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AN APPLIED METHOD TO SOLVE SOME TANGENT PROBLEMS

Abstract: *In descriptive geometry tangent problems are solved by using specific methods such as: changing the projection plans method or the rotation method. In some cases, solving such problems is not quite facile, for it involves the use of multiple changes of projection plans or rotations. Thus, another method, defined generically as the "trajectory method", has been widely considered as a highly relevant method, especially if we have different geometric surfaces. Within this present paper we envisage the implementation of the "trajectory method" in order to solve some tangent problems while applying graphical and analytical solutions.*

Key words: *tangent problems, trajectory, geometric surfaces.*

1. INTRODUCTION

Tangent problems are of salient and complex importance in almost all scientific domains. Thus, the problem that rises in most fields of research, from mechanical engineering to architecture, aims at determining the contact points between different types of surfaces, i.e. the determining of the tangent point.

Hence, tangent problems are to be encountered in the field of medicine as well. The study and the modelling of the phenomena occurring at the contact between the different bones of the human skeleton is of paramount importance in designing the geometry of some the prostheses and other medical devices used in bone reconstruction. [6]

Within this context, tangent problems can be solved both by applying descriptive geometry graphical methods as well as by means of software such as SolidWorks software and even analytically.

ñé Descriptive Geometry is a method to study 3D geometry through 2D images. It provides insights into structure and metrical properties of spatial objects, processes and principles. Typical for Descriptive Geometry is the interplay a) between the 3D situation and its 2D representation, and b) between intuitive grasping and rigorous logical reasoningé . As postulated by Monge, the two main objectives of Descriptive Geometry are the imaging and analysis of 3D objects, objectives which date back to its founder. These two central aims can also be found in recent encyclopaedias such as Brockhaus ñDescriptive Geometry = subject of mathematics. The aim of DG is the representation of 3D objects . . . as well as the interpretation of given images . . . ñ. However, the importance of images cannot be overestimated. They contain highly compressed information: which often cannot be delivered unambiguously in words or ð conversely; the same information expressed in words needs maximum concentration to be understood, though truly understandable all over the word. [5]

By solving the tangent problem, we can determine the optimum in the case of volume positioning, the optimum with respect to the use of available space. [4]

The challenging issue most frequently encountered regards the situation where all bodies are placed identically and symmetrically as indicated in figure 1, a situation which is straightforwardly solved and easily predicted by students. [7]

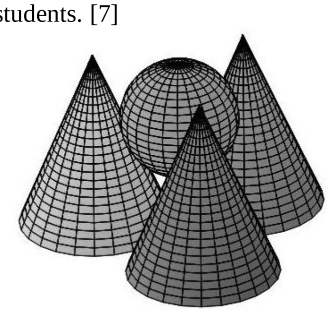

Fig. 1. Determining the tangent position of a sphere with three identical cones [7]

A first approach of the present paper envisages the tangent problem in the situation of three different right circular cones of different dimensions with a sphere of a known radius. Hence, this approach can be generalized, for as it can be applied to other geometric forms as well, all of different forms. The second case to be investigated focuses on a cylinder, a cone and a hemisphere, all of different dimensions.

2. SOLVING THE TANGENT PROBLEM BY MEANS OF DESCRIPTIVE GEOMETRY

Generally, tangent problems are solved by means of the well-known methods of descriptive geometry. Nonetheless, there are situations in which the uses of such methods prove to be problematic. [1], [3].

For the situation presented in the present paper we will apply such a rational in order to find a rapid solution, which can be defined as a ñmethod of trajectoriesò.

How could we define this method and why would it be much easier to apply it in order to solve the problem graphically?

First, this method is more convenient as the tangent problem can be solved by means of only two projections without using other descriptive geometry methods. Thus, we start from the determination of the trajectories where a sphere is ñmovingò on condition that the tangent restriction is being retained to one cone.

In order to draw a vertical trajectory where the centre of a sphere, which maintains its tangency condition, would move, we have used several level plans. Basically, particularly useful are those plans with a rate higher than the radius of the sphere, hence, for a more accurate determining, we do not exclude the other plans either. (Fig.2.)

Thus, two trajectories are obtained T1-2 ï for the tangent to cones 1 and 2, and T2-3 ï for the tangent to cones 2 and 3. Accordingly, the solution of problem will be revealed by the intersection of the two trajectories.

The tangent points are to be observed only in those planes containing the axis of the cone and the centre of the sphere, i.e. the plans [P1], [P2] and [P3], while the visualizing of the tangent points will be achieved by using changing planes projection method.

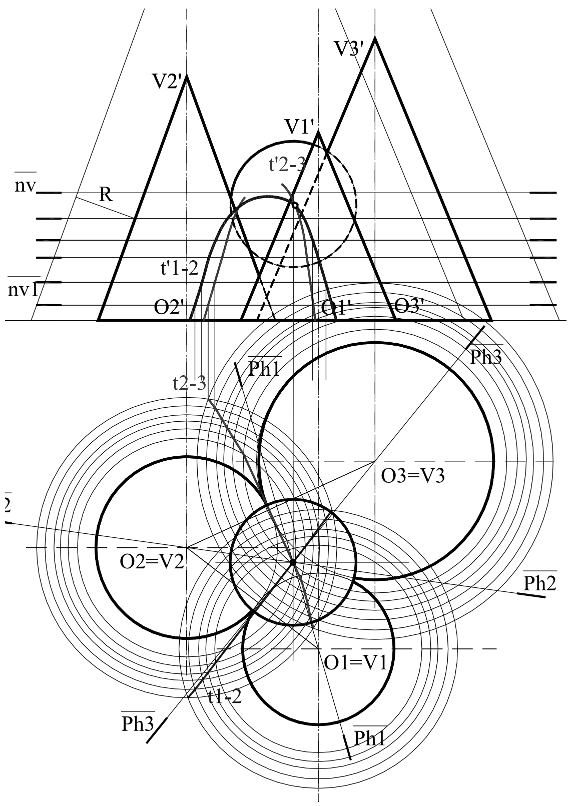

Fig. 2. Determining the positioning of a sphere tangent to three different cones

3. SOLVING THE TANGENT PROBLEM BY MEANS OF COMPUTER-ASSISTED GRAPHICAL METODS

If applying a computer-assisted graphical solution by means of the graphical modelling software Solid Works, we start from a simple geometrical modelling of elementary geometric bodies to which, subsequently, we impose the restrictions required by the problem. [8]

Admittedly, the solution of the problem is reduced practically to the identification of the junction of the three cones.

For each concentric cone we have considered an initial cone of r*, h* dimensions, as in Figure 3, equations (1), (2) and (3) indicating the area on which the sphere centre of radius R could be found in any position tangent to the cone.

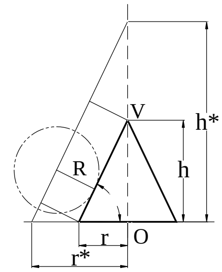

Fig. 3. The geometrical elements of the cone

The radius of the cone is obtained by:

$$r^* = r + \frac{R}{\sin\alpha}, \tag{1}$$

The cone height is:

$$h^* = h + \frac{R}{\sin(90-\alpha)} \tag{2}$$

where:

- R ï the sphere radius;
- r ï the cone radius;
- h ï the cone height;
  - Ůï the angle between the apparent generator in the frontal plane and the base of the cone

$$\alpha = arctg\frac{h}{r} \tag{3}$$

Figures 4, 5 and 6 comprehensively indicate the assembly of the geometric units generated by the data of the problem.

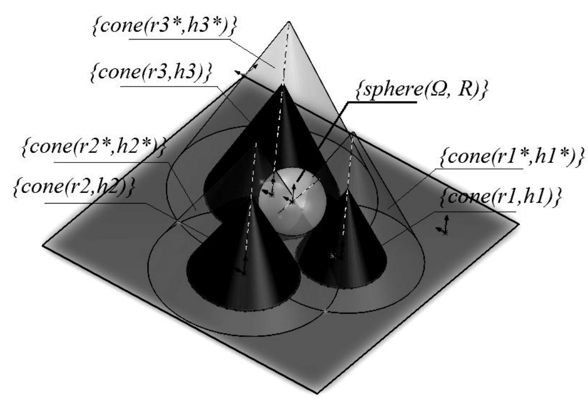

Fig. 4. The tangent problem graphical solution

Hence, the intersection of the two cones determines the two curves on which the centre of a sphere, that simultaneously satisfies the tangent requirement, is placed.

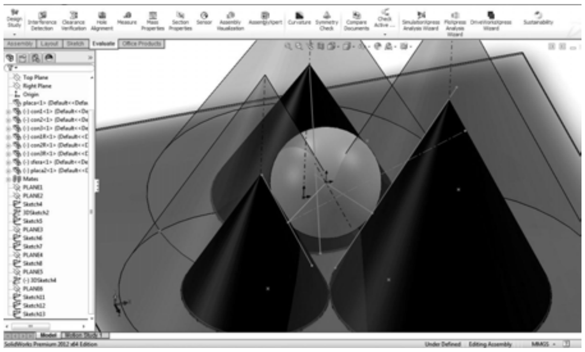

Fig. 5. The tangent problem graphical solution in Solid Works main window

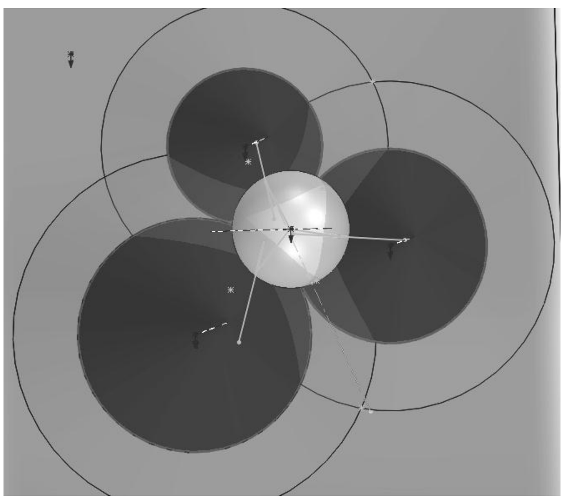

Fig. 6. Overview of the intersection of the three conesô blades, he positioning of the sphere centre

Figure 7 highlights the position of the sphere centre placed at the intersection of the two curves, the sectioning plane containing the axis of the cone 2 and the centre of the sphere, thus the tangent between the two bodies can be observed.

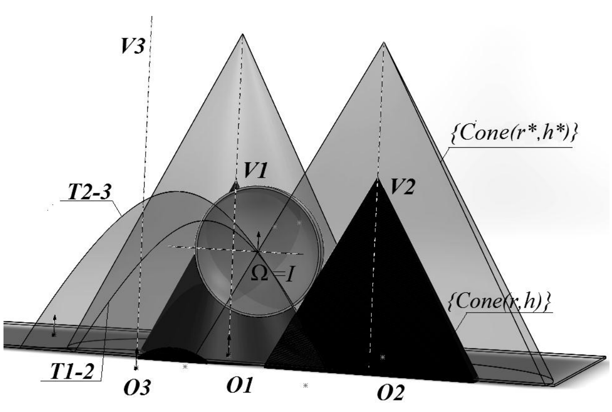

Fig. 7. Visualising the intersection curves of three cones

Similarly to the situation where despriptive geormety has been applied, the tangent points are placed on the conesô generators, i.e. in planes containing the cone axis and the sphere centre. (Fig.8)

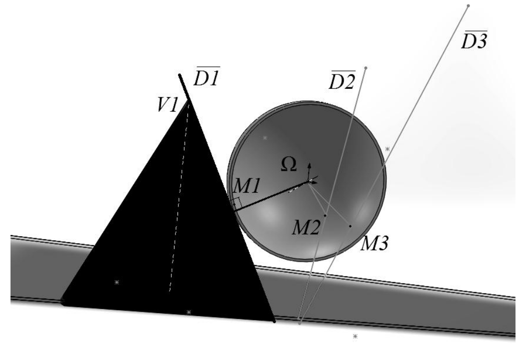

Fig. 8. Visualising the tangent point M1, M2 and M3 with the three cones provided

4. THE GENERALIZATION OF THE METHOD AND METHOD-TESTING BY MEANS OF THE COMPUTER-ASSISTED METHOD

As previously mentioned, we can assert that the method may be applied to other geometric bodies as well; Figure 9 exemplifies the situation in which a cone, a hemisphere and a cylinder of predefined dimensions are considered and whose positions are determined by the tangent conditions initially set. The position of the sphere centre, whose radius is known, can be established by applying the same method of ñtrajectoryò; taking into account the type of the surfaces placed in the centre of the sphere, provided the tangent conditions with each of the units.

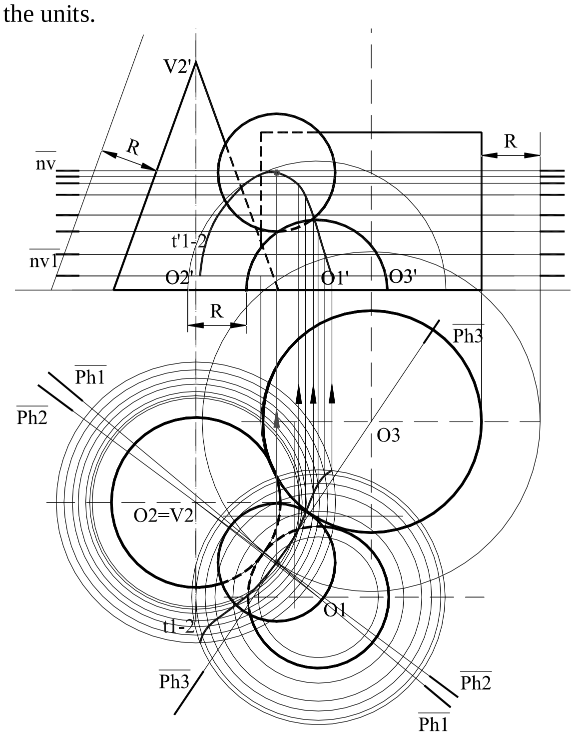

Fig. 8. Determining the position of a sphere tangent to three different geometric units

Subsequent to the modelling of the geometric units and the imposed restrictions, figure 9 illustrates the assembly obtained.

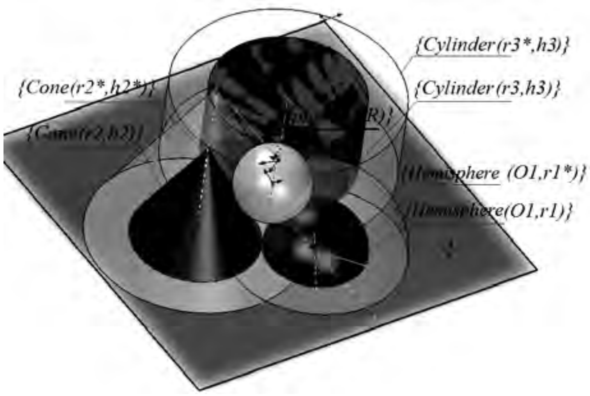

Fig. 9. The graphical solution obtained by means of Solid Works software

The sphere centre determination is located at the intersection three geometrical units derived from the original ones, whose dimensions are determined by the above-mentioned method. Figure 10 below indicates intersection point.

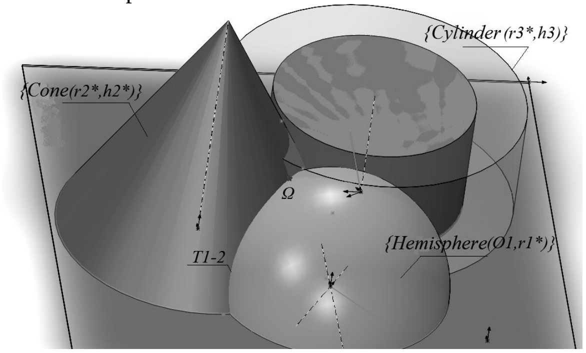

Fig. 10. Establishing the intersection point of the three geometric units

The tangent points can be seen in the plans [P1], [P2] and [P3], determined by the sphere centre and by each specific element of the geometric units considered. Figure 11 indicates the section of a plane containing the axis of the cylinder and the sphere centre, highlighting their tangent and their positioning on the intersection curve between the cone and the hemisphere.

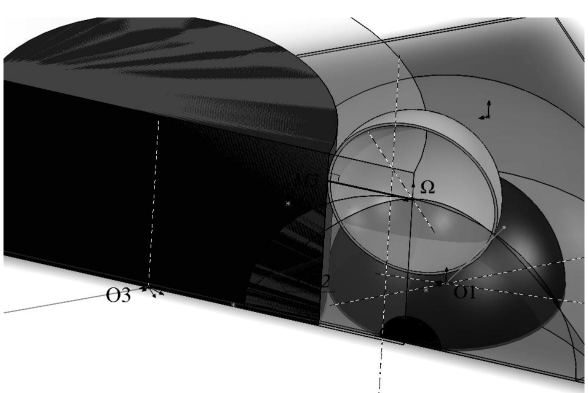

Fig. 11. The tangent between the cylinder and the sphere

5. CONCLUSION

The method envisaged within this present paper has proven to be relatively easy to apply and can provide the solution without resorting to other Descriptive Geometry methods. Consequently, the students will perceive much easier and faster the idea of moving on a trajectory as imposed by the fulfilment of the restrictions imposed by study data, thus aiming at establishing some connections with the technic environment and beyond.

Testing the solution by means of computer-assisted graphic methods validates its accuracy.

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