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GRAPHO-ANALYTICAL STUDIES OF LINIAR EQUATION SYSTEMS

Abstract: *The paper presents the solving of linear three variables system using two methods: analytic and graphic methods. The solving of a such system implies four cases: compatible system with unique solution, simple-indeterminate compatible system, double-indeterminate compatible system and incompatible (impossible) system. The correspondence between the analytic (synthetic) method and the graphic (visual) method is evident for the three space fundamental elements: point, line and plane.*

Key words: *equation, in/compatible system, plane, line, point.*

1. INTRODUCTION

The linear three variables system and three unknowns quantities implies four cases:
- a unique solution compatible system,
- simple - indeterminate compatible system;
- double - indeterminate compatible system;
- incompatible (impossible) system.

Could the high school students imagine a graphic solution when they solve a linear three equations and three unknown quantities system?

Could first year technical universities students imagine that the three fundamental elements of the space: point, line and plane (used in descriptive geometry) have an analytic correspondence?

We think that the answer is negative in a great proportion for both questions.

This paper presents a study and a correspondence between analytic and graphic method.

2. COMPATIBLE SYSTEM WITH UNIQUE SOLUTION

We have the following system:

$$\begin{cases} x+y+z=3 \\ x+y+2z=4 \\ x+2y=3 \end{cases} \tag{1}$$

The system solution is unique and it can be obtained by using every known method:

$$x=1, y=1, z=1 \tag{2}$$

The three equations of the system (1) are the three planes ([P], [R] and [S]) equations (Fig.1) used in analytic geometry. The plane [P] equation is:

$$x+y+z=3 \tag{3}$$

The planes [H], [V] and [L] equations are:

$$z=0; y=0; x=0 \tag{4}$$

[P], [H] and [V] planes intersection is the P_x point on Ox axis.

$$\begin{cases} x+y+z=3 \\ z=0 \\ y=0 \end{cases} \tag{5}$$

System (5) solution is:

$$x=3=OP_x \tag{6}$$

We can similarly obtain P_y and P_z point's position:

$$y=3=OP_y; z=3=OP_z \tag{7}$$

The [P] and [H] planes intersect in line P_xP_y, the plane horizontal trace:

$$\begin{cases} x+y+z=3 \\ z=0 \end{cases} \tag{8}$$

System (8) solution is:

$$x+y=3 \tag{9}$$

meaning P_xP_y horizontal trace equation.

We can similarly obtain vertical and lateral trace equations of the [P] plane.

$$P_xP_z \rightarrow x+z=3 \tag{10}$$

$$P_yP_z \rightarrow y+z=3 \tag{11}$$

[R] plane equation is:

$$x+y+2z=4 \tag{12}$$

[R], [H] and [V] planes intersection is R_x point on Ox axis (Fig.2):

$$\begin{cases} x+y+2z=4 \\ z=0 \\ y=0 \end{cases} \tag{13}$$

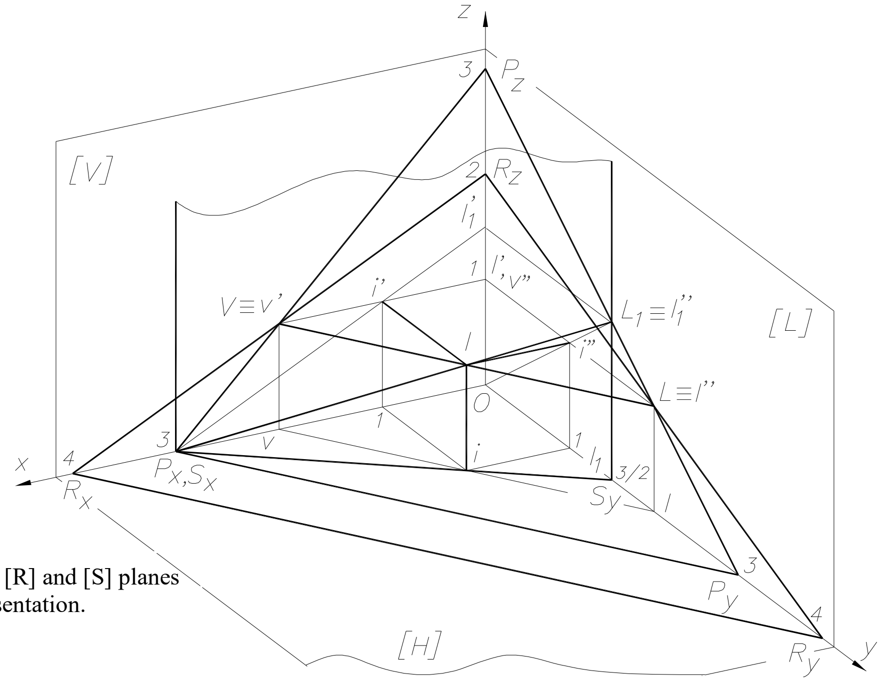

Fig. 1 3D - [P], [R] and [S] planes representation.

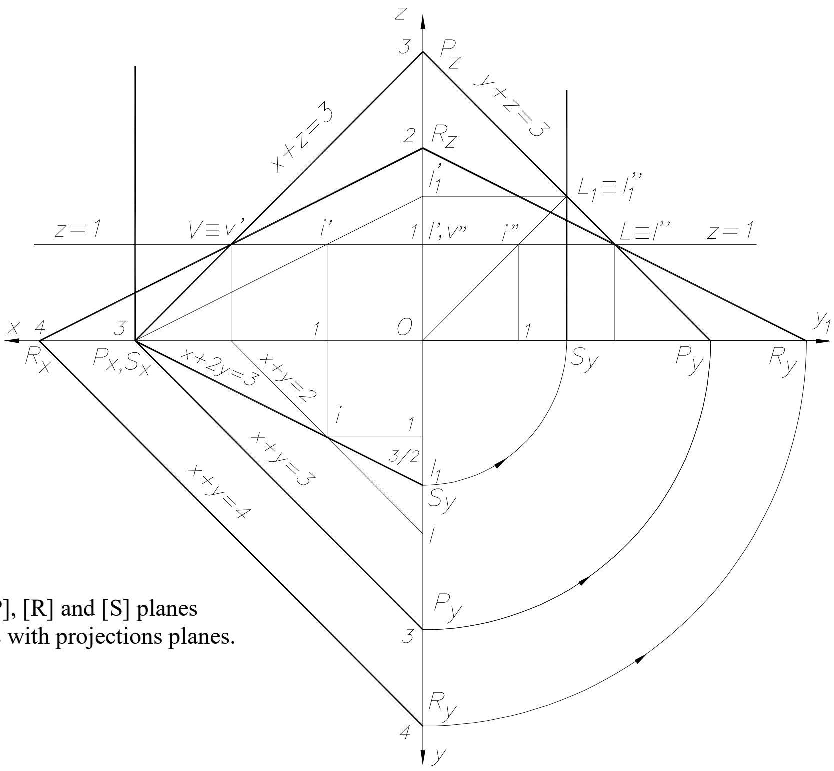

Fig. 2 [P], [R] and [S] planes intersections with projections planes.

System's (13) solution is:

$$x = 4 = OR_x \tag{14}$$

We can similarly obtain R_y and R_z points position:

$$y = 4 = OR_y\,;\, z = 2 = OR_z \tag{15}$$

The [R] and [H] planes intersection is line $R_x R_y$, the plane horizontal trace (Fig.2):

$$\begin{cases} x + y + 2z = 4 \\ z = 0 \end{cases} \tag{16}$$

$$\Rightarrow x + y = 4 \tag{17}$$

meaning $R_x R_y$ horizontal trace equation.

We can similarly obtain vertical and lateral trace equations of the [R] plane:

$$R_xR_z \rightarrow x + 2z = 4 \tag{18}$$

$$R_yR_z \rightarrow y + 2z = 4 \tag{19}$$

The [P] and [R] planes intersection is level line $VL(vl, v'l', v''l'')$ because their horizontal traces are parallel:

$$\begin{cases} x+y+z=3 \\ x+y+2z=4 \end{cases} \quad (20)$$

System (20) solution is:

$$z=1 \quad (21)$$

meaning vertical projection equation ($v'l'$) of level line VL (Fig.2).

Horizontal projection equation (vl) of level line VL (Fig.2) is:

$$x+y=2 \quad (22)$$

[S] vertical plane equation is:

$$x+2y=3; \forall z \quad (23)$$

[P] and [S] planes intersection is line S_xL_1.

VL and S_xL_1 line intersection is the point $I\ (i, i'i'')$.

The point $I\ (1,1,1)$ is the unique solution in our case.

3. SIMPLE-INDETERMINATE COMPATIBLE SYSTEM

We have the system:

$$\begin{cases} x+y+z=3 \\ x+y+2z=4 \\ x+y+3z=5 \end{cases} \quad (24)$$

We notice that the unique solution (using every known method) is:

$$z=1 \quad (25)$$

In this case there is a simple - indeterminate compatible system because we obtain for the three equations:

$$x+y=2 \quad (26)$$

$x=p,\ y=2-p,\ z=1$ are the system solutions and they depend on ***p*** parameter: $p \in (-\infty, +\infty)$. The three planes in (24) system intersect in a level line because their horizontal traces are parallel (Fig. 3). Vl level line equation is:

$$\begin{cases} x+y=2 \\ z=1 \end{cases} \quad (27)$$

In Figure 3, we did not write all notations in order to simplify the graphic representation. In Figure 4, the level line VL is obvious because there is an infinity system's solution. For example: for $p=1$ results $M(1;1;1)$; for $p=0{,}5$ results $N(0{,}5;\ 1{,}5;\ 1)$.

4. DOUBLE - INDETERMINATE COMPATIBLE SYSTEM

We have the system:

$$\begin{cases} x+y+z=3 \\ 2x+2y+2z=6 \\ 3x+3y+3z=9 \end{cases} \quad (28)$$

The three equations are equivalent, so the system is reduced to a single equation, the [P] plane equation (Fig.5).

$$x+y+z=3 \quad (29)$$

The system is a double- indeterminate compatible because its solutions depend on two parameters:

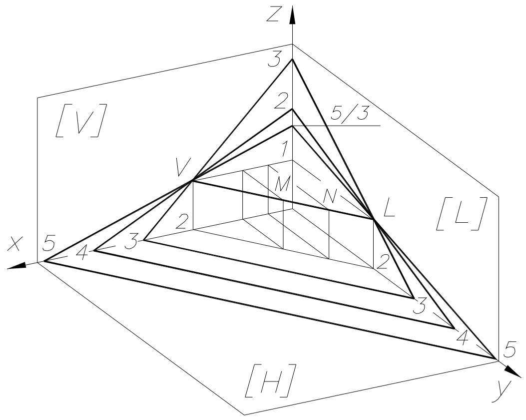

Fig. 3 The three planes intersection.

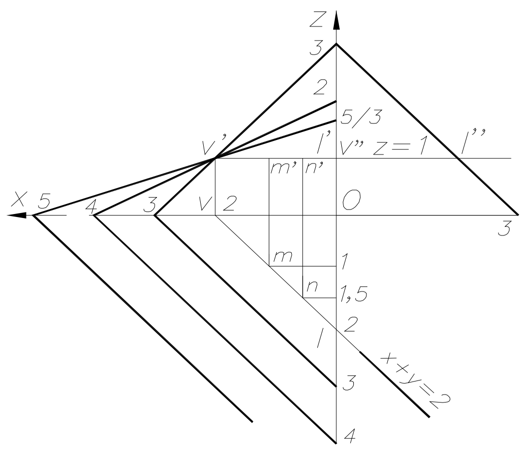

Fig.4 The level line VL solution.

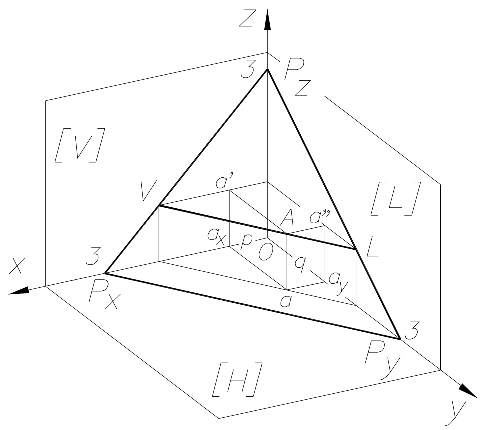

Fig. 5 The double- indeterminate system.

$$x = p;\; y = q;\; z = 3 - p - q \tag{30}$$

In Figure 5, the point *A* has the coordinates: $x = Oa_x = p$; $y = Oa_y = q$ and it is placed on a level line.

In Descriptive Geometry, the point *A* is placed in [P] plane if two of its coordinates are known.

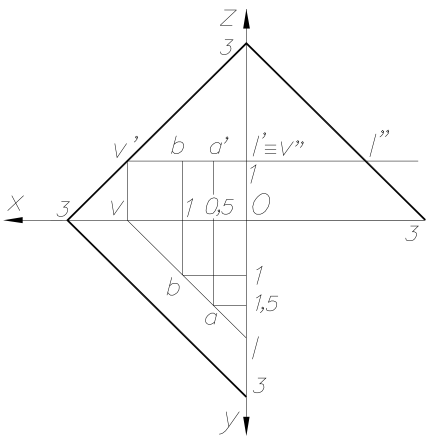

Fig. 6 The points *A* and *B* placed in [P] plane, using a level line.

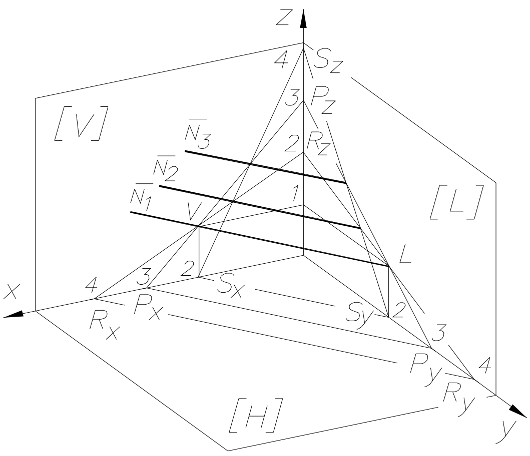

Fig. 7 The three planes intersection.

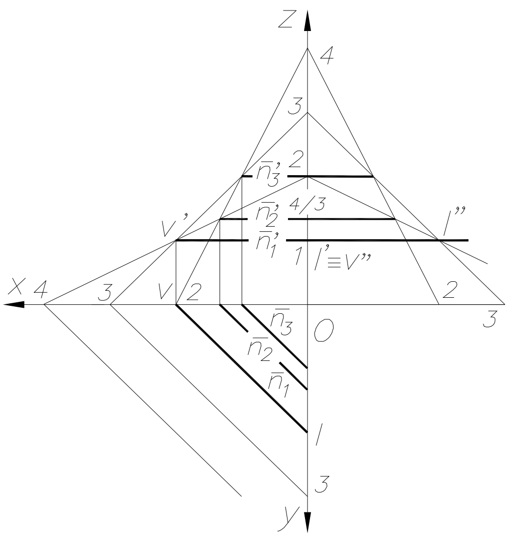

Fig. 8 The three planes intersection.

For example: *A(1,1,z); x=1, y=1,* $\Rightarrow$ *z=1; B(0.5,y,1) x=0.5, z=1* $\Rightarrow$ *y=1.5* (see Figure 6). The points *A* and *B* were placed in [P] plane, by using a level line.

5. INCOMPATIBLE (IMPOSSIBLE) SYSTEM

We have the system:

$$\begin{cases} x + y + z = 3 \\ x + y + 2z = 4 \\ 2x + 2y + z = 4 \end{cases} \tag{31}$$

We obtain three distinct values for *z (z=1; z=2; z=4/3)*, by solving the system equations. In this case the system is impossible. The three planes intersect in three parallel level lines, because their horizontal traces are parallel: $P_x\, P_y \,//\, R_x\, R_y \,//\, S_x\, S_y$; $N_1 \,//\, N_2 \,//\, N_3$ (Fig.7 and Fig. 8).

Observation: We used simple equations for the three planes in order to a clear graphic representation. We only used the first trihedral representation (*x, y, z* positive).

6. CONCLUSIONS

- In this paper we used: algebra, analytic geometry, descriptive geometry notions in order to demonstrate (to prove) the connection between these disciplines and to understand the mathematical essential elements.
- The paper has a didactic character and it is especially useful for technical universities' students. They can better understand the correspondence between analytical equations and their graphical representations.
- The method used in this paper can be extended at the graphical-analytical study of the other surfaces: cylinder, cone, sphere a. s. o.

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