be mentioned that, in the case of a section perpendicular to the axis of the body, the axis of rotation is contains a center of the circular section (Figure 4b). In such case, it may be said that in the inclined rotating cones there are also two axes - an axis of rotation and an axis of the body. The latter is the one that is dropped from the apex of the body to the centre of the elliptical base (by definition). This axis may not be referred to as a rotational one.

Such changes are well visible in the SolidWorks visualization, but may also be seen quite well when solving the problem by the descriptive geometry methods.



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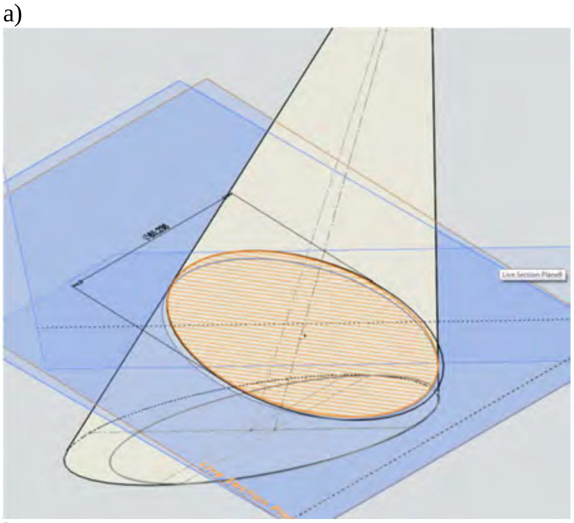

b)

Fig. 5 Shifting of the axis of rotation (Θ). A section of an inclined rotating cone with planes perpendicular to its axes (b).

4. A STUDY AND COMPARISON OF AN INCLINED CIRCULAR AND A ROTATING CONICAL SURFACE BY DESCRIPTIVE GEOMETRY METHODS

4.1. Specific features of inclined circular cones

Figure 6Θ illustrates the intersecting of an inclined circular cone with a plane forming an angle of 90^0 with the axis dropped from the apex of the cone to the centre of the base.

After intersecting the cone, the true size of the cross-section is obtained by the method of rotating. The figure obtained in the drawing shows, that the section is an ellipsis, and not a circle. Therefore, this solution confirms the conclusion made beforehand, that **inclined circular cones are not rotating ones**, unlike right cones. This was also proven by the visualization in Figure 4Θ What is more, in the plane of the section, the centre of the ellipse shifts and does not coincide with the axis of the body any longer.

Figure 7 illustrates how the size of a circular cone progressively decreases (in the one direction) ï from 63 units in the case of a right circular cone, through 55 for a cone forming an angle of 60^0 with its own base, to 33 units for an inclined cone forming an angle of 30^0 with the plane in which lies its own base. At the same time, **the axis of the inclined circular cone shifts towards the inclined.** The angle of the section formed with the axis of the body is still 90^0, and the angle of the axis passing through the centre of the elliptic section formed with the secant plane is constant in value, in this case ï 92^0.

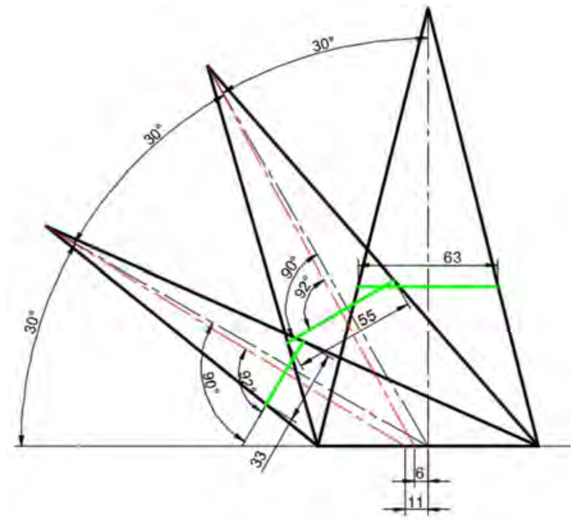

Fig. 7 Shifting of the axis of the body of an inclined circular cone and the change in the size of the cone.

4.2. Specific features of an inclined rotating cone

Figure 6b illustrates the intersection of an inclined rotating cone with a plane perpendicular to its axis. Again, after applying a method of rotating, the true size of the section with the plane is obtained, and the section is a circle. Again, it should be noted that the axis of the body shifts, but certainly this does not apply to the axis of rotation passing exactly through the centre of the section - a circle. Therefore, in the section the centre of the circle coincides with the axis of rotation, but it does not coincide with the axis of the body. This means that the situation is exactly opposite to that in the case of an inclined circular cone.

Figure 8 shows what changes occur when tilted a rotating cone at 60^0 and 30^0 toward the projection plane in which lies its base. We make the clarification that the section is situated at a constant distance from the apex of the cone. Changes to the size of the section are not observed, it remains the same for the three obliquities of the body that are studied. There is an extreme shifting of the axis of the body ï from 8 units at an obliquity of 60^0 toward the base, to 52 units at an obliquity of 30^0. The angle between the axis of the body and secant plane changes (decreases progressively) and is not constant, as it is in the case of the inclined circular cone.

The base of the strongly inclined rotating cone changes drastically from a circle into an ellipse with a

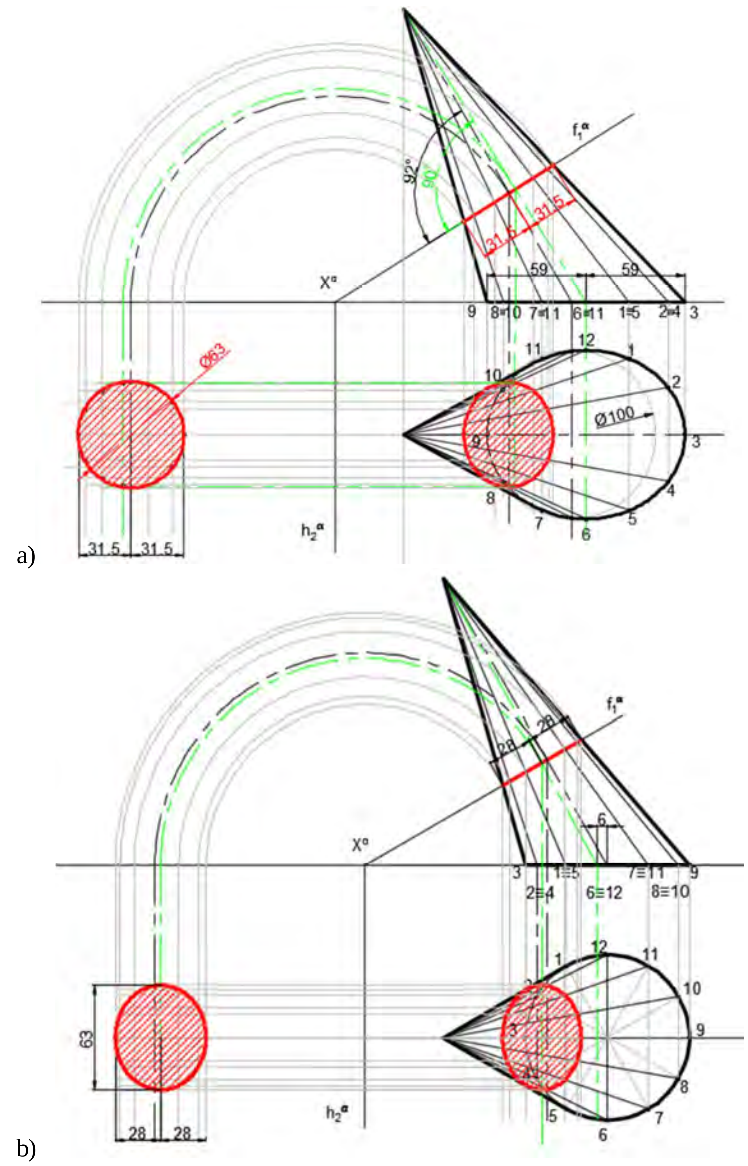

Fig. 6 A section of an inclined circular (a) and inclined rotating (b) cone with a plane perpendicular to their axes.

very big difference in the two axes. The major axis is almost two and a half times larger than the minor one.

4.3. Application in engineering graphics

In engineering graphics it is important what kind of changes are made to the conical surface of the inclined rotating and circular cones in the orthogonal projection on the main projection planes.

It has already been clarified that the orthogonal projection of the right circular and the inclined rotating cone with bases lying in the horizontal projection plane, are projected in the frontal one without changing the diameter of the bodies. The same does not apply to the inclined circular cone that is projected in the frontal projection plane with a reduced distance between the end generatrices in the zone of the section.The latter is again at a constant distance from the top of the cone.

For the sake of comparison, Table 1 shows the numerical values of the size of the studied cones depending on the change in obliquity.

Table 1

Numerical values of the size of the studied cones depending on the change in obliquity

Type of cone	90^0	60^0	30^0
A right cone (diameter)	63	-	-
An inclined circular cone (minor axis)	63	55	33
An inclined rotating cone (diameter)	63	63	63

Table 2 contains the results for the Volume and Surface area, again for the three types of studied cones, as it is specified that the measurement is done at obliquity of the bodies of 60^0. It may be seen that the inclined rotating cone has a larger Volume and Surface area.

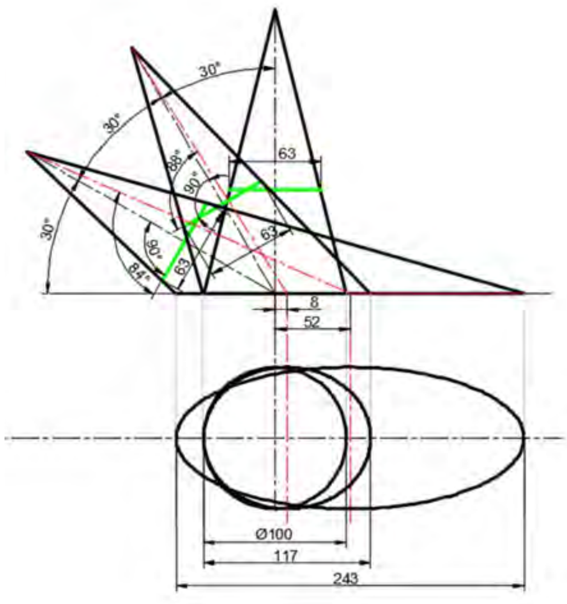

Fig. 8 Axis shifting in an inclined rotating cone

Table 2

Volume and Surface area results for the three types of studied cones

Type of cone	**Volume [mm^3]**	**Surface area [mm^2]**
A straight cone	1519025.26	121230.99
An inclined circular cone 60^0	455034.80	38212.74
An inclined rotating cone 60^0	540391.69	42781.47

5. CONCLUSIONS

On the grounds of the study conducted in this paper, the following conclusions may be drawn:

- There should be a clear distinction between inclined rotating and inclined circular cones.

Conclusions about inclined circular cones:

- **Inclined circular cones are not rotating ones**, unlike right cones, and their cross-section perpendicular to the axis is elliptic.
- The centre of the cross-section is shifted and does not coincide with **the axis of the body.**
- The centre at the base of the inclined circular cone is stationary. The axis of the body passing through the centre of the elliptic section shifts. The direction of shifting is toward the shorter generatrix and, at the same time, toward the obliquity of the body.
- The reduction in the size of the minor axis of the elliptic section is in the direction of the obliquity of the body. The larger the obliquity, the shorter is the minor axis of the elliptic cross-section.

Conclusions for inclined rotating cones:

- The cross-section of an inclined rotating cone is a circle. **The centre of the section coincides with the axis of rotation,** but does not coincide with the axis of the body.
- The centre of the elliptic base of the inclined rotating cone shifts toward the longest generatrix and at the same time in the opposite direction to the obliquity of the body.
- Obviously, the larger the angle of the obliquity, the larger is the shifting of the centre of the elliptical base. Respectively, after its shifting, the axis of the whole body shifts.
- In the orthogonal projection of an inclined rotating cone the base lying in some of the projection planes is projected with an elliptical shape. The minor axis of the ellipse is projected with the size of the true size of the diameter of the cone, and the major axis (always perpendicular to the minor one) is projected viewed from above (in this case in the horizontal projection plane), always in the direction of the axis of the body and in a projection relation with the projection in the adjacent projection plane.
- In the orthogonal projecting of inclined conical surfaces, in any case it should be considered that when projecting inclined **circular conical** bodies is concerned, their section is **elliptical**, and the conical surface **may not** be referred to as a rotating one in the sense of obtained as result of rotation of a generatrix around the axis of the body. When inclined **rotating conical** bodies are projected, their cross-section is a circle, and the bases lying in some of the main projection planes are projected as ellipses.

These conclusions are very important for enriching the prospective engineersô knowledge and introducing them to the specific features of the geometric bodies, from which the real parts are made of.

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