Zoya TSONEVA

INTERSECTION OF INCLINED CONICAL PLANES. APPLICATION OF TRADITIONAL METHODS

Abstract: *One of the goals of teaching applied descriptive geometry to the students of Mechanical Engineering is the acquisition of knowledge and formation of skills in orthogonal projection of basic volumetric primitives and their intersection. The developed elements can be connected in various combinations whose diversity gives infinite opportunities predefined by the specific requirements of design and technology.*

Good knowledge of the properties and actual forms of different types of the geometrical bodies projected is the guarantee for their correct projecting.

The paper presents the specifics of orthogonal projecting of inclined conical bodies and focuses on the relevance of traditional methods for finding the line of bodies' intersection in space.

Key words: *descriptive geometry, orthogonal projecting, intersection the conical planes.*

INTRODUCTION

Descriptive geometry features a wide variety of tasks given to the students of Mechanical Engineering, the aim being the acquisition of knowledge and skills in the subject of projecting intersecting forms.

Cylindrical and conical volumetric primitives intersecting in various ways are one of the most common forms in engineering practice and the purpose is to locate them in an isolated position in drawings. The cases in which intersecting cylindrical and conical forms should be drawn appear to be a great challenge in engineering drawing as these elements are also most intricate to project.

Diverse software exists in contemporary graphics practice offering a rapid solution to the tasks mentioned above, and if there is a need for developments of the adjacent plane, they are generated automatically without designers being aware of how this has happened.

As a matter of fact, this type of geometry is rather intricate for most learners but over-simplification of these concepts would lead to lack of profound knowledge in the fundamentals of orthogonal drawing.

Not being aware of the basics of any scientific field and working only in the area of superficial knowledge is not a good practice in any applied scientific field.

The paper presents the intersection of inclined conical planes, their properties and the relevance of traditional methods used for finding the space curves of intersection between bodies.

2. INTERSECTION OF INCLINED CONICAL PLANES BY THE METHOD OF AUXILIARY CONCENTRIC SPHERES

When looking at the topic of intersecting cylindrical and conical bodies, studentsø textbooks on descriptive geometry make use of elaborate methods such as õThe Method of **Auxiliary Planes though a Straight Line**ö or õThe Method of **a Family of Parallel Planes**ö. These methods may appear repulsive to the learners of descriptive geometry not because of their complexity but

because of the great number of lines which often causes confusion.

One of the easiest and quickest methods used in solving this kind of tasks is õThe Method of **Auxiliary Concentric Spheres**ö. The main peculiarity of the latter is that it is applied only for *rotating bodies with intersecting axes* which is the commonest case in practice. In this case spheres, with their center being the point of intersection of the bodies, act as auxiliary planes. One of the auxiliary spheres intersects each rotating plane in *parallel circumferences* since their centre lies on the axes of both. These circumferences intersect in *points on the line of the intersection* of the rotating planes. They lie on *planes perpendicular to the axes* and hence perpendicular to the plane defined by the axes themselves. In this way, their *projections on the plane of the axes or on a plane parallel to the latter are intercepts and their intersecting points are directly defined*. For this reason, this method is *applied when the plane of the axes is parallel to any of the projection planes*. When the planes of the axes are in general position, it is necessary for them to be rendered into an isolated case by introducing an auxiliary projection plane [1], [2], [3], [4].

Figure 1 illustrates the example most commonly given in the textbooks on descriptive geometry. Here the projection plane is parallel to the plane defined by the intersecting axes of the two cones. The solution to the task is easy and quick to draw.

Let us look at the solution of a task in which two inclined cones intersect. The requirement for the intersecting axes to be parallel to one of the projection planes is met. For greater convenience, the bases of the cones are circumferential. As can be seen, each of the auxiliary spheres intersects the conical surfaces in parallel circumferences whose centres lie on the intersecting axes. The parallel circumferences lie on planes perpendicular to the axis of each of the bodies and therefore they are perpendicular to the plane defined by the axes (fig. 2a).

It turns out that the axes of the inclined circular cones are translated in such a way that the axis which as a rule should be a bisectrix of the projected conical surface
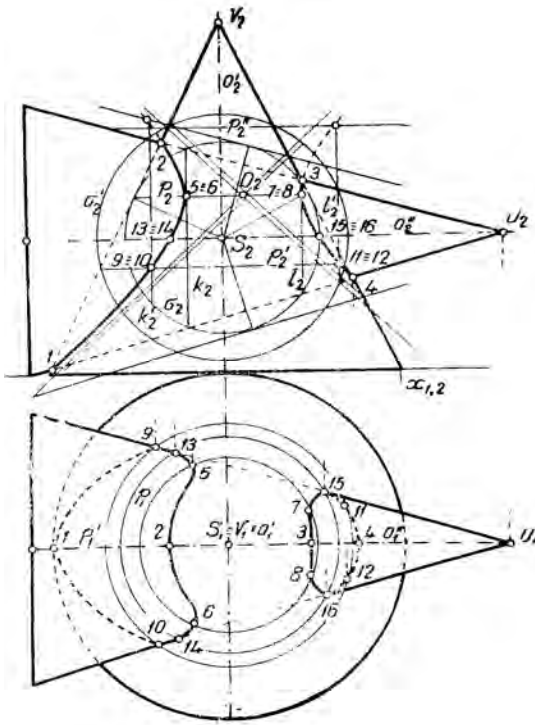
Fig. 1 Intersection of cones. Method of **Auxiliary Concentric Spheres** [1], [2].

does not coincide with the axis drawn from the vertex of the cone to the centre of the base (by definition). For this reason, in order to make it possible to apply the method of the intersecting axes, we have chosen the translated axes (see fig. 2a, point S) as the initial ones are excentrically positioned (i.e. they are not bisectrices to the vertices of the cones). Therefore, in the case of inclined circular cones the axes change their position.

Fig. 2 shows the measurement of the angles resulting from the intersection of the parallel circumferences and the two types of axes. As can be seen, the angle between the axis passing from the vertex of the body to the centre of its base on one hand, and the plane of the parallel circumferences on the other hand is 88^0 (fig. 2b in black), and the angle between the axis appearing to be the bisectrix to the vertex of the cone and the circumference is 90^0 (fig.2b in green).

After obtaining this property of inclined conical cones as a result, we need to answer the following question: Is the solution to the task above accurate as the axes by definition cannot be used?

To find the answer to this question we can use the other well-known methods and compare the results afterwards.

3. INTERSECTION OF INCLINED CONICAL SURFACES BY THE "METHOD OF A FAMILY OF PARALLEL PLANES"

The same task was solved once again this time using the õMethod of a **Family of Parallel Planes**ö (fig. 3). For greater clarity, each of the planes is presented in a

different colour. According to the principles of homothety, the section of each of the inclined cones with each of the intersecting planes parallel to their bases is a circumference. he points of intersection of the obtained pairs of circumferences from each intersecting plane determine the spatial line of intersection between the bodies presented in the adjacent projection. Obviously, the solution obtained by this method is different from the one obtained by the õMethod of Auxiliary Concentric Spheresö. In fig. 3 the solution by the õMethod of a **Family of Parallel Planes**ö is presented in black and the one by the õMethod of Auxiliary Concentric Spheresö is in grey [5], [6].
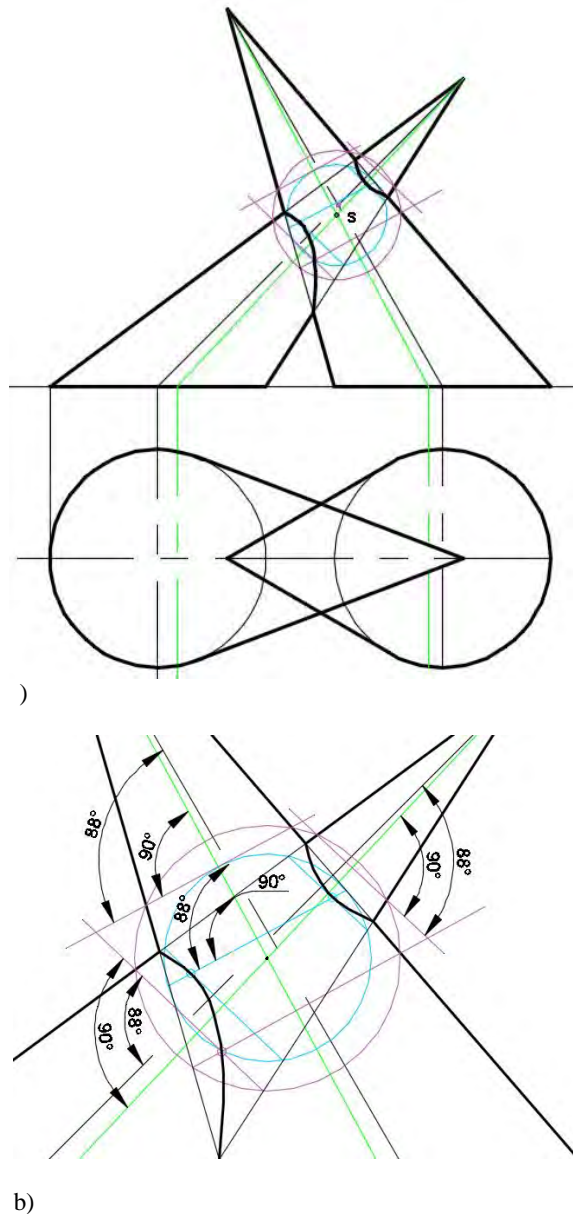

)

b)

Fig. 2 Intersection of inclined circular cones. Method of **Auxiliary Concentric Spheres.**
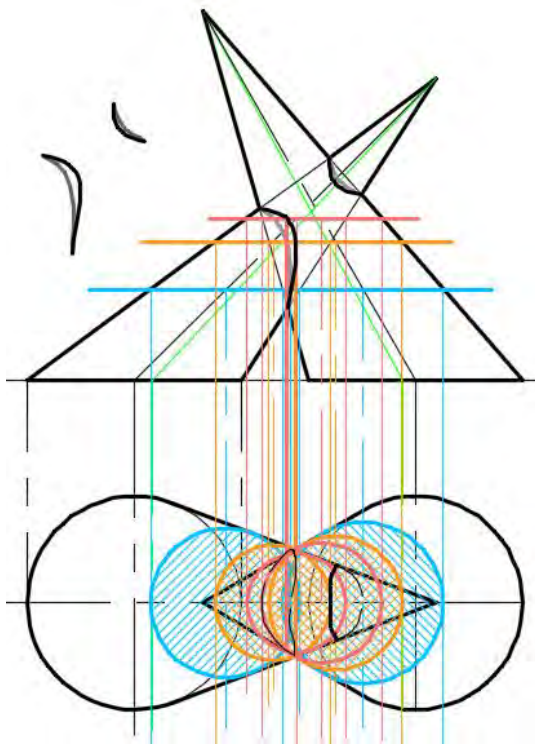
Fig. 3 Intersection of inclined circular cones. Method of **a Family of Parallel Planes.**

4. INTERSECTION OF INCLINED CONICAL SURFACES BY "THE METHOD OF AUXILIARY PLANES THROUGH A STRAIGHT LINE"

In order to be absolutely sure in the accuracy of the solutions we solved the same task by using õThe Method of **Auxiliary Planes through a Straight Line"**.

The drawing in fig. 4 shows the solution to the task by õThe Method of Auxiliary Planes through a Straight Lineö in black, and the one by õThe Method of Auxiliary Concentric Spheresö in grey. The results differ in this case as well.

It is important to note that in this case, the solutions obtained by both õThe Method of a **Family of Parallel Planes**ö and õThe Method of **Auxiliary Planes through a Straight Line**ö coincide and on the other hand they differ from the solution by õThe Method of Auxiliary Spheresö.

The results obtained from all three solutions raise doubts that there must be a problem with the relevance of õThe Method of Auxiliary Concentric Spheresö used or with the dimensions and form of the inclined cones [7], [8], [9].

5. STUDY OF THE DIMENSIONS AND FORM OF AN INCLINED CIRCULAR CONE

With the purpose of the study, an interception of an inclined circular cone placed in a general position with a plane intersecting its axis perpendicularly has been done. The solution of the task is given in fig. 5. From the solution obtained by õThe Method of Rotationö is seen

that it is elliptical and the axis of the body (in black) does not coincide with the centre of intersection. On the other hand, the centre of intersection coincides with the axis
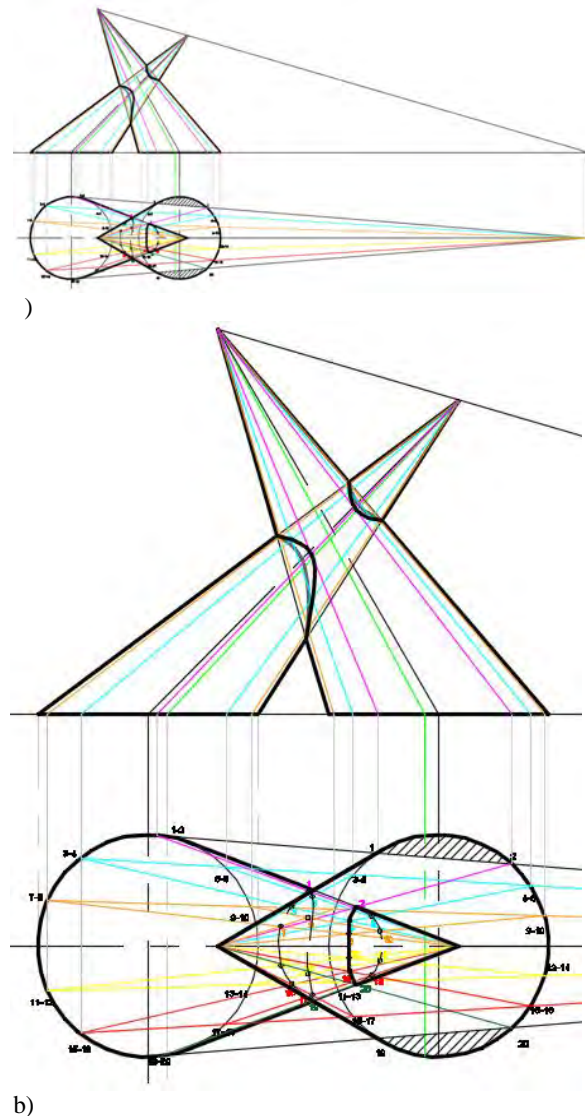

)

b)

Fig. 4 Intersection of inclined circular cones. Method of **Auxiliary Planes through a Straight Line.**

appearing to be the bisectrix to the vertex of the cone (in green). Obviously, the axis has changed its initial position. Now it is clear why the solution to the task by õThe Method of Auxiliary Concentric Spheresö was incorrect.

In this case we can make the conclusion that since the inclined circular cones are not rotating bodies (although we are used to thinking of them as such), then this method is not relevant to be applied because when intersecting the cones with auxiliary spheres there will not be parallel circumferences obtained as a result.

As a matter of fact, in descriptive geometry textbooks for greater convenience, mainly circular cones are used and when they are right, i.e. with their axis perpendicular to their base, they tend to be rotating as well.

Thus the reason why there are no solutions with inclined circular cones by "The Method of Auxiliary Concentric Spheres" has been identified.

obtained from the projection connection whereas the minor axis preserves the length of the diameter of the inclined rotating cone (fig.6).
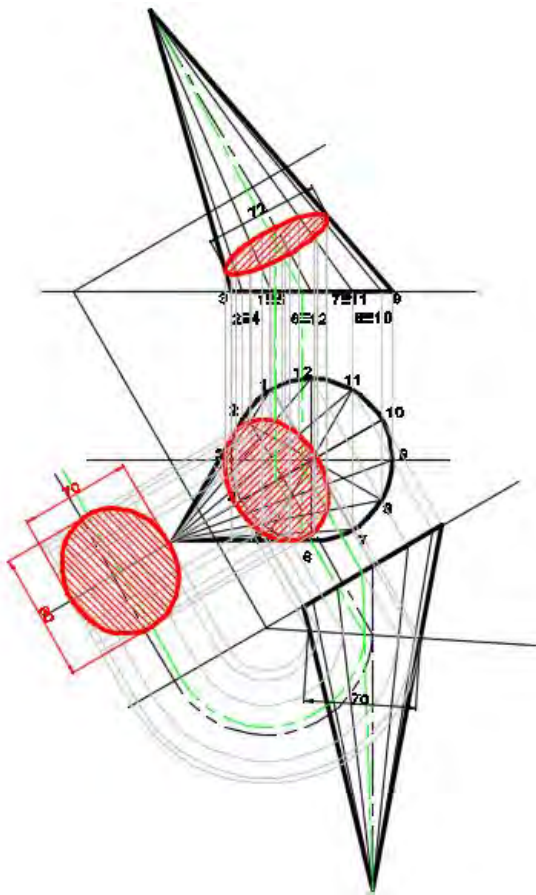

Fig. 5 Intersection of an inclined circular cone with a plane perpendicular to its axis. Defining the real dimensions of the section.
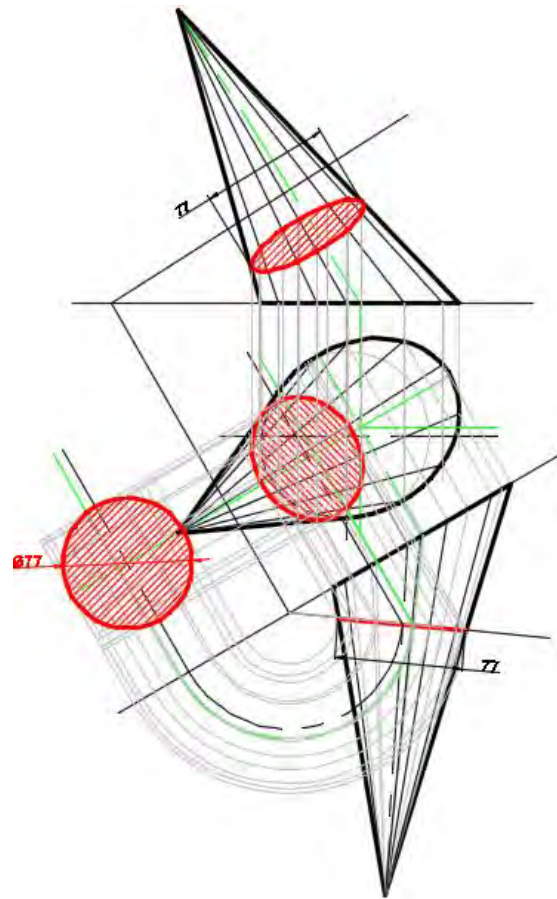

Fig. 6 Intersection of an inclined rotating cone with a plane perpendicular to its axis. Finding the real dimensions of the section.

Logically, a question arises: why there is no solution with inclined rotating cones? Most probably one of the reasons is that they are more difficult to construct.

Fig. 6 illustrates an inclined rotating cone located in general position and it is proved, again by "The Method of Rotation", that its section is a circumference (in black). It can also be seen that in all projections as well as in the section the diameter of the cone is always a constant, unlike the section of the inclined circular cone which is projected with different dimensions on each of the projection planes (fig. 5).

The inconvenience when working with rotating cones is that their bases are projected as an ellipse which is completely explicable since it is known that the intersection of a right cone with a plane positioned at an angle different from 0^0 or 90^0 is an ellipse. However, in each case the elliptical base is oriented in different directions in space depending on the direction of cone inclination which makes its construction difficult. Despite this fact, if the requirement for the axis of rotation to be a bisectrix at the vertex at the same time is met, then it would be easier to construct such a rotating cone. The length of the major axis of the ellipse will be
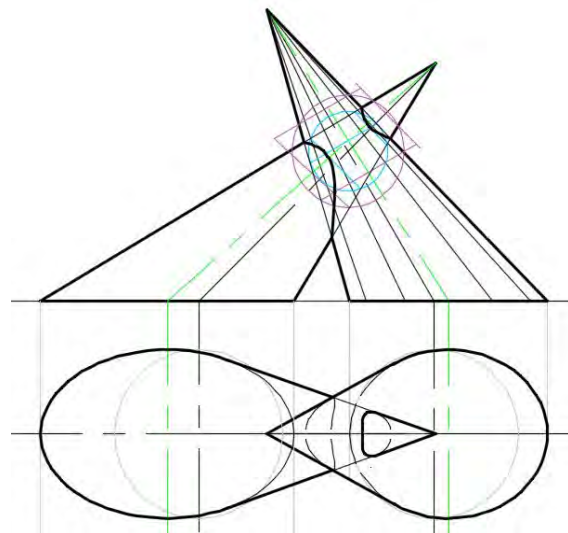

Fig. 7 Intersection of inclined rotating cones. The Method of **Auxiliary Concentric Spheres**.
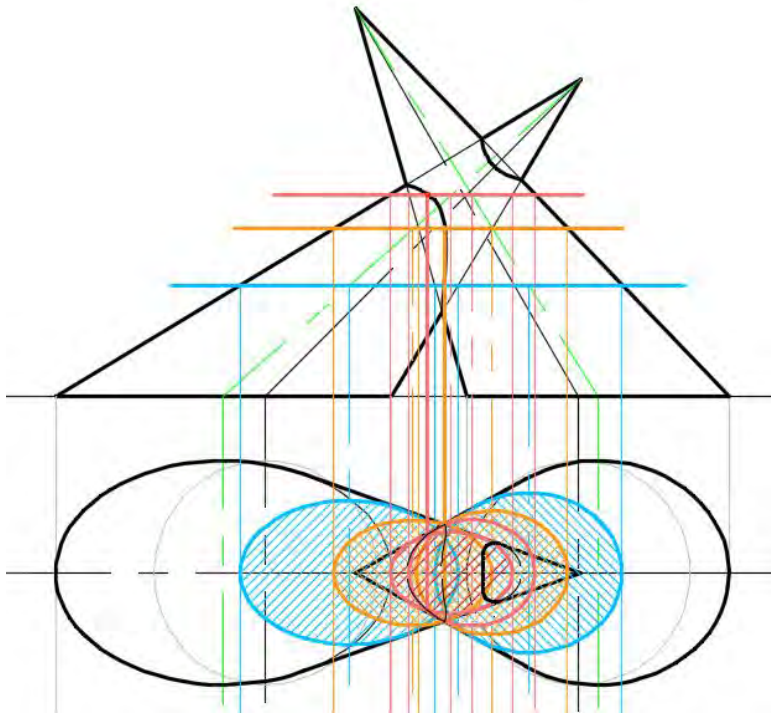
Fig. 8 Intersection of inclined rotating cones. Method of a
Family of Parallel Planes.

Fig. 7 presents the intersection of inclined rotating cones, the solution being obtained by õThe Method of Auxiliary Concentric Spheresö. As can be seen, all the requirements of the method have been satisfied and the intersecting inclined cones are rotating which is proved by the fact that the projections of their bases on the horizontal plane are ellipses.

The same case of inclined cones is illustrated in fig. 8 but this time for finding the intersection line õThe Method of a Family of Parallel Linesö has been used. Following the principles of homothety, the sections obtained are also ellipses.

The solution is obtained by the points of intersection of the elliptical sections. The spatial curve of the intersection between the bodies is obtained by translating the sections in the adjacent projection and connecting them in succession. Comparing the curves obtained by the two methods, we can see that they coincide completely.
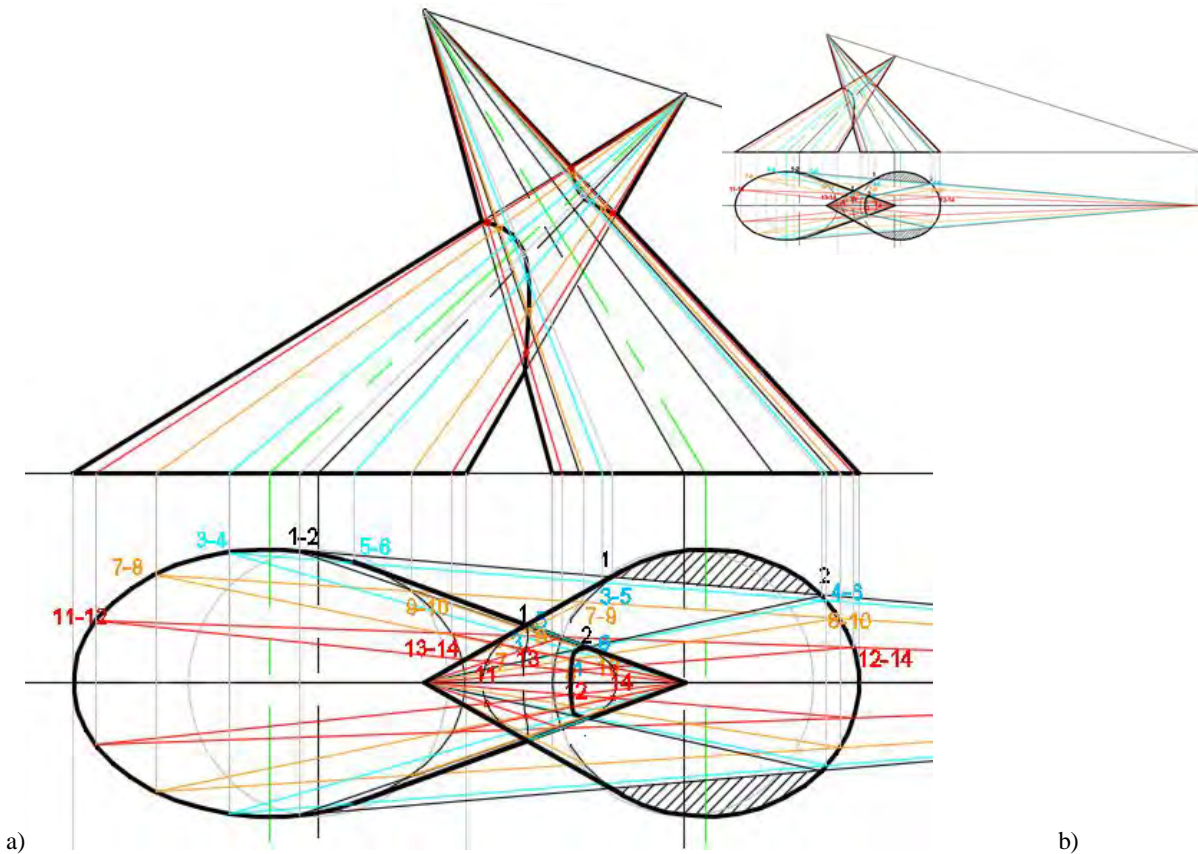

a)   b)

Fig. 9   Intersection of inclined rotating cones. Method of **Auxiliary Planes through a Straight Line**.

Fig. 9 shows the intersection of the same rotating cones as in the other figures but this time the solution is obtained by õThe Method of Auxiliary Planes through a Straight Lineö. Finding the solution by this method is most time-consuming and confusing for the person doing the task. For the purposes of clarity different colours have been used. As can be seen, the solution obtained completely coincides with the solutions of both previous methods.

6. CONCLUSIONS

The only reason for obtaining an inaccurate solution by õThe Method of **Auxiliary Concentric Spheres**ö is the insufficient knowledge of the real form and dimensions of the various types of cones and hence the unawareness of the relevance of the traditional methods for finding the spatial line of intersection between the bodies.

As it has been proved, õThe Method **of Auxiliary Concentric Spheres**ö can be applied only for inclined rotating cones and the inclined circular ones do not pertain to that class although we are accustomed to thinking of all cones as rotating bodies.

õThe Method of Auxiliary Concentric Spheresö is quick and easy to use. Despite this, incorrect solutions might be obtained due to failure to meet the following requirements for the relevance of the method:

1. This method is relevant for rotating bodies only, the circular cones being excluded. Therefore the method is irrelevant for them;

2. The bodiesø axes must intersect;

3. The axes of the intersecting bodies must lie on a plane parallel to any of the projection planes.

All these requirements could not be met if there is no profound knowledge of the real dimensions and form of different types of cones.

REFERENCES

[1] Uzunov N. Petrov G. Dimitrov S. (1963). *Descriptive Geometry, Part 1 (Дескриптивна геометрия, част 1)*, Tehnika Publishing House, volume No 341/I-4, Sofia.

[2] Petrov D. (1971). *Descriptive Geometry (Дескриптивна геометрия)*, publishing house š echnologyõ,, 515 (075.8), Sofia.

[3] Posivyanskiy A. (1965). *A short course of descriptive geometry (Краткий курс начертательной геометрии)*, publishing house "Graduate School", publishing of/164 order 1280 , Moscow.

[4] G€gów V.V., Grinyova B.M., Hnatiuk M.O. (1978). *Descriptive geometry on the basis of algorithmic (Начертательная геометрия на алгоритмической основе)*, publishing house "Graduate School", publishing 362 order 3662, Lions.

[5] F.G. Higbee, M.E. (1915*). The essentials of descriptive geometry*, Stanbope press F. H. GILSON COMPANY BOSTON, U.S.A.

[6] Millar V. Adam, Maclin S. Edward, Markwardt J. Lorraine (1919). *Descriptive geometry*, publishing house "Tracy & Kilgore, Printers", Madison, Wisconsin.

[7] Bartlett W.F., Johnson W. Theodore (1920). *Engineering Descriptive Geometry, A Treatise on Descriptive Geometry as the Basis of Mechanical Drawing, Explaining Geometrically the Operations, Customary in the Draughting Room*, publishing house "The Lord Baltimore Press", Annapolis, MD., USA.

[8] Minor Clyde Hawk (1962). *Schaum`s outline of theory and problems of Descriptive Geometry*, publishing house "McGraw ó Hill Book Company ", publishing 27290 MHUN 8210698. New York, St. Louis, San Francisco, Toronto, Sydney.

[9] Di Paola F., Pedone P., Pizzurro R. M . (2013). *Digital and interactive Learning and Teaching methods in descriptive Geometry,* Social and Behavioral Sciences Volume 106 (2013) 873-885, 4th International Conference on New Horizons in Education, Imprint: ELSEVIER, ISSN: 1877-0428, Chicago.

Author:

Ch. Assist. Prof. PhD Zoya TSONEVA, Technical University ó Varna Department of Industrial Design, E-mail: zoya_tsoneva@abv.bg, tel. +359 894 612359