Ligia PETRESCU, Dumitru MARIN

ANALYTICAL AND MATHCAD INTERSECTION BETWEEN TWO PERPENDICULAR AXES CYLINDERS

Abstract: *The intersection of two surfaces is a common line to both surfaces. This line is often called transition line. The authors propose an analytical and a computer graphics intersection between two perpendicular axes cylinders. The analytical expression of such intersection will be obtained and transposed on the development of one cylinder. The distance between the two cylinders axes is essential in the aspect of the transition line. This transition line can be also obtained, using the computer, by the MathCAD programme. Some samples are relevant in this subject, according to the distance between the cylinders.*

Key words: *geometry, intersection, transition line, surface development, computer graphics intersection.*

1. INTRODUCTION

As result of the visual perception, the representations of the reality have known a various range of solutions in trying to present them under a coherent and convincing form. The graphic representations [1], [2] can be perceived and analyzed by a multitude of geometric parameters, which, through their ways of being arranged, ordinate and placed, make up spatial geometric structures, which are studied by the geometry. The simple presence of a geometric structure or of a purely formal geometry cannot solve the problems of representations. In the modern representations, besides geometry, there are to be encountered a series of other disciplines, in connection with the field in which the representation is made.

Physics and geometry are two of the oldest sciences which put a mark on human evolution. Geometry is the *key* which permit the access to the imagination and further to the creation. The mathematician Dan Barbilian, alias the poet Ion Barbu said that ñgeometry and poetry meet together somewhere up in high sphereò. Component of mathematics but not in contradiction with analyse and algebra, *geometry* represents in technical field an art, the power to analyse and synthesis, of analyse and creation. It gives information about plane and spatial known

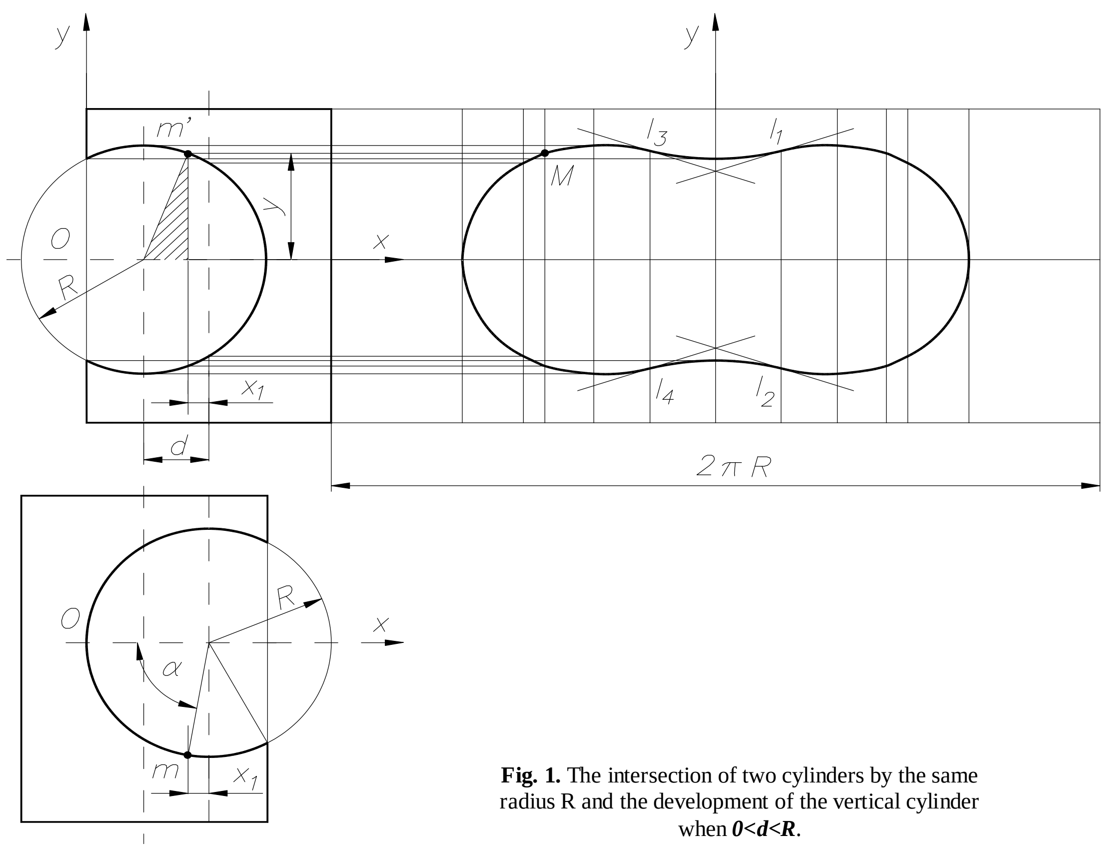

Fig. 1. The intersection of two cylinders by the same radius R and the development of the vertical cylinder when $0<d<R$.

figures, relationship between them, geometrical transformations, plane and spatial structures which can be made with these figures, the properties and possibilities offered by them.

In the technical field the drawing as graphic representation is used as means of communication. The basic of these drawing concerns in geometrical structures, structures made up from different geometric elements that are found in certain position from one another. The cylindrical surface is one of the most used in technique, like element of definition for the shape of different parts. The intersection of cylinders is present in the design of many mechanical parts.

2. THE DEVELOPEMENT OF THE TRANSITION LINE, IF THE TWO CYLINDERS HAVE PERPENDICULAR AXES AND THE SAME RADIUS.

The intersection of two surfaces [1], is a spatial line, named transition line. In Fig 1 are given two cylinders which elements are [3]:

- *R* ï the cylindersô bases radius;
- *d* ï the distance between the two cylinders axes; *(0<d<R).*

The cylinder 1 is a projecting one to the vertical [V] plane, and the cylinder 2, to the horizontal [H] plane.

On the development [3] of the cylinder 2 is taken the point M(m, mô) of the intersection line.

The position of the point M on the intersection is:

$$x = R \propto \tag{1}$$

and

$$x = \frac{x}{R} \tag{2}$$

From the vertical projection, in the crosshatched triangle:

$$y^2 = R^2 - (d - x_1)^2 \tag{3}$$

where:

$$x_1 = R\cos\frac{x}{R} \tag{4}$$

⇒

$$y = \sqrt{R^2\sin^2\frac{x}{R} + 2Rd\cos\frac{x}{R} - d^2} \tag{5}$$

The equation (5) represents the transition line (the intersection line) between the two cylinders, transposed on the development of the vertical cylinder.

The second differential of the expression (5), which solutions are the inflexion points of the transition line, is very complicated, than some particular cases will be studied:

2.1. The case *d=R* (Fig. 2**).** The relation (5) becomes:

$$y = \pm R\sqrt{2\cos\frac{x}{R} - \cos^2\frac{x}{R}} \tag{6}$$

and

$$y'' = R^2\left(\cos^2\frac{x}{R} - 1\right)^2 + R^2 > 0 \tag{7}$$

That means there are no inflexion points on the development of the intersection line (Fig. 2).

for *d=R* and $\frac{x}{R} = 0 \;\Rightarrow\; y_{max} = \pm R$

for *d=R* and $\frac{x}{R} = \pm\frac{\pi}{2} \;\Rightarrow\; y_{min} = 0$.

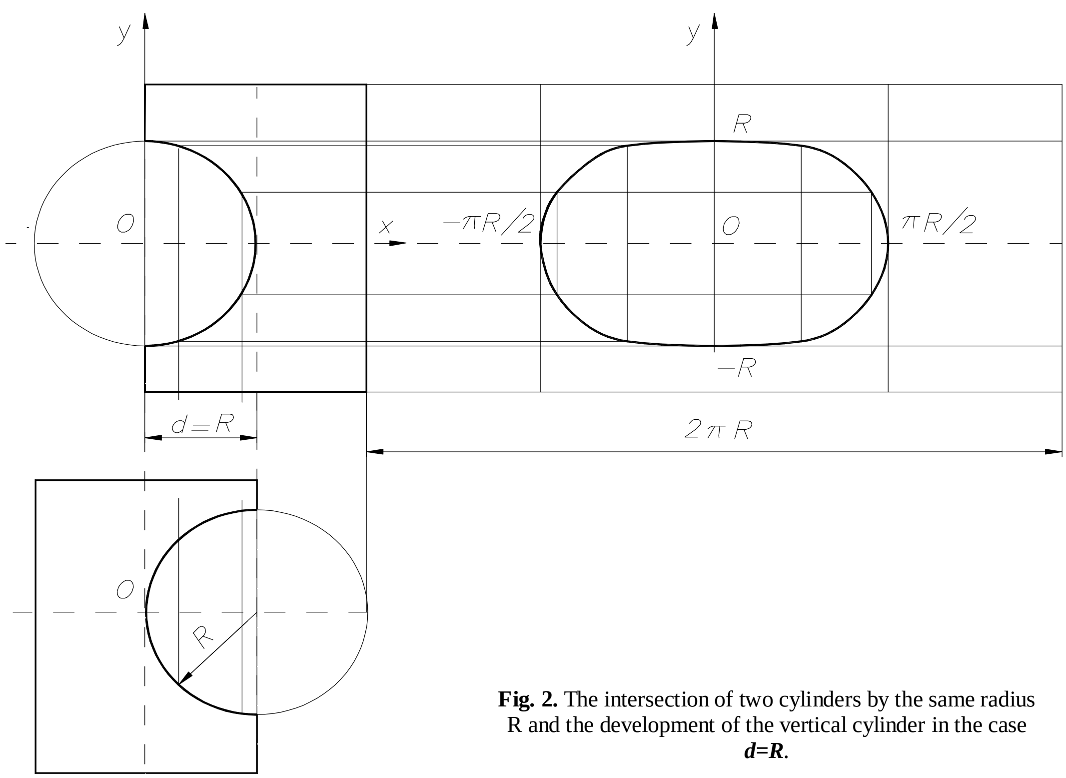

Fig. 2. The intersection of two cylinders by the same radius R and the development of the vertical cylinder in the case ***d=R***.

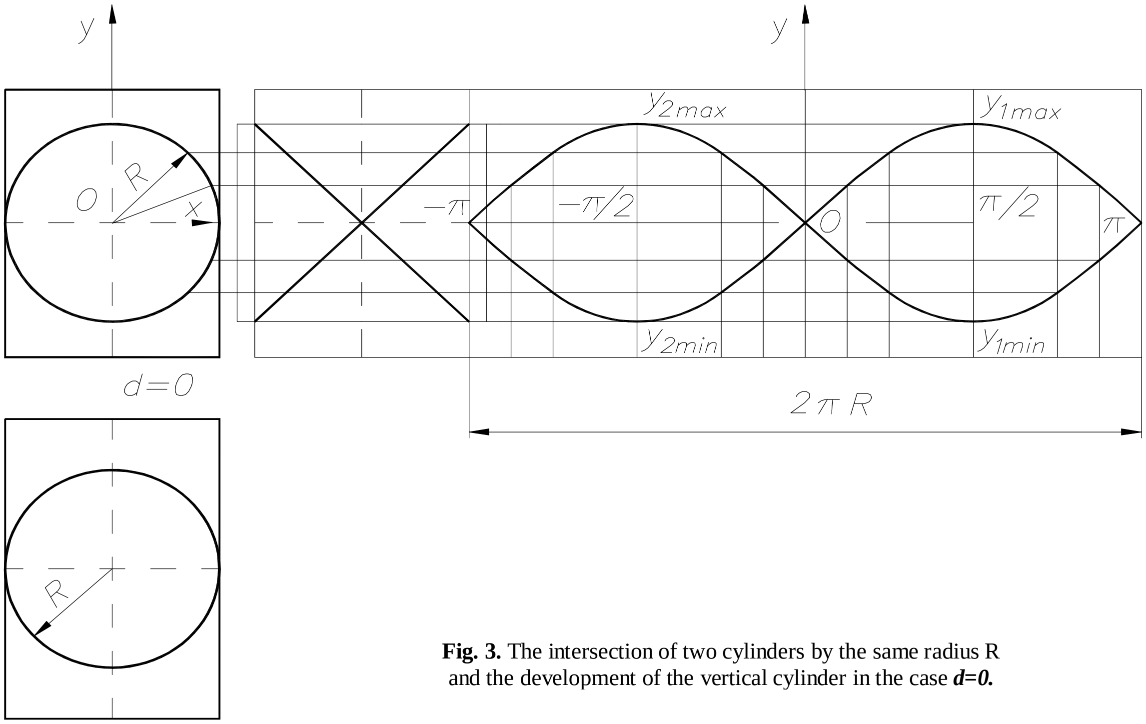

Fig. 3. The intersection of two cylinders by the same radius R and the development of the vertical cylinder in the case ***d=0.***

In the case *d=R,* the intersection line is a closed one, without inflexion points (Fig. 2). The coordinateös axes Ox and Oy are symmetry axes.

2.2The case *d=0* (Fig. 3). In this case the relation (5) becomes:

$$y = \pm R \sin\frac{x}{R} \quad (8)$$

The (8) equations represent two sinusoids in mirror towards *Ox* axis.

The two cylinders which intersect admit two common, tangent planes.

Results that the two transition lines are plane-curves, ellipses. They have two common points which are visible on the development of the intersection line (Fig. 3). Giving particular values to *d* and *x/R* is obtain:

for *d=0* and $\frac{x}{R} = \frac{\pi}{2} \Rightarrow \begin{cases} y_{1max} = R \\ y_{1min} = -R \end{cases}$

for *d=0 and* $\frac{x}{R} = -\frac{\pi}{2} \Rightarrow \begin{cases} y_{2max} = R \\ y_{2min} = -R \end{cases}$

These values are very clear put in view in the Fig. 3.

3. THE COMPUTER ANALYZE OF THE TRANSITION LINE

The graphic tools have evaluated from the first attempts of communications of the people through graphic representations. As the engineering theory and practice have known a continuous evolution, the specific tools developed and brought to a perfect stage so as the engineers and the designers to keep up with the requirements of the progress. Today, there is a relatively new tool, indispensable in projecting: the computer and the graphic stations. One simple fact that is frequently overlooked is that computers are not new. Charles Babbage, an English mathematician, developed the idea of a mechanical digital computer in the 1930s, and many of the principles used in Babbageös design are the basis of todayös computers. The computer appears to be a mysterious machine, but it is nothing more than a tool that just happens to be a highly sophisticated electronic device. It is capable of data storage, basic logical functions, and mathematical calculations. Starting from the mathematiciansô interest in visualizing the graphics of some functions [1], as well as from the engineersô and physicistsô wish of getting information presented as drawings and diagrams from the computer, the graphics obtained by the means of computer become not only a discipline of informatics, but also of the visual arts, of the industrial design and of the projecting.

The CAD techniques, using specialized programs led to the increase of the ñqualityò of realism and precision contained in the drawing realized by means of computer.

In the case of two cylindersô intersection, using the *y* expression (3) is possible to obtain the transition line by the help of PTC ® Mathcad.

In Fig. 4 are plotted eleven transition lines, for eleven particula values of the distance *d=[0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]*, according to the three cases analysed in Fig. 1, 2 and 3.

The algoritm contain the base equation and the specific additional elements of the Mathcad progrrame. The cases presented in Fig. 1, 2 and 3, can be vizualised in the Fig. 5, 6 and 7. Every time the value of *d* distance define the case.

4. CONCLUSIONS

The authors proposed an analytical and a computer graphics intersection between two perpendicular axes

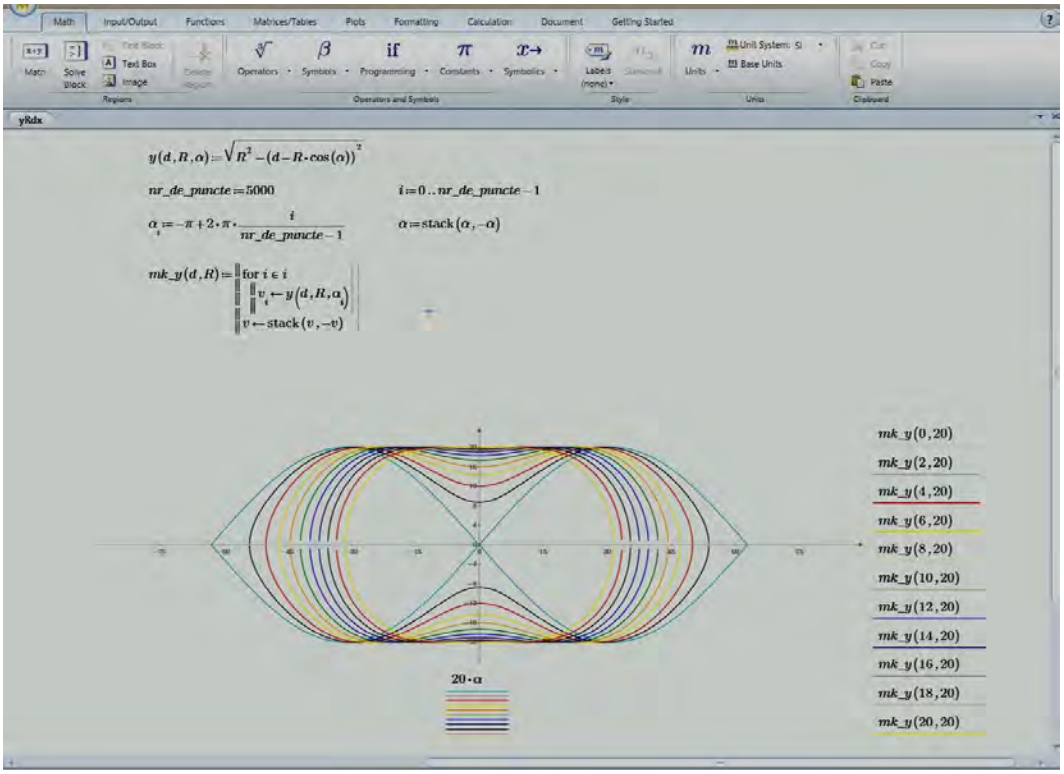

Fig. 4 The development of the transition line of two cylinders with the same radius ***R=20*** and the distance ***d∈[0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]***

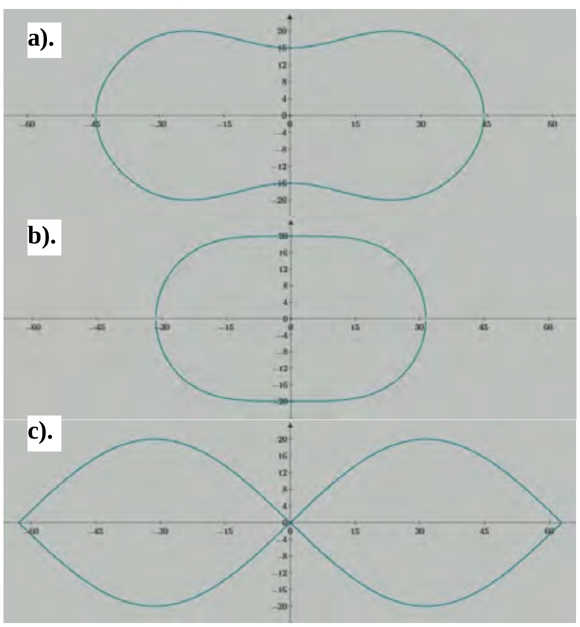

Fig. 5. The intersection of two cylinders by the same radius **R=20** and **a)** ***d=6***, **b)** ***d=20***, **c)** ***d=0.***

cylinders. The analytical expression of such intersection was obtained and transposed on the development of one cylinder. The distance between the two cylinders axes is essential in the aspect of the transition line.

This transition line was also obtained, using the computer, by the MathCAD programme. The samples (Fig. 4) are relevant in this subject, according to the distance *d* between the cylinders.

- for $0 < d < R$ - there are four points of inflexion on the development of the transition line (Fig. 1, Fig. 5a);
- for dÓR - there are no inflexion points on the development of the transition line (Fig. 2, Fig. 5b);
- for *d=0* - the two sinusoids in mirror towards *Ox* axis have two double points which are inflexion points too (Fig. 3, Fig. 5c).

REFERENCES

[1] Petrescu, Ligia, (2015) *Engineering Graphics* - Editura BREN, ISBN 978-606-610-132-5, Bucharest.

[2] Petrescu, Ligia, (2014), *Descriptive Geometry and Engineering Graphics*, Editura BREN, ISBN 978-606-610-080-9, Bucharest.

[3] Marin, D., Petrescu, L. (2013) *Engineering Graphics* ï Editura BREN, ISBN 978-606-610-040-3, Bucharest.

Authors:

1. Assoc prof. Ligia PETRESCU, Ph.D. eng., Department of Engineering Graphics and Industrial Design, Politehnica University of Bucuresti, Romania. E-mail: ligiapetrescu@yahoo.com.

2. Prof. Dumitru MARIN, Ph.D. eng., Department of Engineering Graphics and Industrial Design, Politehnica University of Bucuresti, Romania. E-mail: dumarpub@yahoo.com.