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CONSIDERATIONS ABOUT APPLICATION OF GRAPHICAL CALCULUS

Abstract: *Some typical engineering problems can be solved using graphical methods in an integrated way for knowledge of design, mathematics, mechanics and physics.*
This paper presents some graphical methods which improve student's performances on streigth of intuitive interpretation, visualization and understanding solutions of engineering problems.
The increased use of graphical methods adds another layer of interpretation to a given task requiring the coordination of different knowledge, in order to produce a correct solution and to improve the product design.

Key words: *graphical calculus, graphical differentiation, graphical integration, ray polygon method, tangent method, Simpson's rule.*

1. INTRODUCTION

Graphical calculus is an important chapter of technical drawing, storing and communicating essential information. There are many problems that cannot be solved analytically and have only graphical solution.

The use of graphical and semigraphical methods improves studentôs performances on strength of intuitive interpretation, visualization and understanding solutions of engineering problems.

However, graphical methods give approximate answers and the question of accuracy arises.

Neither graphical or algebraic methods can be used exclusively. Both methods must be familiar to the engineer, an integrated way is essential to improve his analyses and design solutions.

2. CONSIDERATIONS ABOUT GRAPHICAL AND SEMIGRAPHICAL METHODS

Referring to graphical differentiation, the slope law is a very used semigraphical methods and it is equivalent of differentiation of the calculus [1].

Since the differential calculus is focused on determination of the rate of change, this method is appropriate to finding the change of an physical quantity related to a second quantity different in size.

Graphical differentiation uses a common handling to drawing lines tangent to a given curve, with graphical or semigraphical methods.

Semigraphical methods of differentiating transfer the computed tangent values to the ordinate scale of the derived curve.

If the curve is circular, the point of tangency is on the perpendicular of the chord mediator and the tangent is parallel to the chord.

If the given curve is ellipse, parabola or hyperbola, the tangent point is the intersection of the curve and the midpoint of two parallel chords and the tangent line is parallel to the chords.

If the curve havenôt a known analytical form, drawing the tangent is more difficult and requires graphical solutions, of which methods of ray, string or funicular are more convenient.

3. APPLICATION OF GRAPHICAL DIFFERENTIATION

One of the must frequent tasks for students is the study of the motion of a cam.

We propose to find graphically the derived curve, using the ray polygon method, without involving computation of the slope values.

In figure 1 is shown a given curve which describe the motion of a cam, with a harmonic-rise and a parabolic fall.

First, we divided the given curve in short arcs, with equidistant ordinates.

The locations of the origin *O'* and of the pole point *P* are chosen convenient, below the first curve.

On the ordinate scale of the unknown derived curve, are plotted the slope values of tangent points *A, B,* é on the given curve.

The y value per unit x value are computed. If d is multiple of 1, the ordinates are proportionally reduced, to transfer the appropriate slope value.

The tangent points on the given curve is located and projected to the derived curve until the intersection of horizontals and the process is repeated.

The derivate curve is drawn like a smooth curve through the specific points.

The figure 2 shows the tangent method of determination of the tangent points and lines.

We have chosen for given curve a parabola. Rays are drawn from the pole *P* to tangent lines.

The points of the derived curve are determined by the vertical projections until the intersection to horizontal lines of the required curve.

The pole is at a distance of 3/2 unit scale. By succesive differentiation, the second derrived curve is the result of differentiation of the derrived curve.

In our application, the first derived curve is a inclined straight line, and the second is a horizontal.

For more clarity, the curves derived succesively can be drawn one of top of the other, even if each curve requires different scales.

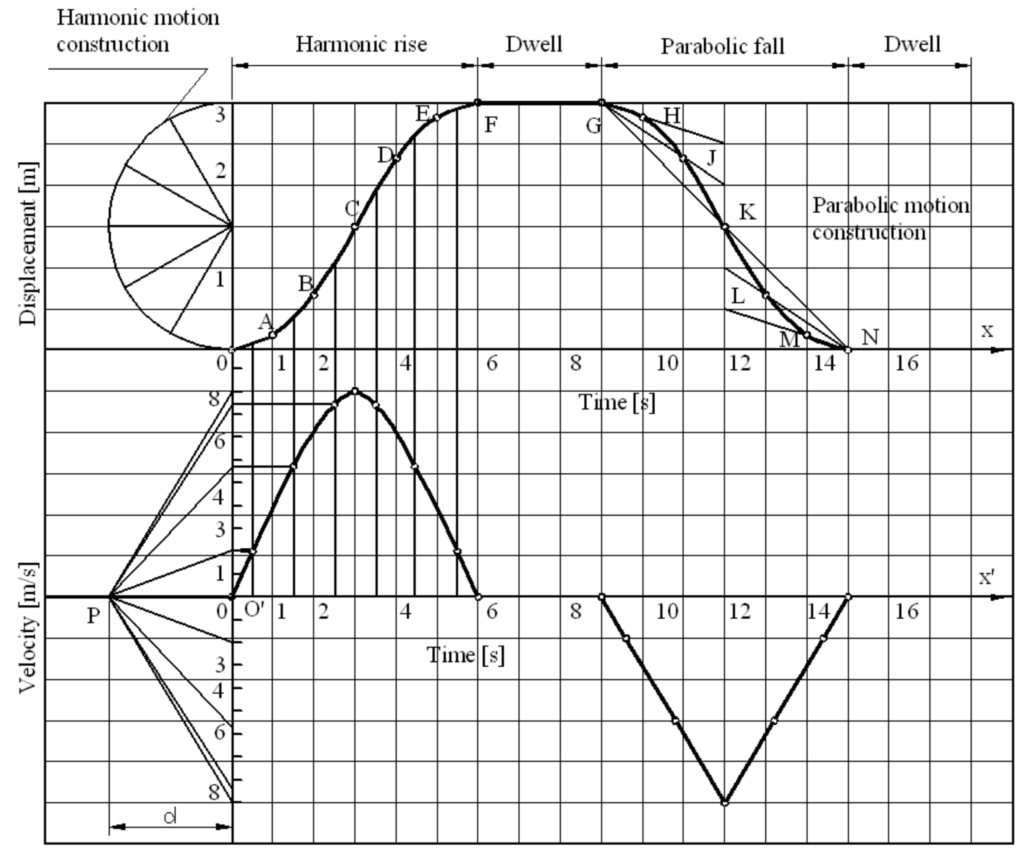

Fig. 1 Application of ray polygon method to the motion of a cam

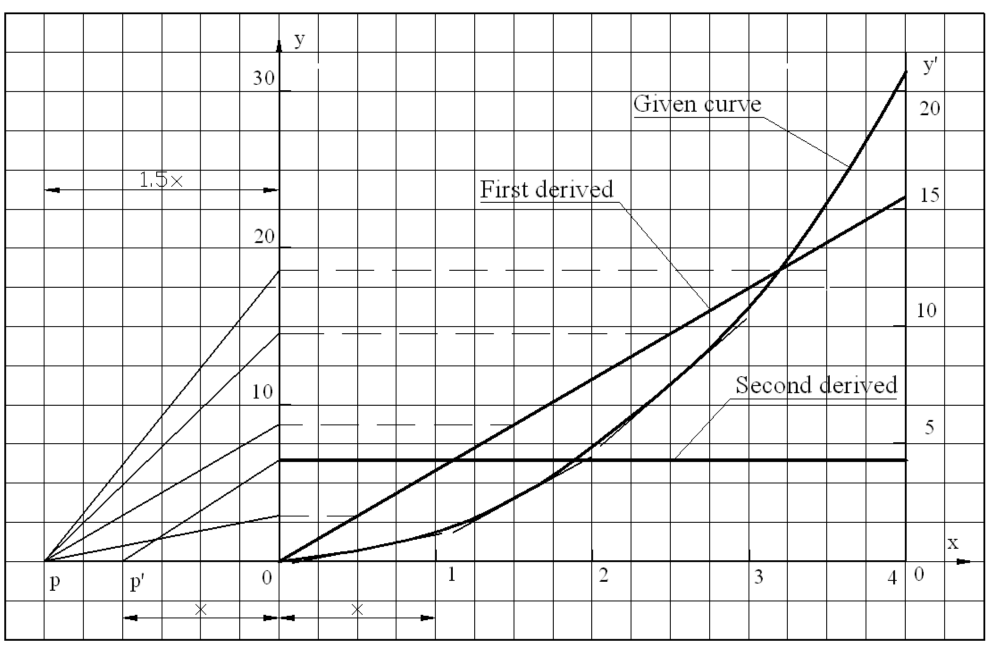

Fig. 2 Application of tangent method

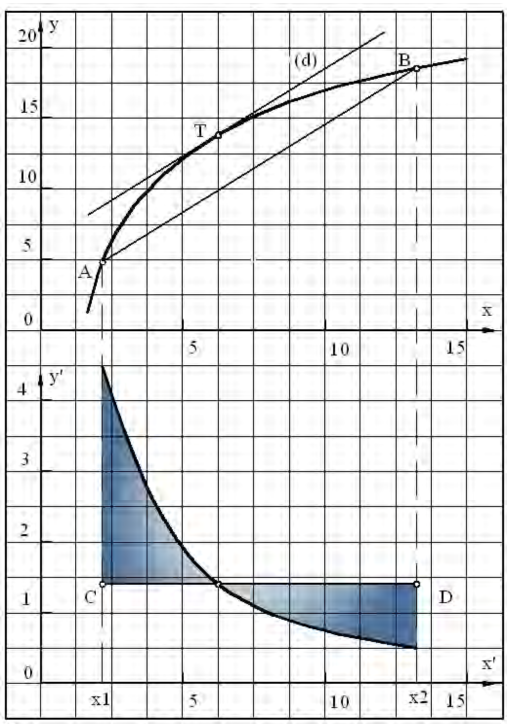

Fig. 3 Determination of derivative curve with Area Low

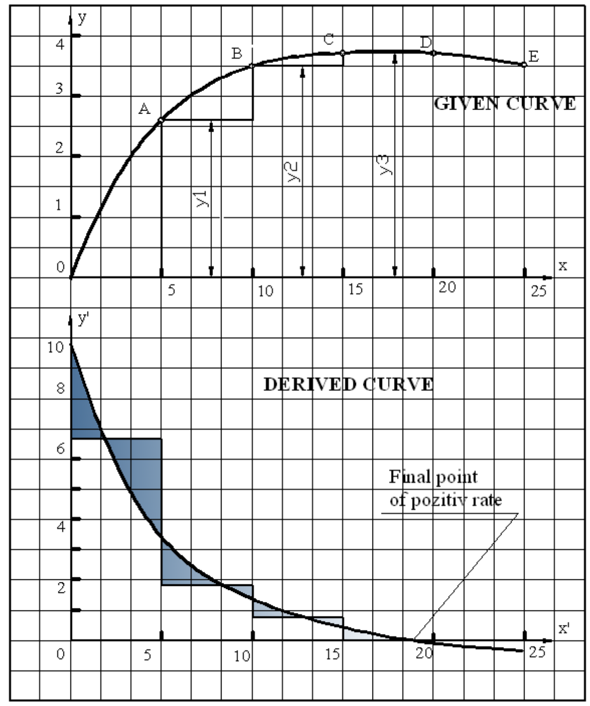

Fig. 4 Application of Area Low

4. SOME PRACTICAL GRAPHICAL SOLUTIONS

Solutions for some of the problems that students must resolve, can be determinate by graphical methods.

One of this method uses Area Low, which stipulates: ñThe difference between the length of any two coordinates of a continuous curve is equal to the total area between the corresponding ordinates of the next lower curve.ò [1].

For example, figure 3 shows a given curve and a tangent to the arc, drawn in *T*. Through *A* and *B* is constructed a chord. A tangent in *T* is parallel to the segment *AB*, with equal slopes.

In figure 4 we have applied this law to an current case study for the students of management: a practical solution of maximize the rate of interest of an investment in a building with an optimum number of stories.

The given curve is drawn using coordinate values (price of building, lot, plans, preliminaries, a.s.o.) and plots the interest. The derived curve plots the rate of interest for the number of stories and it is determined applying area low.

The initial curve is divided in equidistant ordinates y_1, y_2 , y_3,é The chord of arcs trough the points *A, B, C*,é are parallel to the tangent lines.

The solution is semigraphical and the differentiation with area low found that the optimum number is 18 stories.

The next practical graphical solution for integration is the method of the string polygon.

In the next example, the given curve is of low degree and the area between *x* axis and the outline *ABCDEF* is required (figure 5).

The location of the pole *P* is convenient chosen and the vertical projections of points *A, B*, é are founded and the corresponding rays are drawn. Through point *G* is drawn a string parallel to *P6* till the intersection *H* to the vertical *CB*. The next string is parallel to *PC'*, a.s.o. The result is the outline *GHIJ*, which is a higher degree integral: the segment *KH* is equal to the area under *AB*, the segment *IL* is equal to the area under *ABCD*, a.s.o.

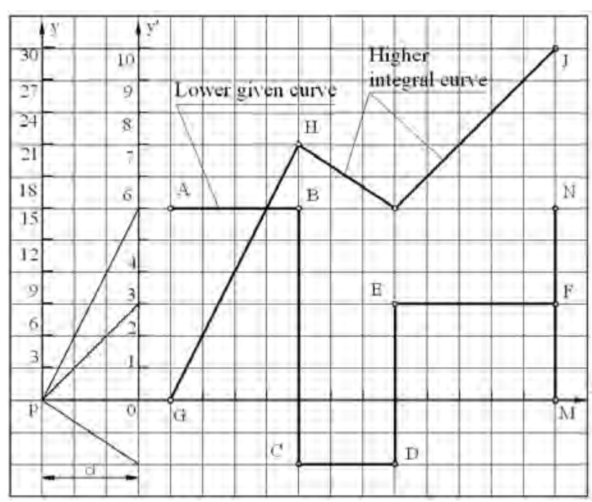

Fig. 5 Graphical integration with Polygon Method

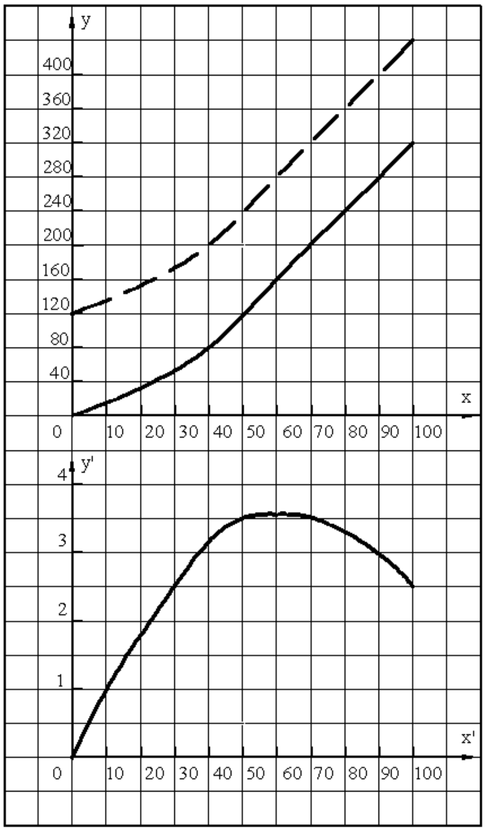

Fig. 6 Constants of integration

In the precedent examples, we assumed that integral curves begin at zero, but, in the general case, the location corresponding the x axis is required by the initial conditions and must be assumed [2].

A practical problem exemplifies this case and it is solved graphically in figure 6. The integral curve describes the volume of fuel consumed when the tank of known capacity was initially full. The integral curve must be translated until the value of the tank capacity and the fuel remained in the tank is the constant value.

When graphical mathematics are affected by the low accuracy, one of useful method is Simpsonós Rule. It is a semigraphical integration and is applied to determine the area, not an integral curve. This method can be applied by students for integrate graphically many laboratory experience, for example, in explosion stroke, piston stroke/piston force or in impact extrusion of soft copper the curve punch stroke, extruding force.

Figure 7 shows a part of a parabola *ABC* and it is required the area *0ABCD*. An essential condition for applying Simpsonós Rule is to divide the given curve into an even number of intervals. The required area is ñthe sum of extreme ordinates, plus four times the sum of odd ordinates, plus twice the sum of even ordinates, all multiplied by one-third the common distance.

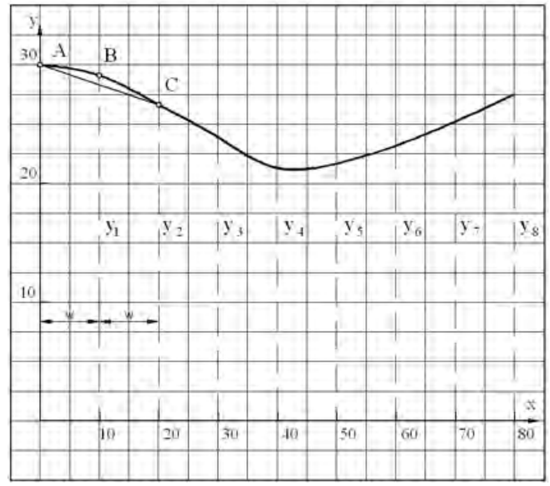

Fig. 7 Semigraphical integration with Simpsonós Rule

5. CONCLUSIONS

Some typical engineering problems can be solved by students using graphical methods, in an integrated way for knowledge of design, mathematics, mechanics and physics.

In differential applications, using independent variables like: time, displacement, quantity and dependent variables like: velocity, energy of work can be determinate derivate curve for: acceleration, power, force, rate of cost.

In integrative applications, using independent variables like: time, area, displacement, volume, quantity and dependent variables like: acceleration, velocity, pressure, force, power can be determinate integrals of integration curve for: force, momentum, impulse, energy.

The increased use of graphical methods adds another layer of interpretation to a given task requiring the co-ordination of different knowledge, in order to produce a correct solution and to improve the product design.

6. REFERENCES

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