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SOME SURFACES OF SECOND ORDER AS EXAMPLES OF WEBER`S SURFACES

Abstract: *In this paper we consider some surfaces of second order as surfaces generated from focal-directorial elements (point, line, plane). Extraction of parts of the surface according to appropriate algebraic-geometric conditions will also be defined. In the paper was conducted classification of Weber's surfaces of second order with two generative elements. Also considered was application of surfaces of second order in architectural design.*

Key words: *focus, derectrix, directorial plane, Weber`s surfaces.*

INTRODUCTION

In paper [1] Weber`s surfaces are determined as parts of algebraic surfaces defined by following equation

$$\begin{aligned}&\alpha_1 R_1 + \alpha_2 R_2 + \alpha_3 R_3 + \beta_1 r_1 + \beta_2 r_2 + \beta_3 r_3\\ &+\gamma_1\rho_1 + \gamma_2\rho_2 + \gamma_3\rho_3 = \delta.\end{aligned} \qquad (1)$$

In this paper we shall consider reduced form of equation:

$$\alpha_1 R_1 + \alpha_2 R_2 + \beta_1 r_1 + \beta_2 r_2 + \gamma_1\rho_1 + \gamma_2\rho_2 = \delta, \qquad (2)$$

where R_i ($i=1,2$) is distance from the point X of locus to focal point X_i

$$R_i = \left|X - X_i\right|; \qquad (3)$$

where r_j ($j=1,2$) is distance from the point X of locus to the j^{th} directrix line d_j $[X_1^{(d_j)}, X_2^{(d_j)}]$

$$r_j = \frac{\left|\left(X_2^{(d_j)} - X_1^{(d_j)}\right)\times\left(X_1^{(d_j)} - X\right)\right|}{\left|X_2^{(d_j)} - X_1^{(d_j)}\right|}; \qquad (4)$$

where ρ_k ($k=1,2$) is distance from the point X of locus to the k^{th} directrix plane D_k $[X_1^{(D_k)}, X_2^{(D_k)}, X_3^{(D_k)}]$

$$\rho_k = \left|\frac{\left(X_2^{(D_k)} - X_1^{(D_k)}\right)\times\left(X_3^{(D_k)} - X_1^{(D_k)}\right)}{\left|\left(X_2^{(D_k)} - X_1^{(D_k)}\right)\times\left(X_3^{(D_k)} - X_1^{(D_k)}\right)\right|}\circ\left(X - X_k^{(D_k)}\right)\right|; \qquad (5)$$

and $\alpha_i, \beta_j, \gamma_k, \delta \in R$ $(i, j, k = 1,2)$.

2. WEBER`S SURFACES OF ORDER TWO

In this section we shall analyse algebraic surfaces of second order (6) which are generated as focal-directrix generated Weber`s surfaces (1) with at most 2 elements (2). Each surface of second order is given by following equation:

$$\begin{aligned}&A_{11}x^2 + A_{22}y^2 + A_{33}z^2 + 2A_{12}xy + 2A_{13}xz + 2A_{23}yz\\ &+2A_{14}x + 2A_{24}y + 2A_{34}z + A_{44} = 0.\end{aligned} \qquad (6)$$

Some of them are considered in [1], [2], [4], [8] and [9]. In the paper [3], we present a method for extraction of arcs of the generalized Weber`s curve.

2.1 Ellipsoid

Let fixed points $F_1(-c,0,0)$ and $F_2(c,0,0)$ be the foci with parameter $c = \sqrt{a^2 - b^2}$ for $a \geq b > 0$. The ellipsoid is the locus of points such that the sum of the distances to two foci is constant; in other words:

$$R_1 + R_2 = \delta \qquad (7)$$

where R_1 and R_2 are the distances to corresponding foci and $\delta = 2a$. Equality (7) is equivalent to:

$$\sqrt{(x+c)^2 + y^2 + z^2} + \sqrt{(x-c)^2 + y^2 + z^2} = 2a. \qquad (8)$$

By squaring equality (8) we obtain:

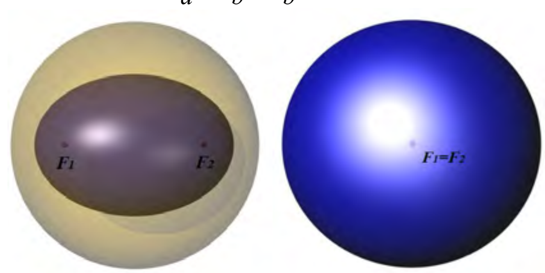
$$\begin{aligned}&2\sqrt{\left((x+c)^2 + y^2 + z^2\right)\left((x-c)^2 + y^2 + z^2\right)} = 4a^2 - 2c^2\\ &-2x^2 - 2y^2 - 2z^2.\end{aligned} \qquad (9)$$

The squaring is correct if $4a^2 - 2c^2 - 2x^2 - 2y^2 - 2z^2$ Ó0, i.e.

$$x^2 + y^2 + z^2 \leq R^2, \qquad (10)$$

where $R = \sqrt{2a^2 - c^2} = \sqrt{a^2 + b^2}$. All points of ellipsoid (Fig. 1 *a*) are in the interior area of the sphere $x^2+y^2+z^2=R^2$. By the second squaring the equality (9) we obtain the canonical form of special ellipsoid-spheroid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1. \qquad (11)$$



a) *b)*

Fig. 1 *a)* Ellipsoid;
b) Sphere.

In the case of $F_1 = F_2$ we obtain sphere (Fig. 1 *b*).

2.2 A two-sheets hyperboloid

Let fixed points $F_1(-c,0,0)$ and $F_2(c,0,0)$ be the foci with parameter $c=\sqrt{a^2+b^2}$ for $a \geq b > 0$. A two-sheet hyperboloid is the locus of points such that the absolute value of difference of the distances to two foci is constant; in other words:

$$\left|R_1 - R_2\right| = \delta \tag{12}$$

where R_1 and R_2 are the distances to corresponding foci and $\delta = 2a$. Equality (12) is equivalent to:

$$\left|\sqrt{(x+c)^2+y^2+z^2}-\sqrt{(x-c)^2+y^2+z^2}\right| = 2a. \tag{13}$$

By squaring equality (13) we obtain:

$$2\sqrt{\left((x+c)^2+y^2+z^2\right)\left((x-c)^2+y^2+z^2\right)} = -4a^2+2c^2 +2x^2+2y^2+2z^2. \tag{14}$$

The squaring is correct if $-4a^2+2c^2+2x^2+2y^2+2z^2$ Ó0, i.e.

$$x^2+y^2+z^2 \geq R^2, \tag{15}$$

where $R=\sqrt{2a^2-c^2}=\sqrt{a^2-b^2}$. All points of a two-sheets hyperboloid (Fig. 2) are in the exterior area of the sphere $x^2+y^2+z^2=R^2$. By the second squaring the equality (14) we obtain the canonical form of a special two-sheets (circular) hyperboloid:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{b^2}=1. \tag{16}$$

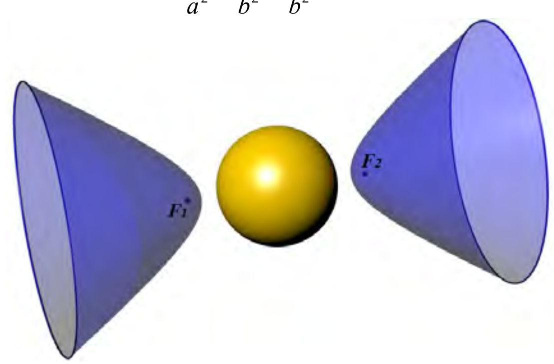


Fig. 2 A two-sheets hyperboloid.

2.3 An elliptic cylinder

Let fixed parallel lines $d_1[P_{11}(-c,0,0), P_{12}(-c,0,1)]$ and $d_2[P_{21}(c,0,0), P_{22}(c,0,1)]$ be the two directrices with parameter $c=\sqrt{a^2-b^2}$ for $a \geq b > 0$. The elliptic cylinder is the locus of points such that the sum of the distances to two directrices is constant; in other words:

$$r_1 + r_2 = \delta \tag{17}$$

where r_1 and r_2 are the distances to corresponding directrices and $\delta = 2a$. Equality (17) is equivalent to:

$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a. \tag{18}$$

By squaring equality (18) we obtain:

$$2\sqrt{\left((x+c)^2+y^2\right)\left((x-c)^2+y^2\right)} = 4a^2-2c^2 -2x^2-2y^2. \tag{19}$$

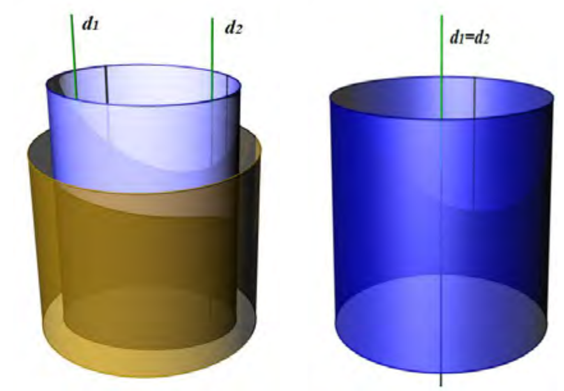
The squaring is correct if $4a^2-2c^2-2x^2-2y^2$ Ó0, i.e.

$$x^2+y^2 \leq R^2, \tag{20}$$

where $R=\sqrt{2a^2-c^2}=\sqrt{a^2+b^2}$. All points of elliptic cylinder (Fig. 3 *a*) are in the interior area of the cylinder $x^2+y^2=R^2$. By the second squaring the equality (19) we obtain the canonical form of elliptic cylinder:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1. \tag{21}$$

In the case of $d_1 = d_2$ we obtain circular cylinder (Fig. 3 *b*).



a) *b)*

Fig. 3 *a)* An elliptic cylinder;
b) Circular cylinder.

2.4 A hyperbolic cylinder

Let fixed parallel lines $d_1[P_{11}(-c,0,0), P_{12}(-c,0,1)]$ and $d_2[P_{21}(c,0,0), P_{22}(c,0,1)]$ be the two directrices with $c=\sqrt{a^2+b^2}$ for $a \geq b > 0$. A hyperbolic cylinder is the locus of points such that the absolute value of difference of the distances to two directrices is constant; in other words:

$$\left|r_1 - r_2\right| = \delta \tag{22}$$

where r_1 and r_2 are the distances to corresponding directrices and $\delta = 2a$. Equality (22) is equivalent to:

$$\left|\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}\right| = 2a. \tag{23}$$

By squaring equality (23) we obtain:

$$2\sqrt{\left((x+c)^2+y^2\right)\left((x-c)^2+y^2\right)} = -4a^2+2c^2 +2x^2+2y^2 \tag{24}$$

The squaring is correct if $-4a^2+2c^2+2x^2+2y^2$ Ó0, i.e.

$$x^2+y^2 \geq R^2, \tag{25}$$

where $R=\sqrt{2a^2-c^2}=\sqrt{a^2-b^2}$. All points of a hyperbolic cylinder (Fig. 4) are in the exterior area of the cylinder $x^2+y^2=R^2$. By the second squaring the equality (24) we obtain the canonical form of a hyperbolic cylinder:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1. \tag{26}$$

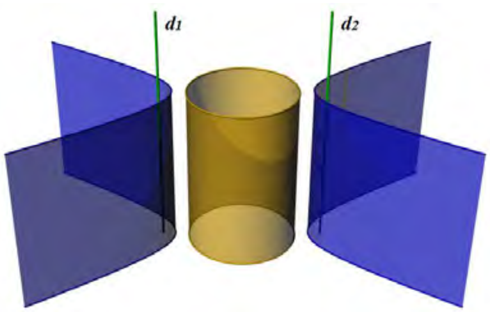


Fig. 4 A hyperbolic cylinder.

2.5 Hyperbolic paraboloid

Let fixed skew lines $d_1[P_{11}(-1,0,0),\ P_{12}(1,0,0)]$ and $d_2[P_{21}(0,-1,c),\ P_{22}(0,1,c)]$ be the two directrices, parameter $c \neq 0$. A hyperbolic paraboloid is the locus of points such that the distances to directrices are same; in other words:

$$r_1-r_2=0 \tag{27}$$

where r_1 and r_2 are the distances to corresponding directrices ($\delta=0$). Equality (27) is equivalent to:

$$\sqrt{y^2+z^2}=\sqrt{x^2+(z-c)^2}. \tag{28}$$

By squaring equality (28) we obtain:

$$y^2+z^2=x^2+(z-c)^2 \tag{29}$$

Finally, we obtain the canonical form of a hyperbolic paraboloid (Fig. 5):

$$x^2-y^2=2cz-c^2. \tag{30}$$

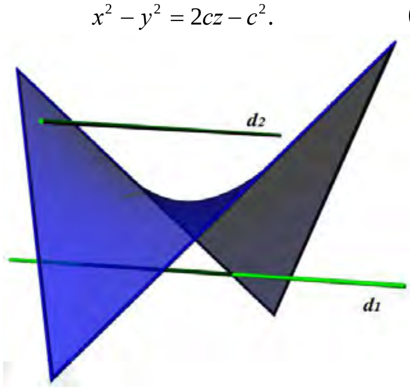


Fig. 5 Hyperbolic paraboloid.

2.6 Degenerate surface (two planes)

Let fixed intersecting lines $d_1[P_{11}(-1,0,0),\ P_{12}(0,0,c)]$ and $d_2[P_{21}(1,0,0),\ P_{22}(0,0,c)]$ be the two directrices ($d_1 \cap d_2 = P_{12} = P_{22}$), parameter $c \neq 0$. The locus of points, such that the distances to directrices are same, is degenerate surface (two planes); in other words:

$$r_1-r_2=0 \tag{31}$$

where r_1 and r_2 are the distances to corresponding directrices. Equality (31) is equivalent to:

$$\frac{\sqrt{y^2(c^2+1)+((x+1)c-z)^2}}{\sqrt{c^2+1}}=\frac{\sqrt{y^2(c^2+1)+((x-1)c+z)^2}}{\sqrt{c^2+1}} \tag{32}$$

By squaring equality (32) we obtain:

$$\frac{y^2(c^2+1)+((x+1)c-z)^2}{c^2+1}=\frac{y^2(c^2+1)+((x-1)c+z)^2}{c^2+1} \tag{33}$$

Finally, we obtain the degenerate surface - two planes (Fig. 6):

$$x(z-c)=0 \Leftrightarrow x=0 \vee z=c. \tag{34}$$

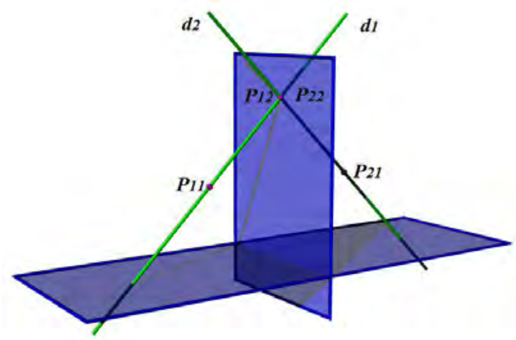


Fig. 6 Degenerate surface (two planes).

2.7 A parabolic cylinder

Let the line $d_1[P_{11}(-p/2,0,0),\ P_{12}(-p/2,0,1)]$ be the directrix and let the point $F_1(p/2,0,0)$ be the focus, for parameter $p > 0$, $F_1 \notin d_1$. A parabolic cylinder is the locus of points such that their of the distance to directrix is same as distance to focus; in other words:

$$R_1-r_1=0 \tag{35}$$

where R_1 is distance to the focus F_1 and r_1 is the distance to directrix d_1. Equality (35) is equivalent to:

$$\sqrt{\left(x-\frac{p}{2}\right)^2+y^2+z^2}=\sqrt{\left(x+\frac{p}{2}\right)^2+y^2}. \tag{36}$$

By squaring this equality, we obtain the algebraic equation for parabolic cylinder (Fig. 7):

$$z^2=2px. \tag{37}$$

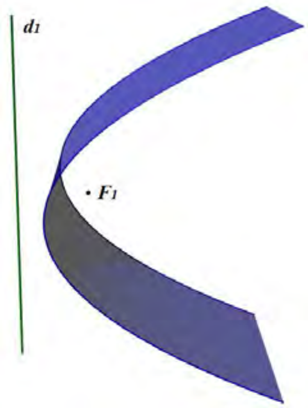
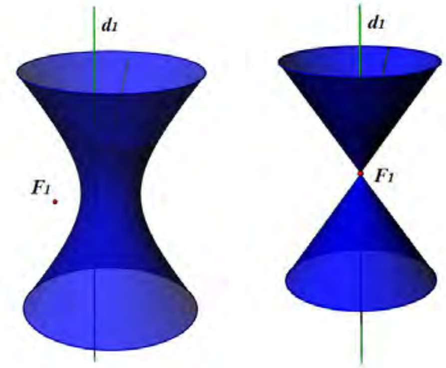


Fig. 7 A parabolic cylinder.



a) *b)*

Fig. 8 *a)* A one-sheet hyperboloid.
b) Conical surfaces.

2.8 A one-sheet hyperboloid

Let the fixed line $d_1[P_{11}(0,0,0),\ P_{12}(0,0,1)]$ be the directrix and let the fixed point $F_1(c,0,0)$ be the focus, $c \neq 0,\ F_1 \notin d_1$. A one-sheet hyperboloid is the locus of points such that the distance to focus equals some multiply of the distance to diretrix; in other words:

$$R_1 - k \cdot r_1 = 0 \tag{38}$$

where R_1 is distance to the focus F_1 and r_1 is the distances to directrix d_1, $k > 1$. Equality (38) is equivalent to:

$$\sqrt{(x-c)^2 + y^2 + z^2} = k \cdot \sqrt{x^2 + y^2}\ . \tag{39}$$

By squaring equality (39) we obtain:

$$(x-c)^2 + y^2 + z^2 = k^2 \cdot (x^2 + y^2), \tag{40}$$

i.e.

$$(k^2 - 1)\ x^2 + 2cx - c^2 + (k^2 - 1)\ y^2 - z^2 = 0\ . \tag{41}$$

If $c^2 = k^2 - 1$ then equality (41) gives canonical form of special one-sheet (circular) hyperboloid (Fig. 8 *a*):

$$\frac{(x + 1/c)^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{b^2} = 1, \tag{42}$$

where $a^2 = 1 + 1/c^2$, $b^2 = k^2$.

If $c = 0$ i.e. $F_1 \in d_1$ then equation (41) becomes:

$$x^2 + y^2 - \frac{z^2}{k^2 - 1} = 0, \tag{43}$$

the canonical form of special (circular) conical surface (Fig. 8 *b*).

2.9 Paraboloid

Let the plane D_1: $x = -p/2$ be the directorial plane and let the fixed point $F_1(p/2,0,0)$ be the focus, for parameter $p > 0,\ F_1 \notin D_1$. Paraboloid is the locus of points that have the same distance from the directorial plane as distance to focus; in other words:

$$R_1 - \rho_1 = 0 \tag{44}$$

where R_1 is distance to the focus F_1 and $\mathbf{r}_1$ is the distances to directrix plane D_1. Equality (44) is equivalent to:

$$\sqrt{\left(x - \frac{p}{2}\right)^2 + y^2 + z^2} = \left|x + \frac{p}{2}\right|. \tag{45}$$

Let us notice that

$$\sqrt{\left(x - \frac{p}{2}\right)^2 + y^2 + z^2} \geq \left|x - \frac{p}{2}\right| = -x + \frac{p}{2} > -x - \frac{p}{2} = \left|x + \frac{p}{2}\right|$$

for $x < -p/2$. Therefore, the equality (45) can be considered only for the half-space $x \geq -p/2$, and in that case, by squaring the equality (45), we obtain the algebraic equation for paraboloid

$$y^2 + z^2 = 2px. \tag{46}$$

Previously determined parabola is completely inside the half-space $x \geq -p/2$ (Fig. 9).

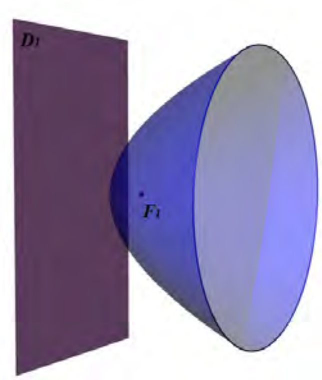


Fig. 9 Paraboloid.

2.10 A two-sheets hyperboloid

Let the plane D_1: $x=0$ be the directorial plane and let the fixed point $F_1(c,0,0)$ be the focus, $c \neq 0$, $F_1 \notin D_1$. A two-sheets hyperboloid is the locus of points that have distance to the focus as multiply of distance from the directorial plane; in other words:

$$R_1 - k \times \rho_1 = 0 \qquad (47)$$

where R_1 is distance to the focus F_1 and $\mathbf{r}_1$ is the distances to directorial plane D_1, $k > 1$. Equality (47) is equivalent to:

$$\sqrt{(x-c)^2 + y^2 + z^2} = k \times |x|. \qquad (48)$$

By squaring equality (48) we obtain:

$$\left(k^2 - 1\right) x^2 + 2cx - c^2 - y^2 - z^2 = 0. \qquad (49)$$

If $c^2 = k^2 -1$ then equality (49) gives canonical form of special two-sheet (circular) hyperboloid (Fig. 10):

$$\frac{(x+1/c)^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{b^2} = 1, \qquad (50)$$

where $a^2 = 1+1/c^2$, $b^2 = k^2$.

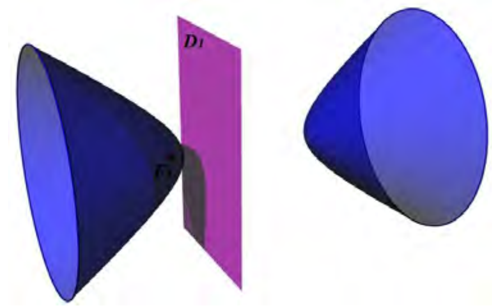


Fig. 10 A two-sheets hyperboloid.

If $c = 0$ i.e. $F_1 \in D_1$ then equation (48) becomes:

$$x^2 - \frac{y^2}{k^2-1} - \frac{z^2}{k^2-1} = 0, \qquad (51)$$

the canonical form of special (circular) conical surface (Fig. 11).

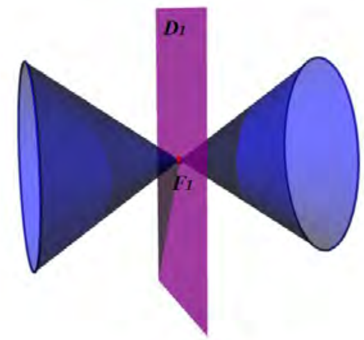


Fig. 11 Conical surfaces.

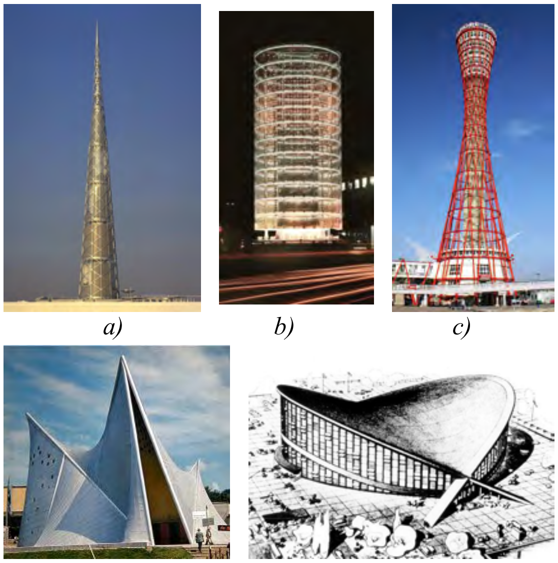
Table 1

Classification of Weber`s surfaces second order with two generative elements.

a_1	a_2	b_1	b_2	g_1	g_2	d	Weber`s surface (2)	Relations	Surface of second degree	Number of figure
1	1	0	0	0	0	>0	$R_1+R_2=$**d**	$F_1 \neq F_1$	Ellipsoid	1 *a)*
1	1	0	0	0	0	>0	$R_1+R_2=$**d**	$F_1 = F_1$	Sphere	1 *b)*
1	-1	0	0	0	0	>0	$/R_1-R_2/=$**d**	$F_1 \neq F_1$	Hyperboloid (two-sheets)	2
0	0	1	1	0	0	>0	$r_1+r_2=$**d**	$d_1 \parallel d_2$	Elliptical cylinder	3 *a)*
0	0	1	1	0	0	>0	$r_1+r_2=$**d**	$d_1 = d_2$	Cylinder	3 *b)*
0	0	1	-1	0	0	>0	$/r_1-r_2/=$**d**	$d_1 \parallel d_2$	Hyperbolic cylinder	4
0	0	1	-1	0	0	=0	$r_1-r_2=0$	skew d_1, d_2	Hyperbolic paraboloid	5
0	0	1	-1	0	0	=0	$r_1-r_2=0$	$d_1 \cap d_2$	Degenerate surface (2 planes)	6
1	0	-1	0	0	0	=0	$R_1-r_1=0$	$F_1 \notin d_1$	Parabolic cylinder	7
1	0	$-k$	0	0	0	=0	$R_1-kr_1=0$, $k>1$	$F_1 \notin d_1$	Hyperboloid (one-sheet)	8 *a)*
1	0	$-k$	0	0	0	=0	$R_1-kr_1=0$, $k>1$	$F_1 \in d_1$	Conical surface	8 *b)*
1	0	0	0	-1	0	=0	$R_1-\mathbf{r}_1=0$	$F_1 \notin D_1$	Paraboloid	9
1	0	0	0	$-k$	0	=0	$R_1-k\mathbf{r}_1=0$, $k>1$	$F_1 \notin D_1$	Hyperboloid (two-sheets)	10
1	0	0	0	$-k$	0	=0	$R_1-k\mathbf{r}_1=0$, $k>1$	$F_1 \in D_1$	Conical surface	11

2.11 Quadric surfaces in architecture design

With development of technique and technology on the end of XX century and on the beginning of XXI century, usage of algebraic surfaces of second and higher order in design of architectural objects become more frequent. Because of its simple geometrical construction, but also and simplified aesthetic and elegance, cylinder, cone, hyperboloid, paraboloid, sphere (Fig. 12), are just some of surfaces that attract attention in research, from mathematicians as well as architects. Application of these forms in architecture is significant for its ergonomic and aerodynamic value, [5]-[7].



a) *b)* *c)*

d) *e)*

Fig. 12 Quadric surfaces in architecture design.
a) Conical surface (Millenium Tower, Sir Norman Foster)
b) An elliptic cylinder (Tower of Winds, Toyo Ito)
c) A one-sheet hyperboloid (Kobe Port Tower, Nikken Sekkei)
d) Hyperbolic paraboloid (The Philips Pavilion, Le Corbusier)
e) A parabolic cylinder (The J.S.Dorton Arena, Matthew Nowicki) - pictures taken from [10]-[14]

3. CONCLUSION

In this paper was shown that some surfaces of second order can be generated as Weberĝs surfaces and areas where they are located are defined. Result of this paper is a classification of Weber`s surfaces of second order. Surfaces of second order, defined in that way, are, by its origin, very appropriate for optimization problems, [1], [4]. Mentioned Weber`s surfaces of higher order (2) will be subject of some further considerations in geometry of architectural form.

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