Carmen POPA, Ivona PETRE, Adriana CÎRSTOIU

UNFOLDING METHODS OF THE CYLINDRICAL SURFACES

***Abstract**: The aim of this paper is to make a comparison between the classical methods of the descriptive geometry and mathematical methods, for obtain the unfolding of cylinders. We establish the intersection curves between cylinders, by using the mathematical program. This thing can be obtained by introducing the curves equations, which are inferred, in mathematical program. This paper take into discussion two right cylinders and another inclined to 45 degrees. The intersection curves are be obtained by using the classical methods of the descriptive geometry, too.*

***Key words:** unfolding, intersection curve, cylinder, projection vertical, mathematical methods.*

1. INTRODUCTION

The calculation of the unfolding has a wide applicability, especially in the connection of the pipes with equal of different diameters. This calculation can be done by using several methods among which: the use of classical methods, the use of analytical methods, the use of mathematical methods with a view to automating their drawing and numerical methods using the ñsplineò [1]. We took three cylinders with the following diameters: C of diameter D =48 mm, C_1 of diameter D_1 = 30 mm, C_2 of diameter D_2=20 mm. The angle $\varphi = 45^o$.

2. THE MATHEMATICAL METHOD OF ESTABLISHING THE INTERSECTIONS CURVES

The projection of the intersection curves necessitates the solving of the following phases [2, 3]: the writing of the curves equations resulted from the intersections of the areas that can be unfolded and the writing of the transformations equations by the unfolding of the intersection curve.

2.1 The calculation of the intersection curve γ_1 of the cylinder C and γ_2 of the cylinder C_1

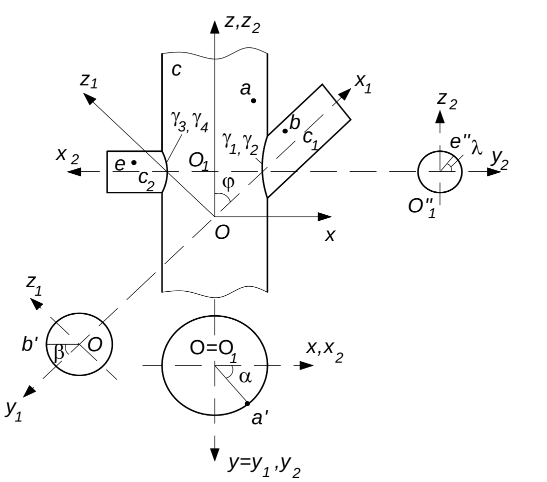

Fig. 1 The geometrical elements of the cylinders

In accordance with the Fig. 1 we take the cylinder C, of diameter D, and its reference system Oxyz and the cylinder C_1, of diameter D_1, and its reference system $Ox_1y_1z_1$, where $y = y_1$. The cylinders equations expressed in the chosen reference systems are:

$$x^2 + y^2 = R^2 \tag{1}$$

$$y^2 + z_1^2 = R_1^2 \tag{2}$$

The two reference systems are rotated, one given another, by the angle φ. The transformation formula of the coordinates, to passing from the system Oxyz into $Ox_1y_1z_1$ and viceversa are:

$$x_1 = x \cdot \cos\varphi + z \cdot \sin\varphi \tag{3}$$

$$z_1 = z \cdot \cos\varphi - x \cdot \sin\varphi \tag{4}$$

$$x = x_1 \cdot \cos\varphi - z_1 \cdot \sin\varphi \tag{5}$$

$$z = x_1 \cdot \sin\varphi + z_1 \cdot \cos\varphi \tag{6}$$

We relate the equations of the both cylinders to system Oxyz and by eliminating the variable y, we obtain the equation of the vertical projection of the intersection:

$$z^2 - 2x \cdot tg\varphi \cdot z + \frac{R^2 - R_1^2}{\cos^2\varphi} - x^2 = 0 \tag{7}$$

The equation of the transformation curve γ_1, border of the cylinder C, is obtained by applying the transformations (8), (9) to the equation (7).

$$x = R \cdot \cos\alpha = R \cdot \frac{\cos x_d}{R} \tag{8}$$

$$z = z_d \tag{9}$$

where x_d and z_d are the coordinates of the point A in unfolding. This point A is indicated by its projections a and aĉ.

In this case the following equation is obtained:

$$z_d^2 - 2Rz_d\cos\frac{x_d}{R}\cdot tg\varphi + \left[\frac{R^2 - R_1^2}{\cos^2\varphi} - R^2\cos^2\frac{x_d}{R}\right] = 0 \tag{10}$$

Then:

$$z_d = R\cos\frac{x_d}{R}\cdot tg\varphi \pm \frac{1}{\cos\varphi}\sqrt{R_1^2 - R^2\sin^2\frac{x_d}{R}} \tag{11}$$

$$x_d \in \left[-R\cdot\arcsin\frac{R_1}{R}, R\cdot\arcsin\frac{R_1}{R}\right] \tag{12}$$

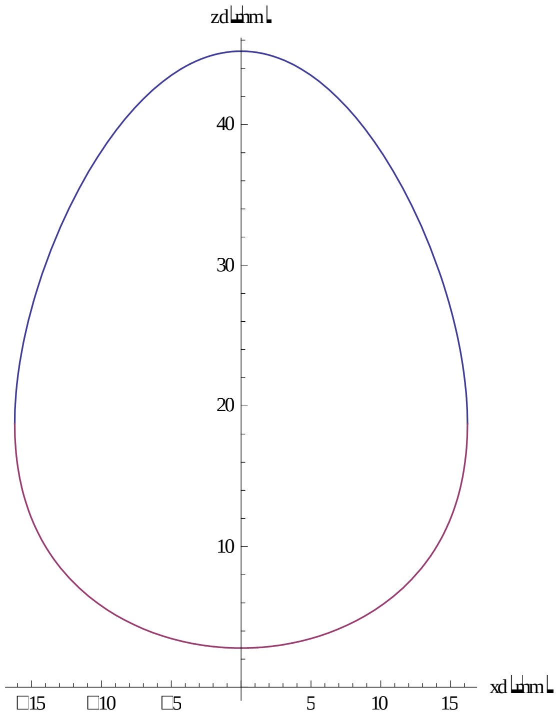

Fig. 2 The unfolding of the intersection curve γ_1 of the cylinder C

We obtain the Fig. 2, by introducing the equations (11), (12), into mathematical program [3].

The equation of the transformation curve γ_2, border of the cylinder C_1, is obtained by applying the transformations (13), (14) to the equation (7):

$$x_1 = x_{d1} \tag{13}$$

$$z_1 = R_1\cdot\sin\beta = R_1\cdot\frac{\sin z_{d1}}{R} \tag{14}$$

where x_{d1} and z_{d1} are the coordinates of the point B(b, b′, b″) in unfolding.

The following equation is obtained:

$$x_{d1}^2 + 2R_1\sin\frac{z_{d1}}{R_1}x_{d1} - R_1^2\sin^2\frac{z_{d1}}{R_1} - \frac{R^2 - R_1^2}{\cos^2\varphi} = 0 \tag{15}$$

Then:

$$x_{d1} = -R_1\sin\frac{z_{d1}}{R_1} \pm \frac{1}{\cos\varphi}\sqrt{R^2 - R_1^2\cos^2\frac{z_{d1}}{R_1}} \tag{16}$$

$$z_{d1} \in [0, 2\cdot\pi\cdot R_1] \tag{17}$$

The Fig. 3 is obtained by introducing the equations (16), (17) into mathematical program.

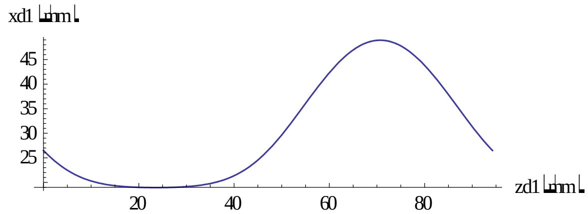

Fig. 3 The unfolding of the intersection curve γ_2 of the cylinder C_1

2.2. The calculation of the intersection curve γ_3 of the cylinder C and γ_4 of the cylinder C_2

In this last case we took the cylinder C_1, of diameter D and C_2, of diameter D_2, with their reference system $O_1x_2y_2z_2$.

The cylinders equations expressed in the chosen references systems are:

$$x_2^2 + y_2^2 = R^2 \tag{18}$$

$$y_2^2 + z_2^2 = R_2^2 \tag{19}$$

The variable y_2 is eliminated and the equation of the vertical projection of the intersection is:

$$x_2^2 + R_2^2 - z_2^2 = R^2 \tag{20}$$

The equation of the transformation curve γ_3, border of the cylinder C, is obtained by applying the transformations (8), (9) to the expression (20):

$$z_{d2}^2 + R^2 - R^2\cos^2\frac{x_{d2}}{R} - R_2^2 = 0 \tag{21}$$

The solutions are:

$$z_{d2} = \pm\sqrt{R_2^2 - R^2\sin^2\frac{x_d}{R}} \tag{22}$$

$$x_{d2} \in \left[-R\cdot\arcsin\frac{R_2}{R}, R\cdot\arcsin\frac{R_2}{R}\right] \tag{23}$$

The Fig. 4 is obtained by introducing the relations (22), (23) into mathematical program.

The equation of the transformation curve γ_4, border of the cylinder C_2, is obtained by applying the transformations (24), (25) to the equation (20).

$$x_2 = x_{d3} \qquad (24)$$

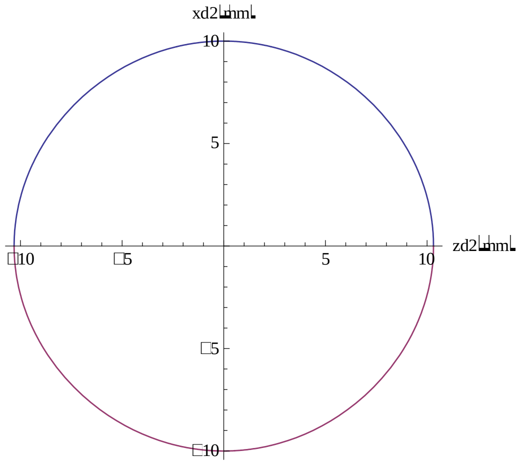

Fig. 4 The unfolding of the intersection curve γ_5 of the cylinder C

$$z_2 = R_2 \cdot \sin\lambda = R_2 \cdot \frac{\sin z_{d3}}{R_2} \qquad (25)$$

where x_{d3}, z_{d3} are the coordinates of the point E(e, eô,eôô) in unfolding.

The equation (26) is obtained:

$$x_{d3}^2 + R_2^2 - (R_2 \sin\frac{z_{d3}}{R_2})^2 - R^2 = 0, \qquad (26)$$

which has the solutions:

$$x_{d3} = \pm\sqrt{R^2 - (R_2 \cos\frac{z_{d3}}{R_2})^2}\ , \qquad (27)$$

where: $z_{d3} \in [0, 2 \cdot \pi \cdot R_2]$ (28)

The Fig. 5 is obtained by introducing the relations (27), (28) into mathematical program.

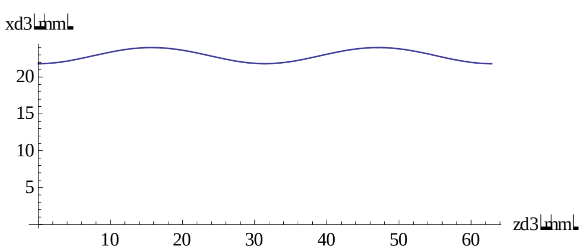

Fig. 5 The unfolding of the intersection curve γ_6 of the cylinder C_3

The establishment of the unfolding of the intersection curves has a great significance in the calculation of the pipes which will be weld.

3. THE DESCRIPTIVE GEOMETRY METHOD OF ESTABLISHING THE INTERSECTIONS CURVES

To determine the curve of intersection of the C, C_2 cylinders, in vertical projection, we use the cylinders projection in the $[H]$ horizontal plane (Fig. 6). In this plan, the base circle of the C_2 cylinder is rabated, which is divided into a number of $a, b, \ldots, g, h$ equal parts in the $[H]$ horizontal plane, which in vertical projection corresponds to the $a', b', \ldots, g', h'$ points. The generators of the base circle intersects in horizontal projection the C cylinder at the $1, 2, \ldots, 8$ points. With the aid of these points, in the $[V]$ vertical plane, at the intersections of the two cylinders generators, are obtained the $a'1', b'2', \ldots, h'8'$ points of the intersection curve.

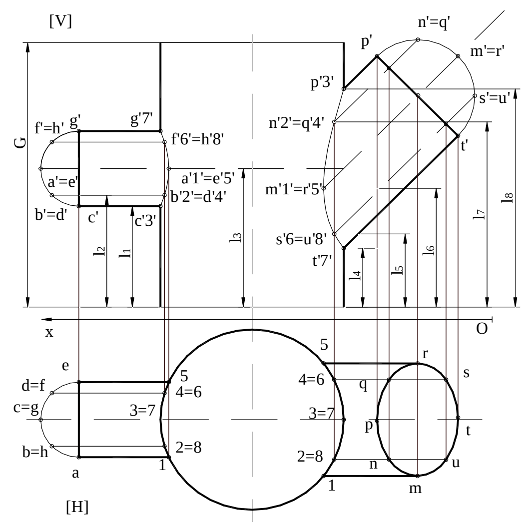

Fig. 6 The intersection curves of the cylinders

To unfold the C_2 cylinder, a segment having length equal to the base circle, which are measured the eight partes to divide the base circle, so $A_oB_o = arc(ab), B_oC_o = arc(bc), \ldots, H_oA_o = arc(ha)$ (Fig. 7).

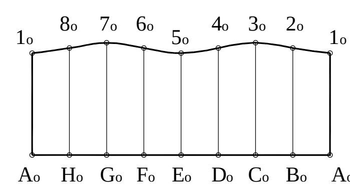

Fig. 7 The unfolding of the C_2 cylinder

From these points, the respective generators lengths are getting up, which are the true size both in vertical and horizontal plane ($\overline{A_o 1_o} = \overline{a1}, \overline{B_o 2_o} = \overline{b2}, ..., \overline{H_o 8_o} = \overline{h8}$).

For unfold the C cylinder (Fig. 8), it is considered a rectangle whose dimension is equal to the length of the G generator, and the other side is half the unfolded length ($\pi 48/2$).

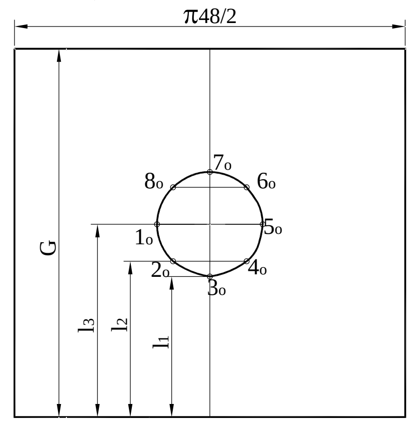

Fig. 8 The unfolding of the C half cylinder

On this, the generator lengths are considered, which are measured in the l_1, l_2, l_3 vertical plane (the others being symmetrical toward the axis) and the measured arc lengths in the horizontal plane, $arc(12), arc(23), ..., arc(81)$. The intersection curve will be $1_o, 2_o, ..., 8_o, 1_o$. In order to determine the curve of intersection of the C, C_1 cylinders, in vertical projection, is done in a similar manner, the $m'1', n'2', ...u'8', m'1'$ points of the intersection curve being obtained at the intersections of the two cylinders generators (Fig. 1). For the C_1 cylinder unfolded, the cylinder unfolded length, divided into the $P_o, Q_o, ..., P_o$ points is drawn (Fig. 9). From these points the generator length measured from the $[V]$ vertical plane are getting up, where the generators are in the true size, thus obtaining the $\overline{P_o 3_o}, \overline{Q_o 4_o}, ..., \overline{P_o 3_o}$ straight lines.

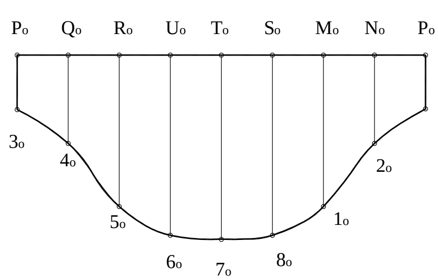

Fig. 9 The unfolding of the C_1 cylinder

For the C unfolded cylinder (Fig. 10), it is considered a rectangle whose dimension is equal to the length of the G generator, and the other side is half of the unfolded length ($\pi 48/2$). On this, the generator lengths measured in the vertical plane, $l_4, l_5, ..., l_8$, and the measured arc lengths in the horizontal plane, $arc(12), arc(23), ..., arc(81)$, are considered. The intersection curve will be $1_o, 2_o, ..., 8_o, 1_o$.

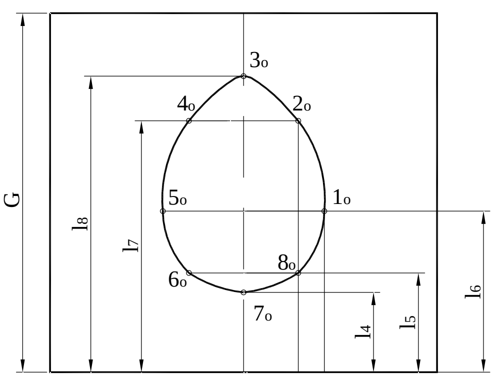

Fig. 10 The unfolding of the other C half cylinder

4. CONCLUSIONS

For the correct execution of some pieces or subassemblies with complex form, which meet the requirements, the methods of descriptive geometry are absolutely necessary. Resolve the difficulties of producing patterns, by determining the types of surfaces that are part of that is very necessary. The presented method is very speedy and exactly and using the program we can obtain the cylinders unfolding for any other dimensions. The two methods have the same results.

5. REFERENCES

[1]Moncea, J. (1989). *The unfolding of surfaces*, Technical Publisher, Bucharest

[2]Popescu, Tr., V. (2004). *Descriptive geometry*, Universitaria Publisher,ISBN: 973ï 8043ï 259-9, Craiova

[3] www.wolfram. com/mathematica

Author(s):

Carmen POPA, Ph.D., Lecturer, Department of Materials Equipments Installations and Robots, Valahia University of Târgoviste, România, E-mail: carmenpopa2001@yahoo.com , tel. 0040744310647.

Ivona PETRE , Ph.D. Associate professor, Department of Materials Equipments Installations and Robots, Valahia University of Târgoviste, România, E-mail: petreivonacamelia@yahoo.com.

Adriana CÎRSTOIU, Ph.D. Lecturer, Department of Materials Equipments Installations and Robots, Valahia University of Târgoviste, România, E-mail: adriana_cirstoiu@yahoo.com.