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CYLINDRICAL-CONICAL SURFACES INTERSECTION USING AUXILIARY SPHERES METHOD WITH MOBILE CENTER

Abstract: *The paper presents: intersection between two cylinders - one is a rotation cylinder and the other is an oblique circular cylinder; intersection between a rotation cylinder with an oblique circular cone. This method can be applied if the axes of the two surfaces are coplanar. The paper also presents the equations of the intersection curves which become simpler when the two surfaces are rotation cylinders. In this case the auxiliary spheres center is fixed at the axis intersection of the two rotation cylinders.*
The auxiliary spheres method with mobile center can be applied when a rotation cylinder is intersected with a torus or an oblique cone. The method is often used because it is rapid and allows intersection curve drawing using only a projection of the two surfaces.

Key words: *intersection, auxiliary spheres method, cylinder, cone, torus.*

1. INTRODUCTION

The intersection between two rotation surfaces (especially cylinders and cones) with intersected axis and parallel with a projection plane is rapidly drawn only in one projection, using the auxiliary spheres method.

This method is possible because a sphere intersects a rotation cylinder or cone by two circles, if the sphere center is placed on a cylinder or a cone axis (Fig.1 and 2)

Figure 3 represents cone-cylinder intersection (the cone gets into the cylinder), both are rotation surfaces with intersected axis, perpendicular and parallel with projection vertical plane.

Rs_2 radius sphere intersects cylinder by C_1 circles and cone by circle C_2. The circles C_1 and C_2 intersect in double points 1 and 2 of intersection curve (two points are on invisible side of intersection curve).

We can find the intersection curve minimum point 3, by using Rs_1 radius sphere, tangent to the horizontal cylinder.

Figure 4 represents cylinder-cone intersection (the cylinder gets into the cone).

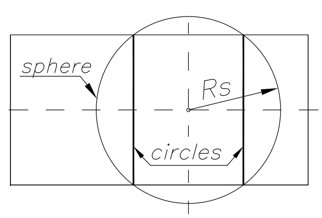

Fig. 1 Intersection sphere-cylinder

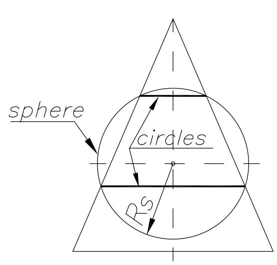

Fig. 2 Intersection sphere-cone

An oblique circular cylinder intersects with a sphere by two anti-parallel sections circles (Fig.5). In this case, the sphere center is not placed on cylinder axis but in perpendiculars intersection drawn through the center of the two anti-parallel section circles, on the circles plane.

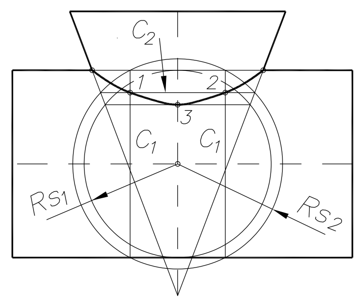

Fig. 3 Intersection rotation cone-cylinder

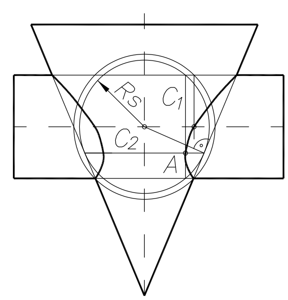

Fig. 4 Intersection cylinder-rotation cone

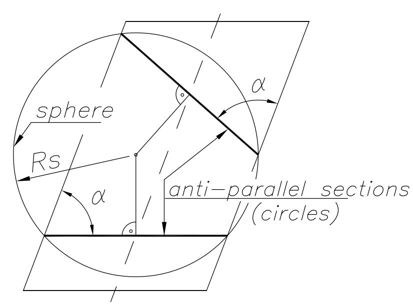

Fig. 5 Intersection sphere-oblique circular cylinder

When a rotation cylinder intersects a circular oblique cylinder, the auxiliary spheres centers must be placed on the rotation cylinder axis, so these get mobile centers.

2. ROTATION CYLINDER INTERSECTION WITH OBLIQUE CIRCULAR CYLINDER

Figure 6 represents two cylinders with intersected axis and parallel with projection vertical plane.

The center of R_S radius sphere is placed on the rotation cylinder axis which is *Ox* axis too. It intersects the two cylinders by C_1 and C_2 circles. C_1 and C_2 circles intersect in *A* points (one of them is placed on the invisible side of the intersection curve).

In figure 6 we can calculate *A* point co-ordinates:

$$x = a \pm \sqrt{R_s^2 - r^2} \tag{1}$$

$$y = atg\alpha \tag{2}$$

but $a = y / tg\alpha$

and $R_s^2 = R^2 + y^2$

In this case the equation (1) is:

$$x = \frac{y}{tg\alpha} \pm \sqrt{R^2 + y^2 - r^2} \tag{3}$$

After calculations, we finally obtain [3]:

$$(x - \frac{y}{tg\alpha})^2 - y^2 = R^2 - r^2 \tag{4}$$

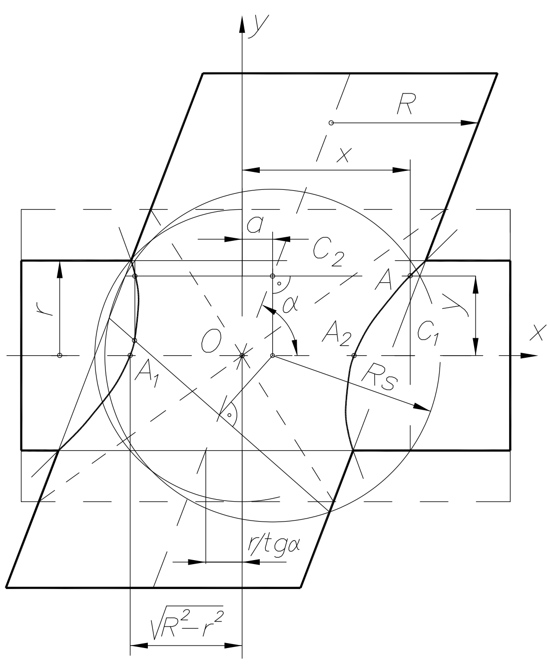

Fig. 6 Intersection rotation cylinder - oblique circular cylinder

The equation (4) is an hyperbole, its branches are symmetrical in comparison with origin *O*.

If the angle $\alpha = \pi / 2$, then $tg\alpha \to \infty$.

The oblique circular cylinder turns into a rotation cylinder and the equation (4) is:

$$x^2 - y^2 = R^2 - r^2 \tag{5}$$

The equation (5) is an equilateral hyperbola , its branches are symmetrical in comparison with *Oy* axis.

If in equation (4) *R=r* we obtain:

$$[x - y(1 + \frac{1}{tg\alpha})][x + y(1 - \frac{1}{tg\alpha})] = 0 \tag{6}$$

The equations (6) are two straight lines which pass through the origin *O* and the asymptotes of the hyperbole (4) at the same time.

For $y = 0$ the equation (4) is:

$$x = \pm\sqrt{R^2 - r^2} \tag{7}$$

that is the co-ordinates of the A_1 and A_2 points on *Ox* axis.

For $y = \pm r$ formula (4) is:

$$x = \pm R \pm \frac{r}{tg\alpha} \tag{8}$$

that is the co-ordinates of the intersection points of the apparent edge generatrix.

If *R<r* then the oblique circular cylinder in figure 6 gets into the rotation cylinder and the relation (4) is:

$$y^2 - (x - \frac{y}{tg\alpha})^2 = r^2 - R^2 \tag{9}$$

The relation (9) is a hyperbole,its branches are symmetrical in comparison with the origin *O* (Fig.7). It is orientated along *Oy* axis and has the same asymptotes in relation (6).

The relation (9) differential in comparison with the variable *y* is:

$$2yy' + 2(x - y / tg\alpha)(1 - y' / tg\alpha) = 0 \tag{10}$$

or: $$y' = \frac{(xtg\alpha - y)tg\alpha}{y + ytg^2\alpha - xtg\alpha} \tag{11}$$

For $y' = 0$ we obtain:

$$y = xtg\alpha \tag{12}$$

that is the hyperbola minimum point is placed on the oblique cylinder axis. In this case the minimum point value is:

$$y_{\min} = \pm\sqrt{r^2 - R^2} \tag{13}$$

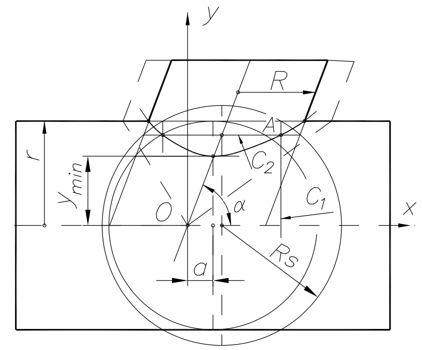

Fig. 7 Intersection oblique circular cylinder-rotation cylinder

If $R>r$ and $\alpha = \pi/2$, the relation (9) is:

$$y^2 - x^2 = R^2 - r^2 \tag{14}$$

Relation (14) is the equation of an equilateral hyperbola with Oy axis as its symmetry axis (Fig.8).

In figure 8 is represented only a hyperbola branch, it is the vertical projection of intersection curve between the two rotation cylinders.

For $R=r$ relation (14) is:

$$(y-x)(y+x)=0 \tag{15}$$

The straight lines $y=x$ and $y=-x$ are equilateral hyperbola asymptotes in figure 8 [4].

On the other hand, the straight lines $y=x$ and $y=-x$ are vertical projections of the intersection curves (two semi-ellipses perpendicular on the projection plane), between two rotation cylinders with the same diameter, their axis are intersected, perpendicular and parallel with the vertical plane.

The intersection between Rs radius sphere and the two rotation cylinders are the circles C_1 and C_2.

The circles C_1 and C_2 intersect in points A and B (two of the points are placed on invisible side of the intersection curve)

The minimum point of the intersection curve is obtained in equation (14), for $x=0$:

$$y_{\min} = \pm\sqrt{R^2 - r^2} \tag{16}$$

Relations (13) and (16) shows that the minimum point has the same value no matter the r radius cylinder which gets into the R radius cylinder is oblique circular or a rotation cylinder and the α angle value is variable.

The ± signs in relations (13) and (16) are used because of the existence of the two intersection curves.

The intersection in figure 8 is often used in different fields: industry, sanitary installations etc.

Figure 9 represents a connection part (T-part for connection)[5].We can notice the intersection between the two interior cylindrical surfaces, the two semi-ellipses in a vertical projection degenerated into two straight lines.

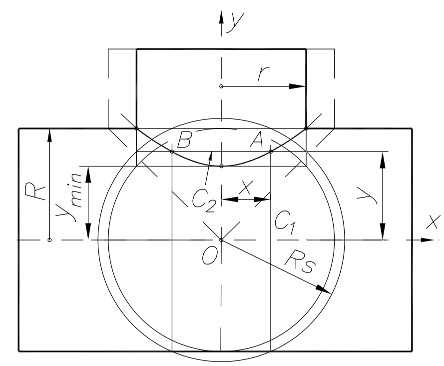

Fig. 8 Intersection of two rotation cylinder

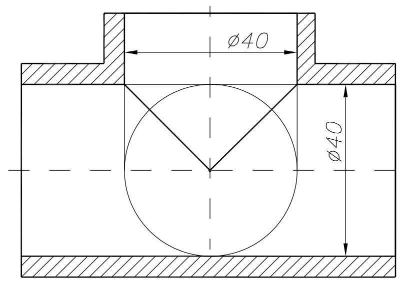

Fig. 9 Intersection of two rotation cylinder (application)

3. OTHER INTERSECTIONS USING AUXILIARY SPHERES METHOD WITH MOBILE CENTER

In figure 10, the oblique circular cylinder gets into a rotation cone. The intersection between Rs radius sphere and the two surfaces are the circles C_1 and C_2. The two circles intersect in point A . The center of the sphere is placed at the intersection of the perpendicular drawn through the center of the C_2 circle on its plane, with the cone axis.

Other intersection points can be found similarly.

In figure 11, the rotation cylinder gets into an oblique circular cone. The intersection between Rs radius sphere and the two surfaces are the circles C_1 and C_2 .The two circles intersect in point A .The center of the sphere is placed at the intersection of the perpendicular drawn through the center of the C_2 circle on its plane, with the cylinder axis.

Other intersection points can be found similarly.

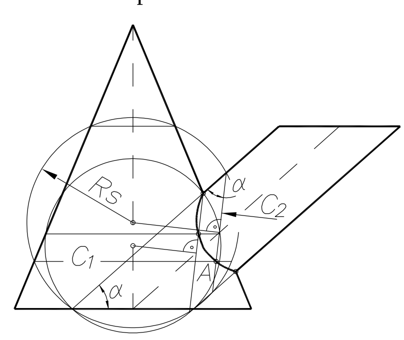

Fig.10 Intersection rotation cone-oblique circular cylinder

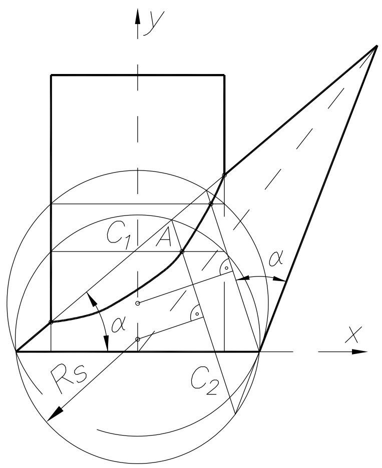

Fig. 11 Intersection rotation cylinder-oblique circular cone

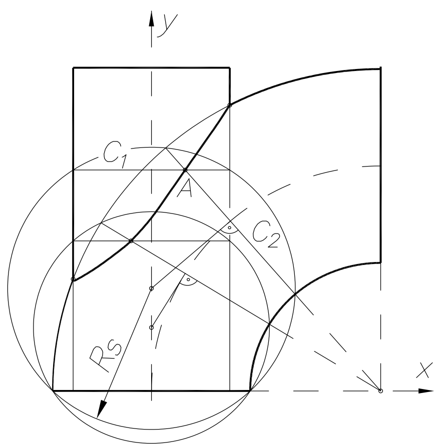

Fig.12 Intersection rotation cylinder-torus

In figure 12, the rotation cylinder gets into a torus.

The intersection between *Rs* radius sphere and the two surfaces are the circles C_1 and C_2 .The two circles intersect in point *A* .The center of the sphere is placed at the intersection of the perpendicular drawn through the center of the C_2 circle on its plane, with the cylinder axis. The C_2 circles are different positions of the torus generating circle [1].

Other intersection points can be found similarly.

4. CONCLUSION

The grapho-analytical studies of cylindrical surfaces intersections by using the auxiliary spheres method with mobile center, proves its validity as well as in general case (when one of surface is a rotation surface) and in particular case (when both surfaces are rotation surfaces)

The auxiliary spheres method with mobile center is often used because it is rapid and allows intersection curve drawing using only a projection of the two surfaces. It can also be applied:

-when an oblique circular cone or a torus intersects a rotation cylinder.

-when an oblique circular cylinder or a torus intersects a rotation cone.

The paper is interesting from a didactical point of view (due to its many thoroughly studied and presented cases) as well as from a practical point of view.

A mathematical algorithm can be conceived using formula (4) for cylindrical-conical surfaces automatic intersection. The parameters *R* and *r* can be considered constant and α , variable.

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