Carmen MÂRZA, Dalia CÎMPEAN, Georgiana CORSIUC

ABOUT THE GEOMETRY AND THE APPLICATIONS OF THE TWISTED SURFACES

Abstract: *The helical and spiral surfaces are used in various fields. These surfaces are obtained by rotating a segment line called generatrix around directrices lines, respectively the spiral surfaces are obtained by composing simultaneously the movements of translation and rotation of a plane figure around an axis - which can be a straight line or a curved line, or around a real or imaginary surface - called core.*
After an overview of these surfaces, the attention of the authors will be focused on the geometrical and graphical analysis of the twisted surfaces in civil engineering branch, such as: columns, helical ramps and stairs, respectively buildings having a futuristic design.

Key words: *helical surface, twisted surface, stair, core, tower.*

1. INTRODUCTION

One of the natural tendency of the people has been that of surrounding themselves with useful objects for everyday living. At the beginning, people were interested only in the functionality of these objects, but, as time went on together with the intellectual and spiritual development, artistic feelings became more refined and the desire of having more beautiful and unique objects around became prevalent, as well as the appeal to a wide variety of shapes.

Retrospectively, one can state that twisted, spiral-shaped shapes, such as curves and then surfaces, attracted people because of their smartness and inspiring mystical character. Among the first spiral-shaped adornments of this kind can be found in the Cucuteni culture, one of the oldest civilisations in Europe, which also covered part of the Romanian territory. Decorations of the mentioned form can also be met with the Chinese, Egyptian and Greek civilisations (see Arhimedeœs spiral) etc.

In a more complex approach, twisted surfaces can be encountered during the Middle Ages. Several examples are significant from this point of view:

- In the Baroque style, defined by excessive ornaments, one can find twisted and spiralled columns, as in the Jesuit churches; a derived variant typical for the Portuguese architecture is the Manueline style, where twisted columns in the shape of a rope are inspired by the ropes used in navigation, as the Iberian used to be great sailors; such elements are seen in monasteries decorations, such as the Jeronimos monastery in Lisbon (Fig.1);

Fig. 1 Jeronimos Monastery of Lisbon [6]

- In medieval France, stranded columns and twisted banisters can be found in the furniture and decorations from the period of Henry the 2nd and Louis the 13th, in a topic to be taken over later on by the French Renaissance ï Fig.2;

Fig. 2 Furniture items with twisted columns

- In our country, the old Romanian style with elements typical of Moldova (Fig.3) and Wallachia we find carved stone, having twisted rope shapes. In Putna Monastery, for instance, the climax of such elements will be met with the Brancovan style buildings (Fig.4).

Fig. 3 Putna monastery spire

Fig. 4 Brancovan detail

If during the ancient and medieval times these shapes were used on the basis of intuition, in an empirical manner, with a minimal geometrical support. In contemporary times, the study of such surfaces with the help of mathematics [3], [5] and computer modelling [2] - by using performant softwares, has expanded their range of application.

2. APPLICATIONS OF HELICOIDAL AND TWISTED SURFACES

The most often met surfaces included in this category are the helicoidal surfaces, which are generated by the motion of a segment of a line called directrix curve generatrix, of which one is a helix. This can be cylindrical, conical or spherical.

In the family of helicoidal surfaces, two main types can be distinguished [1]:

- straight, where the generatrix moves along two directing lines, a curve one - the helix and a straight one ï the cylinder axis, in an angle of 90^o;
- oblique (or askew), generated either by straight lines displacing along the two curves mentioned earlier at an angle < 90^o, or by straight lines that remain permanently tangent to the points of the cylindrical helix and displace along two directing curves that are not situated in the same plane.

The helicoidal surfaces have applications in engineering in thread construction, obtained by moving a plane figure on the surface of a cylinder following the rules previously mentioned. Among the applications of helicoidal surfaces one can mention the square and rectangular thread, and among the oblique surface applications we can exemplify, the triangular and trapezoid thread.

Spiralled surfaces are found by the mixed motions of translation and rotation of a plane figure about a rectilinear or curved axis, or a real or imaginary surface, called core. The plane figure can be a triangle, square, pentagon, hexagon, circle, etc.

2.1 Helicoidal ramps and flights of stairs

In civil engineering, the most frequently met applications of these surfaces are the helicoidal flights of stairs and access ramps for cars.

In Fig.5, are presented two types of ramps: Fig.a - one way ramp and Fig b ï two ways ramp, respectively. The traffic ramp is, in fact, a helicoidal surface with a horizontal director plane, whose generation in MatLab can be seen in Fig. 6.

Fig. 5 Helicoidal ramps [7], [8]

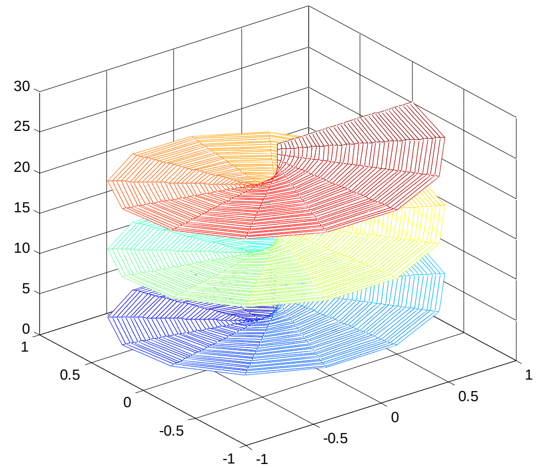

Fig. 6 Generation of a ramp in MatLab

Other helicoidal cylindrical flights of stairs are presented below, with the horizontal director plane perpendicular to the axis of the directing helix. It is noticed that at the bottom or intrados there is a helicoidal surface acting as a director plane perpendicular to the axis.

Fig.7 represents a helicoidal flight of stairs without a core, and Fig.8 shows the building in ArhiCad of a helicoidal flight with core whose mathematical generation can be seen below.

Fig. 7 The helicoidal coreless flight of stairs [9]

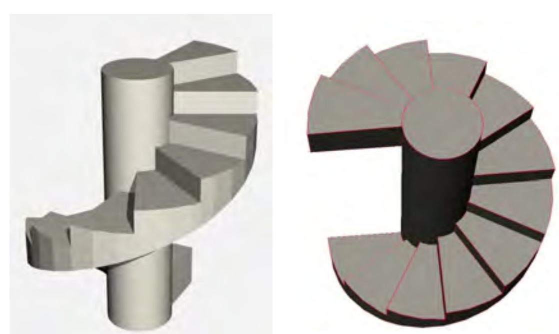

Fig. 8 The helicoidal flight of stairs with core

In this way, we present aspects concerning the geometry of helicoidal flights of stairs having a core in the continuous ramp form, and respectively a ramp interrupted by an in-between stairhead at the middle of the ramp.

a) Continuous helicoidal flight of stairs

Consider the linear space R^3 and the cartesian system of coordinated Oxyz, together with the canonical basis {i,j,k}. First, the position of the spiral staircase, is given as is drawn in Fig. 9.

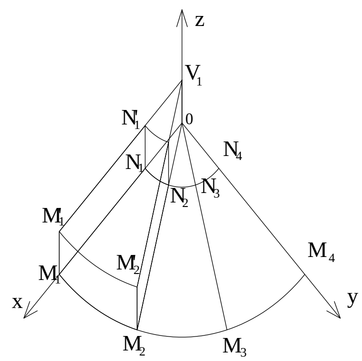

Fig. 9 The geometry of a circular helicoidal stair

The circle (C) is divided in 12 equal parts, starting from the axis Ox and going counterclockwise.

The angle is taken $\theta \in [0, \frac{\pi}{6}]$, and consider a cylinder with the basis (c).

$$\text{(C)}: \; x^2+y^2=R^2, \; z=0; \qquad (1)$$

$$\text{(c)}: \; x^2+y^2=r^2, \; z=0. \qquad (2)$$

Having k=12 stairs which are following a helix of the radius R and the height H=12h, a single stair surface is consisted by:

$$S_{bottom}: \begin{cases} x = R\cos[\theta + (k-1)\frac{\pi}{6}] \\ y = R\sin[\theta + (k-1)\frac{\pi}{6}]\,; \\ z = 2(k-1) \end{cases} \qquad (3)$$

$$S_{top}: \begin{cases} x = R\cos[\theta + (k-1)\frac{\pi}{6}] \\ y = R\sin[\theta + (k-1)\frac{\pi}{6}]\,; \\ z = 2k \end{cases} \qquad (4)$$

$$S_{cil1}: \begin{cases} x = R\cos[\theta + (k-1)\frac{\pi}{6}] \\ y = R\sin[\theta + (k-1)\frac{\pi}{6}]\,; \\ z = h + 2(k-1) \end{cases} \qquad (5)$$

$$S_{cil2}: \begin{cases} x = r\cos[\theta + (k-1)\frac{\pi}{6}] \\ y = r\sin[\theta + (k-1)\frac{\pi}{6}]\,; \\ z = h + 2(k-1) \end{cases} \qquad (6)$$

Here, k= 1,2,...,12, and it is considered $h \in [0,2]$ and $\theta \in [0, \frac{\pi}{6}]$.

For each k we have a different stair. Each stair is limited by two planes from the pencil of planes (the first stair by P_1 and P_2, the second by P_2 and P_3, etc.).

$$P_k : \sin(k-1)\frac{\pi}{6}x - \cos(k-1)\frac{\pi}{6}y = 0\,,\ \text{k=1,2,é ,12.} \qquad (7)$$

The parametric equations of the exterior helix, which is following the stairs, is:

$$\begin{cases} x = R\cos\theta' \\ y = R\sin\theta' \;,\; \theta' \in [0, 2\pi) \\ z = b\theta' \end{cases} \qquad (8)$$

The helix starts at the point $M_1(R,0,0)$ and has the height H=2´ b. Also, the curvature, *k,* of the helix and the torsion, τ , are given by the classical expressions:

$$k = \frac{R}{R^2 + b^2} \qquad (9)$$

$$\tau = \frac{b}{R^2 + b^2}. \qquad (10)$$

The osculating plane of the helix is:

$$\begin{vmatrix} x - R\cos\theta' & y - R\sin\theta' & z - b\theta' \\ -R\sin\theta' & R\cos\theta' & b \\ -R\cos\theta' & -R\sin\theta' & 0 \end{vmatrix} = 0 \qquad (11)$$

The minimal surface of the helix is a helicoid.

b) Helicoidal flight of stairs with in-between stairhead

Consider now, the geometry of a plane stair having an intermediate landing as in Fig.10

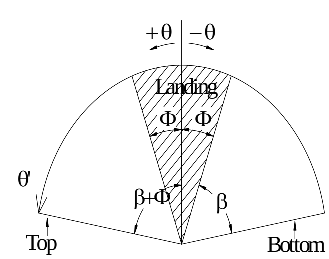

Fig. 10 The geometry of a circular helicoidal stair with an intermediate landing at mid-span (in the plane)

The coordinates at the mid-surface are:

$$x = R\cos\theta$$
$$y = R\sin\theta'$$
$$z = \theta' R\tan\alpha\ ,\ \theta' \in [0, \beta]$$
$$z = \beta R\tan\alpha\ ,\ \theta' \in [\beta, \beta + 2\Phi]$$
$$z = (\theta' + 2\Phi)R\tan\alpha\ ,\ \theta' \in [\beta + 2\Phi, 2\beta + 2\Phi]$$

$$\text{with}\quad \alpha = \tan^{-1}\frac{H}{R2\beta} \qquad (12)$$

Here, dô is the variable angle, ʌ the angle subtended at the centre by half landing, H is the height of the helicoid and R is the centerline radius on horizontal projection.

2.2 Twisted buildings

The present paragraph discusses the geometry of several twisted, rotated buildings, seen as reference points in the contemporary architecture of the last decade.

The torsioned tower called Turning Torso from Malmö, Sweden is the first high rise building in the form of a spiral ever built in the world. The tower was designed by the Spanish architect Santiago Calatrava, who was inspired in his work by his own sculpture (Fig.11); it was called the Twisting Torso and it mimics the twisting of the human body [4].

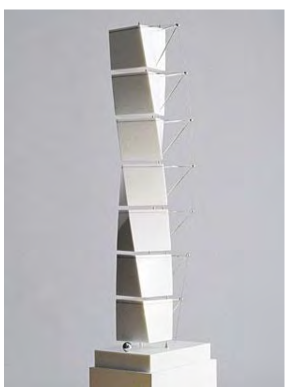

Fig. 11 Twisting Torso [12]

The Turning Torso (Fig. 12) is the highest building in Sweden taking the third place in Europe with its 190 meters. The irregular pentagon in plane has a rotation of 90^{o} between the first and last level. The building is made up of nine modules twisted around a core. Every module has the upper basis twisted by $10^{\ 0}$ relative to the lower basis. The first plane of the upper module overlaps the last plate of the lower module and a certain volume is left between them. The 54 floors host 147 private flats, relaxation and spa rooms as well as a winery.

In Fig. 13, one gives the generation in 3D Max of the volumes forming the tower. It is noticed that, the horizontal projection of every module is twisted by 10^{o} with respect to the previous module and that finally, the rotation reaches 90^{o}.

Fig. 12 Torso Tower, Malmo [10]

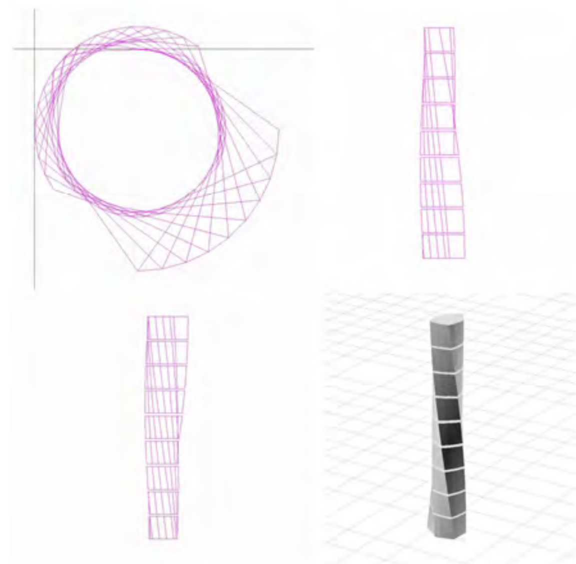

Fig. 13 3D generation of the Torso Tower

Similar to Turning Torso, also with a 90^{0} twist between ground floor and the last floor is the Infinity Tower of Dubai or the Cayan Tower. This is the tallest twisted block in the world, with its height of 306 meters, its 80 floors and five underground parking levels. The plane shape of a level consists in joining two rectangular straight figures rotated to each other ï Fig. 14. Every floor is rotated by 1.2° to get a final rotation of 90°, giving the impression of a helix.

Different from the Turning Torso Tower, made up of successive modules incorporated in the beam around a fixed core, the Infinity Tower has a larger horizontal section and therefore, requires plates to twist at every level as the building gets taller and taller.

The building was designed by the architectsô team Skidmore, Owings & Merrill, which also contributed to the design of the famous tallest building Burj Khalifa (Dubai) and of the Trump Tower (Chicago). The spiral shaped tower is a luxury residential, with 80 floors, housing one or more room flats, and penthouse flats with a panoramic view of the sea.

Fig. 14 Infinity Tower level plan [11]

Fig. 15 Infinity Tower Dubai

In Fig. 16, the generation with the 3DMax software of the Infinity Tower is given.

The Revolution Tower of Panama (Fig.17), also known as F&F Tower, was designed by Pinzón Lozano & Asociados. This office building with 242.9 meters height, has 52 floors above the ground and 4 floors under the ground. The first 13 levels above the ground are used as parking areas, the rest of 39 levels are forming a surface obtained by rotating a square shaped plane (Fig.18) section around a central axis, so that from the bottom to the top a full rotation of 360 degrees is achieved. The shape of a helix is thus created.

The tower is composed of rotated volumes. Every volume represents a floor limited between two plates, the lower plate of the upper floor supporting on the upper plate of the previous level and rotating by an angle of 10º. The difference of the two rotated plates leads to a plate surface limited by a parapet and this gives the impression of an intersection of volumes.

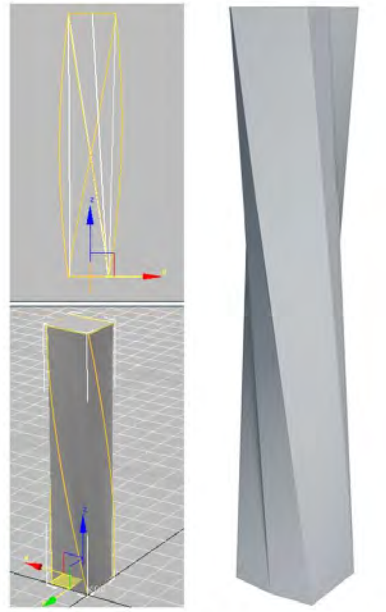

Fig. 16 3D generation of the Infinity Tower

Fig. 17 Revolution Tower, Panama [10]

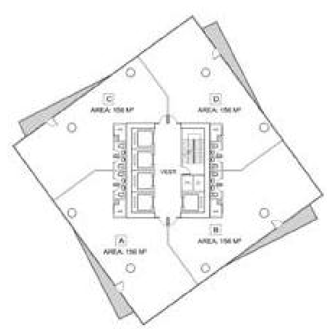

Fig. 18 Level plane of the Revolution Tower

In Fig. 19 can be seen the tower generation stages and the final result, as seen from two points of view.

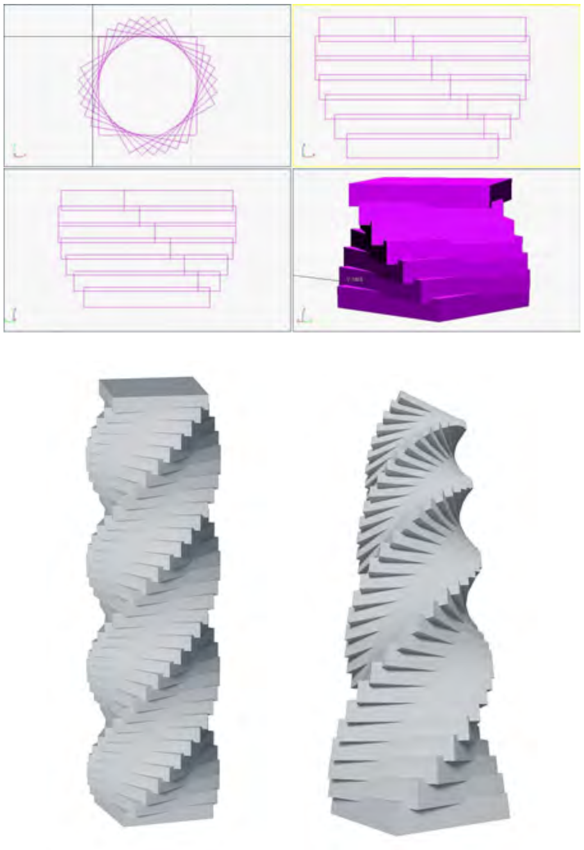

Fig. 19 3D generation of the Revolution Tower

3. CONCLUSION

In this paper, the authors make a brief presentation of the history of using curves and spiral, respectively helicoidal surfaces in various fields. Their focus regards, however, civil engineering field.

Among the contributions of the paper, we can notice:

- the representation of the helix traffic ramp with the help of the MatLab software;
- the geometrical determination of a helix flight of stairs with a central core in the forms of a continuous ramp and in-between stairhead; the flight was represented with the help of ArhiCAD software;
- the study of the geometry of several existing buildings of reference, for which various graphical simulations were also made.

The imagination of the architects and their desire to produce unique works is today possible due to the high performance of the materials and equipment used in constructions, as well as computer modelling.

REFERENCES

[1] Gogu, M., Olariu, F. (1999). *Descriptive Geometry (Geometrie descriptivă),* UTCN, C.Z.U. 514.14:744(075.8), Cluj-Napoca.

[2] Kilibarda, L., Miloġeviĺ, M., Tica, L., Dragoviĺ, M. (2014). *Geometric 3d models of spiral structures inspired by contemporary architecture.* Proceedings of 4th International Scientific Conference on Geometry and Graphics moNGeometrija 2014, Volume 1: Geometry and Graphics applied in Engineering and Architecture, pp. 13-23, ISBN 978-86-88601-13-9, Vlasina, Serbia, June 2014, Faculty of Civil engineering and Architecture in Niġ.

[3] Patriciu, A., M. (2010). *On some H3 - helicoidal surfaces and their parallel surfaces at a certain distance in 3 - dimensional Minkowski space*, Annals of the University of Craiova, Mathematics and Computer Science Series Volume 37(4), 2010, Pages 93-98, ISSN: 1223-6934.

[4] Randelovic, D. (2012). *Surface geometry as an impression of contemporary architectonic forms*, Proceedings of 3rd International Scientific Conference on Geometry and Graphics moNGeometrija 2012, pp. 213-221, ISBN 978-86-7892-405-7, Novi Sad, Serbia, June 2012, Faculty of Technical Sciences Novi Sad.

[5] Wadud, Z., Ahmad, S.(2005), Simple *design charts for helicoidal stair slabs with intermediate landings*, Journal of the Institution of Engineers (India), Vol. 85, No. CV 4, February, pp 269-275.

[6] http://commons.wikimedia.org/wiki/File:JeronimosCloister.jpg

[7] http://natureobservationtower.weebly.com/

[8] http://www.arcspace.com/features/alberto-campo-baeza-/ma-andalucias-museum-of-memory/

[9] http://www.rizziscale.com/eli-le-06-210

[10] http://en.wikipedia.org/wiki

[11] http://imgarcade.com/1/infinity-tower-plan/

[12] http://archidialog.com/tag/twisting-male-torso-sculpture/

Author(s):

Assoc. Prof. Ph.D. Eng. Carmen MÂRZA, Director of Department, Technical University of Cluj-Napoca, Department of Building Services, E-mail: Carmen.Marza@insta.utcluj.ro, tel.0040264202519;

Assoc. Prof. Ph.D. Dalia CÎMPEAN, Technical University of Cluj-Napoca, Department of Mathematics, E-mail: Dalia.Cimpean@math.utcluj.ro, tel.0040264401535

Eng. Drd. Georgiana CORSIUC, Technical University of Cluj-Napoca, Department of Building Services, E-mail: Georgiana@mail.utcluj.ro, tel.0040264202519.