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STEREOGRAPHIC PROJECTION OF THE HEAVENS OVER BELGRADE USED FOR THE CONSTRUCTION OF ASTRONOMICAL CITY CLOCK

Abstract: *There are many different ways of rendering 3D objects into two dimensions. Different kinds of projections are able to represent things like size, areas, distances or perspective, realistically. One particular kind of projection used for representing spheres and circles on spheres in two dimensions (i.e. some maps of the earth or celestial sphere) is stereographic projection. Stereographic projection has two important characteristics that differentiate it from other kinds of projections: it preserves circles and angles. This paper discloses the construction of the stereographic projection of the celestial sphere over Belgrade as the clock face of the astronomical clock. This astronomical clock will be capable to present the motion, as seen from the Earth, of the Sun, Moon, Ecliptic and the phases of the Moon during the year. Moreover, the hours of the sunrise, noon and sunset will be clearly observed on the stereographic dial of the astronomical clock.*

Key words: *stereographic projection, Belgrade, astronomical city clock, celestial sphere, astrolabe*

1. INTRODUCTION

A stereographic projection, or a stereonet, is a powerful method for displaying and manipulating the 3D geometry of lines and planes [1]. It is a type of conformal azimuthal map projection which has found its application in cartography, crystallography, geology, photography [1, 6].

It is impossible to project sphere onto a flat surface in such a way that all distances remain the same. Most map projections adopt one of two following procedures: preservation of angles or preservation of areas. Stereographic projection is distinguished by these two related properties:
1. circles on the sphere correspond to circles on the plane;
2. it preserves the angle between paths (it is conformal) [6, 8].

The first of these two characteristics was known to the Greeks of the Hellenistic period, and can be found in Apollonius' treatise on conic sections [2]. It was crucial in the design of astrolabes. The second seems to have been first discovered by the English mathematician and astronomer Thomas Harriot (1560-1621). He served as a cartographer and navigator for Sir Walter Raleigh (1552-1618) during his expedition to Roanoke Island (today's Dare County, North Caroline), but his proof remained unpublished until long after his death [2].

The stereographic projection is a very ancient geometrical technique. Hipparcus of Nicaea (c. 190 – c. 120 BC), the greatest astronomer of ancient times, was the first one who introduced the stereographic projection [3]. Hipparcus's work was promoted by Alexandrian astronomer Claudius Ptolemy (c. 100 – c. 170 AD) who used it as a means of representing the stars on the heavenly sphere. The original Ptolemy's Greek manuscript is lost, but a Latin translation under the title *The Planispherium* appeared in 16th century [2, 4].

The astrolabe was the ancient equivalent of today's hand-calculator, that was used to solve practical problems such as: determining the times of sunrise and sunset; determining the altitude of stars; determining the time of day and the number of hours of darkness; measuring heights of objects; (in Islam) locating the direction of Mecca and determining the appropriate times for prayer etc [3, 7].

The most used and the most popular was the instrument named *planispheric astrolabe* (Fig.1). It was both an observing instrument that measured angles and a portable analogue computer that could be used to solve astrological, astronomical and geometric problems [5]. Its design was based on a model of the universe in which the sun's motion was tracked against the surface of a large celestial sphere centered on the earth [3, 5].

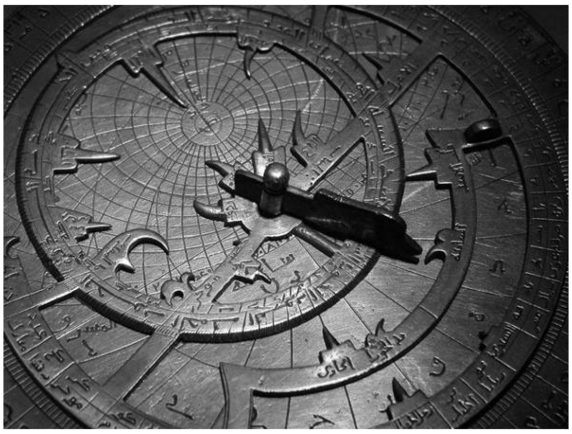

Fig. 1 Planispheric astrolabe. [11]

By the 9th century, the use of *planispheric astrolabe* had spread across the Arab world and was carried into Spain in the wake of Islamic conquest [3]. From the 13th century onwards, astrolabes spread from Spain throughout Europe [3, 4].

Since this paper deals with stereographic projection used for the construction of astronomical clock, we should mention some of the astronomical clocks that are still in use. The most famous one is the Old-Town Hall clock in Prague (Czech Republic), also known as the Prague Orloj (Fig.2). The second one is in Czech Republic as well, but in the city of Olomouc (Fig.3). It is the rare example of a heliocentric astronomical clock.

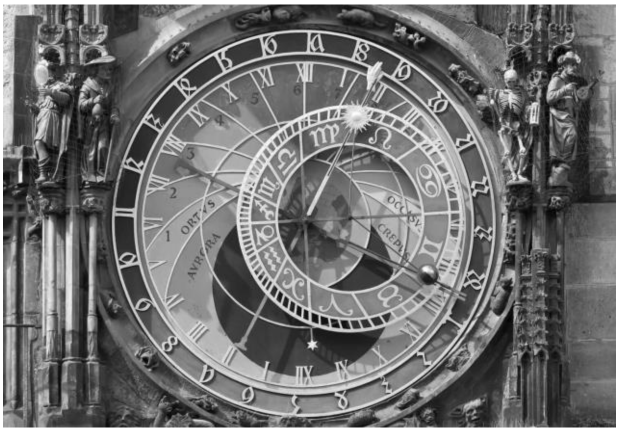

Fig. 2 The Prague Orloj. [12]

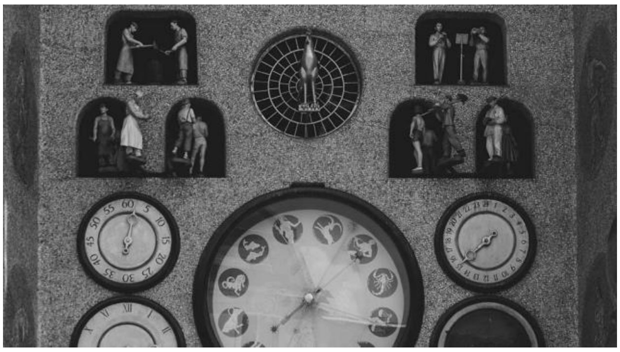

Fig. 3 The Olomouc astronomical clock. [13]

2. DESIGN OF THE DIAL OF THE ASTRONOMICAL CLOCK

Among many different applications of the stereographic projection, one interesting but almost forgotten is its usage in the design of the dial of the astronomical clocks. Since the stereographic projection is conformal and thus preserves angles, this type of clock is capable to exhibit positions and motions of the Sun [9, 10], Moon, Zodiac constellations and planets as they are observed from the Earth. In this paper, the construction of the astronomical dial for the latitude of Belgrade (44.80^0) will be disclosed and explained in all necessary details.

Due to the fact that Celestial sphere rotates around the Celestial axis *NS*, the dial of the astronomical clock can be obtained by the stereographic projection of the Celestial sphere either from the Celestial pole *N* or *S*.
Since Belgrade is on the northern hemisphere, the projection is accomplished from the pole *N*. The stereographic projection, from the Celestial pole *N* to the plane π, of the Celestial equator *e*, Celestial Tropic of Cancer t_N, celestial Tropic of Capricorn t_S, Zodiac circle (Ecliptic) *z* and horizon *h* for the latitude of Belgrade is shown on Fig.4. The circles of Celestial equator *e* and Ecliptic *z* intersect in two points: one of them is the first point of Aries γ and the second one is the first point of Libra λ.

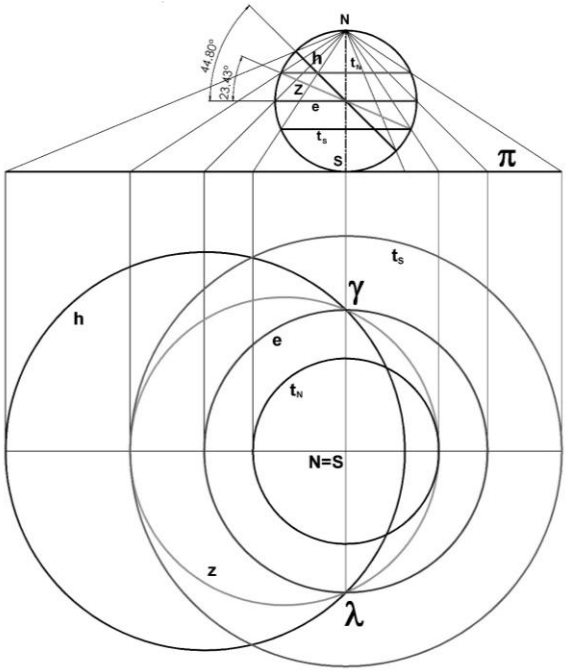

Fig. 4 Stereographic projection of the Celestial equator *e*, Celestial Tropic of Cancer t_N, Celestial Tropic of Capricorn t_S, Zodiac circle (Ecliptic) *z* and horizon *h* for the latitude of Belgrade (44.80^0).

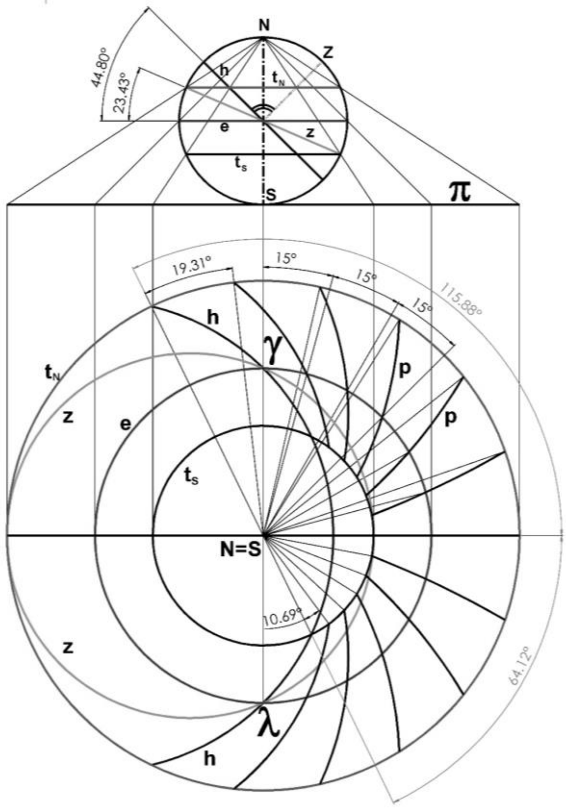

Fig. 5 The complete geometry of the dial of the astronomical clock: the Celestial equator *e*, Celestial Tropics t_N, t_S, Zodiac circle (Ecliptic) *z*, Zenith *Z*, horizon *h* and unequal (proportional) hours *p* for the latitude of Belgrade (44.80^0).

Due to the fact that the observer of the Celestial sphere is located in its center, the angular coordinates of any celestial object in the local Horizontal coordinate system (altitude and azimuth) can be measured directly on the astronomical dial obtained by the stereographic projection of the Celestial sphere.

On Fig.5, the complete geometry of the dial of the astronomical clock for the latitude of Belgrade [10] is disclosed. Since the Sun and the Moon, as observed from the Earth, never surpass the Celestial Tropic of Cancer, the astronomical dial is edged by this circle. The circle of Zodiac (the ecliptic) *z* touches both Celestial Tropics and intersects equator *e* and horizon *h* circles in the same pair of points - γ and λ. Above the horizon *h*, 12 arcs of unequal (proportional) hours *p* and its geometrical construction are also shown. Whereas the hour as the 24th part of the day was not defined and established in medieval ages yet, the common praxis in those days was to divide the time period between sunrise and sunset on 12 equal parts. Because these intervals of time vary during the year, they are called unequal or proportional hours. They are geometrically determined by the approximate construction on Fig.5.

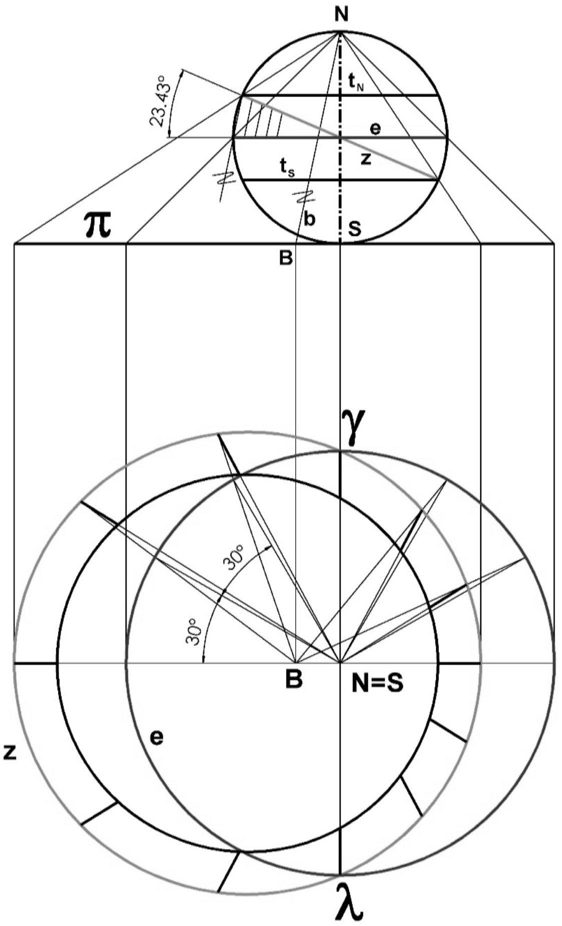

Fig. 6 The stereographic projection of the Zodiac circle and the division of the Zodiac circle on 12 equal segments.

The arcs above the horizon *h* of the Celestial Tropic of Cancer, equator and Celestial Tropic of Capricorn are divided on 12 equal parts. Each tree points of these divisions define one arc of unequal (proportional) hour *p*. Since the astronomical clock displays the positons of the Sun and the Moon, and motion of the stars too, the representation of the Zodiac is an indispensable part of its face and thus it must be geometrically constructed as well. The stereographic projection of the Zodiac circle (ecliptic) and its division on 12 equal segments are given on Fig.6. The Zodiac circle is divided on 12 equal segments by the composition of the affine bisector projection of the equator to the Zodiac circle and its central (stereographic) projection to the plane π. The composition is the central collineation again which center *B* is the vanishing point of the central (stereographic) projection to the plane π of the affine bisector rays. The stereographic projection of 12 equal divisions on the equator, seen without deformation on the plane π, is centrally projected from the point *B* to the stereographic projection of the Zodiac. As is already mentioned, the circle of Zodiac (the ecliptic) intersects equator circle in the pair of points - γ and λ. The point γ is the first point of Aries which represent the vernal equinox and the point λ is the first point of Libra which represent the autumn equinox.

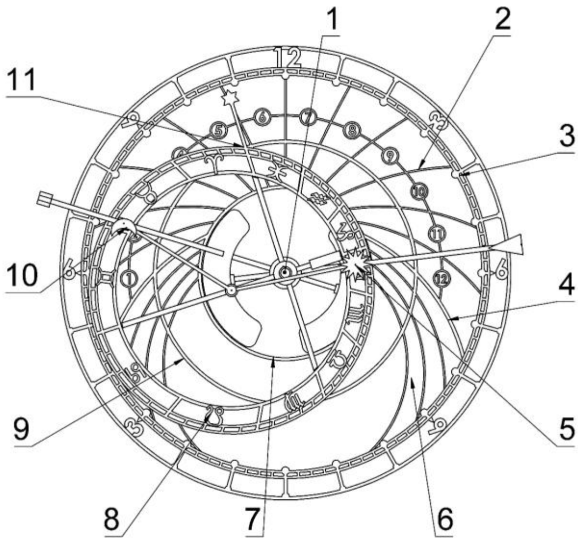

Fig. 7 The Dial of the Astronomical Clock:
1 - axis, observer position; 2 – arc of an unequal (proportional) hours; 3 – Celestial tropic of Cancer; 4 – horizon; 5 – Sun; 6 – twilight; 7 - Celestial Tropic of Capricorn; 8 – circle of Zodiac; 9 – Celestial equator; 10 – Moon; 11 – First point of Aries (vernal equinox).

On Fig.7, the model of the astronomical clock dial created by the using of the SolidWorks application is shown. The Sun (5), Moon (10) and the Zodiac circle (8) are movable clock hands. Common time can be ridden on the 24 numeral division placed at the edge of the Celestial tropic of Capricorn (3), by the position of the Sun pointer. The sidereal time can also be determined on the same division of the Celestial tropic of Capricorn (3), by the position of the star mark (γ point) at the Zodiac circle (8). The position of the Sun (5) relative to the circle of Zodiac (8) discloses the current date of the year. The relative angular distance between the Moon (10) and the Sun (5) determines the Moon phase. The time of the sunrise and sunset is determined regarding the angular distance between the position of the Sun and the intersection point between its apparent trajectory and the arc of the horizon (4). The twilight periods, between sunset and dusk as well as between dawn and sunrise can also be determined (6). The date of the vernal and autumn equinoxes can be disclosed when the Sun (5) is positioned on the circle of equator (9). The date of the summer solstice can be determined when the Sun (5) is located on the circle of the Celestial tropic of Capricorn (3), while the date of the winter solstice is revealed when the Sun (5) is placed on the circle of the Celestial Tropic of Capricorn (7). The duration of the apparent day, as the time interval between the sunrise and sunset, can be disclosed regarding the arcs of the proportional hours (2). Obviously, all the features of the astronomical clock mentioned and explained above can be obtained not just by the proper construction of the dial, but also by the correct design of the clock mechanism. Since the mechanism synthesis of the astronomical clock is not the subject of this paper, it will be disclosed and explained by the future work.

3. CONCLUSION

As all scientific papers, this work is significant mainly for education, particularly in this case in the field of geometry and astronomy. Moreover, information exposed in this paper can be applied for maintenance of numerous old astronomical clocks built in the medieval ages as a part of cultural heritage of the Europe and the world.

REFERENCES:

[1] George H. Davis, Stephen J. Reynolds, Charles F. Kluth (2011), *Structural Geology of Rocks and Regions, 3rd Edition*, Wiley, ISBN: 9780471152316, USA

[2] Bill Casselman (2004), *Mathematical Illustrations: A Manual of Geometry and PostScript*, Cambridge University Press, ISBN: 978-0521547888, UK

[3] Richard J. Howarth (1996), *History of the stereographic projection and its early use in geology*, Terra Nova, Volume 8, pp. 499-513, ISSN: 1365-3121

[4] Daniel Daners (2012), *The Mercator and stereographic projections, and many in between*, The American Mathematical Monthly, Volume 119, (March 2012), pp. 119-210, ISSN: 0002-9890

[5] Edited by Robert Odell Bork, Andrea Kann (2008), *The Art, Science and Technology of Medieval Travel*, CRC Press, ISBN: 9781138259874, USA

[6] Donald Fenna (2006), *Cartographic Science: A Compendium of Map Projections with Derivations*, CRC Press, ISBN 9780849381690, USA

[7] J. D. North (1974), *The Astrolabe*, Scientific American, Volume 230, No. 1, (January, 1974), pp. 96-107, ISSN: 00368733

[8] Erik W. Grafarend, Friedrich W. Krumm (2006), *Map Projections: Cartographic Information System*, Springer, ISBN 9783642364945, USA

[9] Misa Stojicevic, Zorana Jeli, Milos Obradovic, Ratko Obradovic, Gabriel – Catalin Marinescu (2019), *Designs of Solar Concentrators*, FME Transactions, Vol 47, No 2, pp. 273-279, ISSN 1451-2092, Belgrade

[10] Mirjana S. Devetakovic, Dorde D. Dorđevic, Gordana D. Dukanovic, Aleksandra D. Krstic Furundzic, Budimir S. Sudimac, Alessandra Scognamiglio (2019), *Design of Solar Systems for Buildings and Use of BIM Tools: Overview of Relevant Geometric Aspects*, FME Transactions, Vol 47, No 2, pp. 387-397, ISSN 1451-2092, Belgrade

[11] https://www.thefridaytimes.com/tft/allah-dad-the-royal-astrolabe-maker/ (05.02.2019.)

[12] http://www.amazingczechia.com/sights/prague-astronomical-clock-orloj/ (05.02.2019.)

[13] https://miss-sophies.com/olomouc/cs/olomouc/ (05.02.2019.)

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