SYNTHESIS AND ANALYSIS OF A MECHANISM GENERATING THE RATIONAL CIRCULAR CUBIC

Abstract: *Based on the mathematical curve named rational circular cubic [2] and on its animation, it was conceived an original mechanism generating this curve. The mechanism has an additional kinematic chain that provides the drawing point of the curve as the center of a variable length straight segment. There are given the successive positions of the mechanism and the drawn curve, as well as the similar curves obtained at different positions of the fixed vertical straight and the driving's element joint. Such a mechanism can be used in textile machines, as a toy, in kinetic art, etc.*

Keywords: *Rational circular cubic, cubic generating mechanism, cissoids, connecting rod curve, mechanism R-PRR-PRP.*

1. INTRODUCTION

Rational circular cubic are mathematical curves, also called elongated cissoid or hypercissoid (crunodal case) shrunk cissoid or hypocissoid (acnodal case). Forms of this curves are given in [2] (Figure 1).

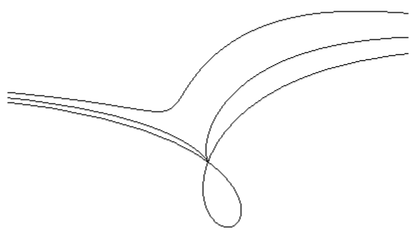


Fig. 1 Forms of the rational cubic circular curve [2]

The rational circular cubics are the circular cubics, that have a real singularity, the point O (Figure 2). The cubic is called crunodal, cuspidal or acnodal, depending on whether this singularity is a double point with different tangents, a cuspidal point or an isolated point [2].

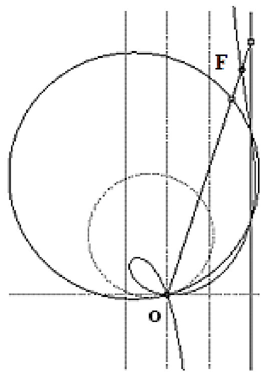


Fig. 2 Cissoid construction in the crunodal case [2]

The cartesian equation of this curve is [2]:

$$x(x^2 + y^2) = (d + 2a)x^2 + 2bxy + dy^2 \qquad (1)$$

Further on, based on the cissoid construction in crunodal case (Figure 2) [2], it was conceived an original mechanism that generates this curve and the synthesis of the mechanism is shown below.

2. SYNTHESIS OF THE GENERATING MECHANISM

It starts from the center circle in D (Figure 3) and the DC radius and a fixed straight line GB, located at the distance AG = XB of the ordinate passing through A and D. The straight line AB, having variable length, have a rotational joint in point A. The point B slides on the fixed line GB, so that it was provided with slider 4 (which ensure the variable length of AB) and 5 (allowing B to slide along BG). The slider in C allows the C point to slide along AB line.

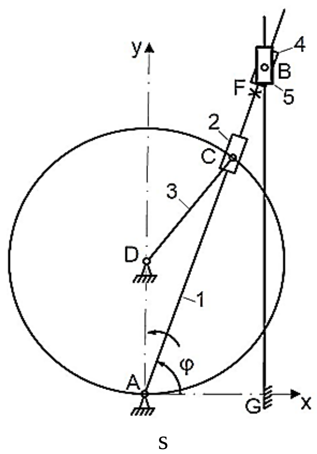


Fig. 3 The first version of the generating mechanism

The structural analysis of the mechanism was made and the resulted structural diagram is given in Figure 4. The mechanism consists of the driving element AB and one PRR type dyad, respectively CCD, and one PRP type dyad, BBB.

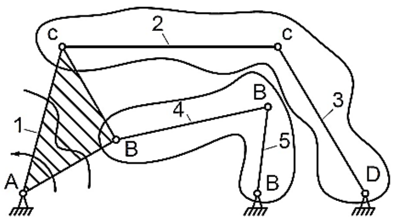


Fig. 4 **S**tructural diagram for the mechanism

Since Figure 2 shows that the traser point of the curve is in the point F, meaning the midpoint of the CB segment (Figure 3), it is used the additional cinematic chain CEBF (Figure 5) for finding the middle of a straight segment, based on the geometric rule. It draws from C an arc with radius CE > CB, resulting in a locus of the point E, then another arc with BE = CE is drawn from B, resulting in their intersection the point E. From this point the perpendicular EF goes on CB, so that F is the middle of CB.

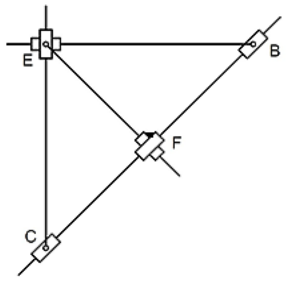


Fig. 5. Additional cinematic chain

This chain has not been attached neither to the kinematic diagram (Figure 3), nor to the structural diagram (Figure 4), to not complicate them. It is just a dependent chain that gives the length of CB variable to the movement of the mechanism, the sliders in Figure 5 permitting variations in the lengths of the chain elements.

In order to find a wide range of curves, another version of the the mechanism was studied, by changing it as in Figure 6, that is, the joint of A was located in another area, different from the origin of the system.

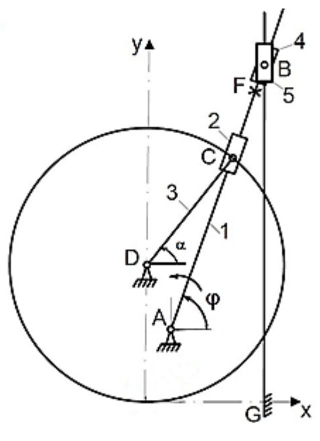


Fig. 6 The second version of the generating mechanism

3. THE POSITIONS OF THE MECHANISM

Based on figure 3 the following equations are written:

$$DC^2 = AC^2 + AD^2 - 2AC \cdot AD \cdot \cos(90 - \varphi) \quad (2)$$

$$x_C = AC \cdot \cos\varphi \quad (3)$$

$$y_C = AC \cdot \sin\varphi \quad (4)$$

$$x_B = const. \quad (5)$$

$$AB = \frac{x_B}{\cos\varphi} \quad (6)$$

$$y_B = AB \cdot \sin\varphi \quad (7)$$

$$x_F = 0{,}5(x_B - x_C)\cos\varphi + x_C \quad (8)$$

$$y_F = 0{,}5(y_B - y_C)\sin\varphi + y_C \quad (9)$$

From (2) we calculate AC. From the equations (3) and (4) we get the coordinates of the point C. The equation (5) defines the position of the line GB. From the equation (6) we get AB and from the equation (7) it results y_B.The equations (8) and (9) establish the position of the point F.

Based on figure 6 the following equations are written:

$$AB = \frac{x_B - x_A}{\cos\varphi} \quad (10)$$

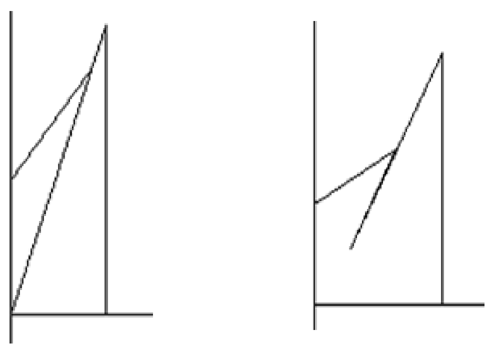
$$y_B = AB \cdot \sin\varphi + y_A \quad (11)$$

$$x_C = x_D + DC \cdot \cos\alpha = x_A + AC \cdot \cos\varphi \quad (12)$$

$$y_C = y_D + DC \cdot \sin\alpha = y_A + AC \cdot \sin\varphi \quad (13)$$

4. RESULTS

We choose two sets of initial data for the mechanism:
a- the first set : XB = 24, DC = 34, AD = DC (dimensions in mm) and for $\varphi = 72^0$ we get the resulting mechanism (Figure 7 a).
b- the second set: : XB = 35, DC = 27, XA=0; YA=0 mm and for $\varphi = 65^0$ we get the resulting mechanism (Figure 7 b).



a **b**

Fig. 7. The resulting mechanism:
a- for the first set of data; **b**-for the second set of data

The successive positions of the mechanism are shown in Figure 8 and the **v**ariation of the coordinates of point F in figure 9.

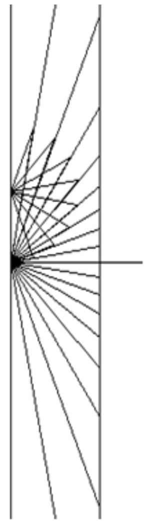


Fig.8. Successive positions

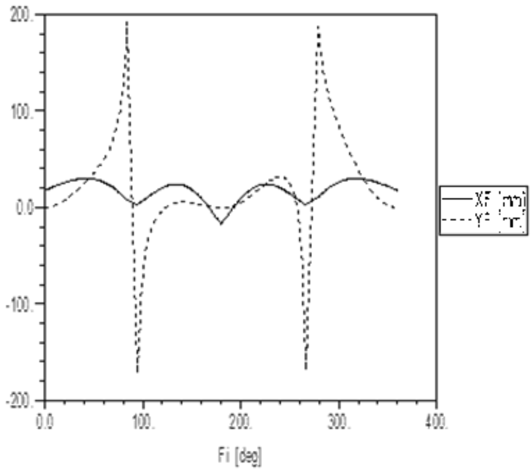


Fig. 9 The **v**ariation of the coordinates of point F

The resulting curve for the initial data XB = 35, XA=0; YA=0 is shown in Figure 10.

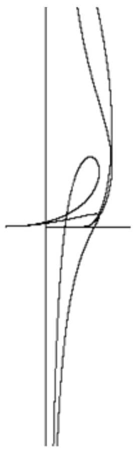


Fig. 10. The resulting curve

We have studied also the influence of the variation of the XB dimension on the curve.

The fixed ordinate passing through G and B may influence the shape of the curve. In Figure. 11 ... 14 are shown the different curves resulting for different values of XB, when the coordinates of point A are constant (XA=0; YA=0).

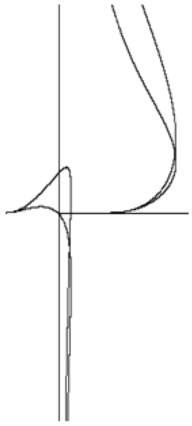
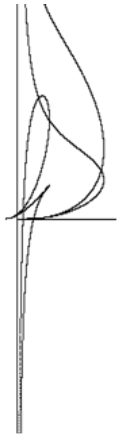



Fig.11. The curve for XB=–50 **Fig.12**. The curve for XB=–10

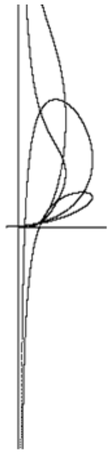
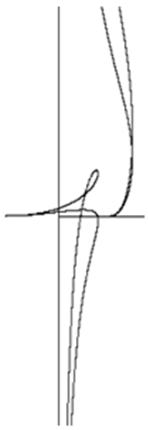



Fig.13. The curve for XB=10 **Fig.14.** The curve for XB=50

When XB is modified the curves are similar but with some deformations, especially in the area around the origin of the axis system.

It is interesting the resulting curve for the case XB = XA=YA=0 (Figure 15).

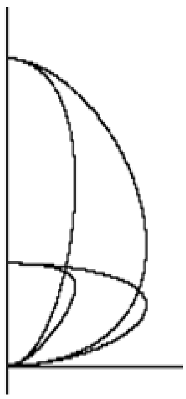
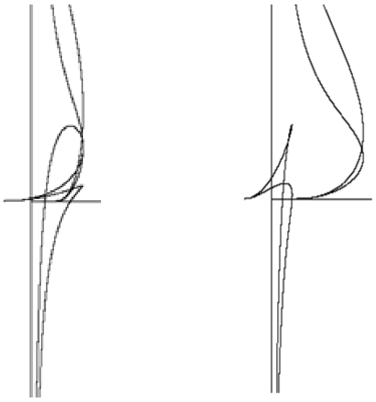


Fig.15 The curve for XB = XA=YA=0

We took also in considerations the cases where the line BG is tangent to the circle with the radius of DC (Figure 16):
a- the case for XB=DC=27; XA=YA=0 ;
b- the case for XB=−27; XA=YA=0



a **b**

Fig.16. The curves where the line BG is tangent to the circle with the radius of DC

Further on the position of the point A was changed to obtain other curves, maintaining XB = 35, i.e. BG is to the right of the circle. In Figures 17 ... 23 examples are given for different cases.

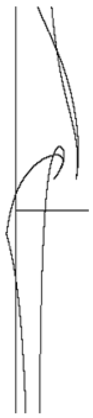


Fig.17. The curve for XA=10; YA=15

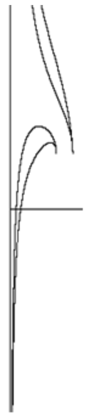


Fig.18. The curve when point A overlaps point D

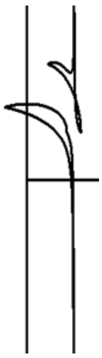


Fig.19. The curve for XA = YA = 27 = DC on the abscissa of D, to the right

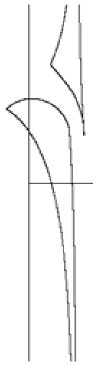
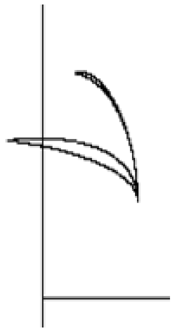
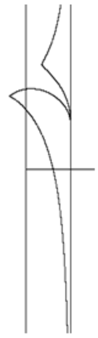


Fig.20. The curve for XA =−27, YA = 27 = DC on the abscissa of D, to the left





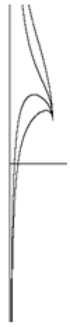


Fig.21. XA=YA=XB=27 (BG is tangent to the circle to the right)

Fig.22. XA=–27; YA=27; XB=27

Fig.23. XB=27; XA=0; YA=27 (A in D)

5. CONCLUSIONS

The paper started from rational circular cubic mathematical curve for which forms and relationships are given in [2]. Also in [2] there is an animation of a three-point line on it, which shows how this curve can be generated. Starting from this animation, the synthesis of the generating mechanism was done, resulting in a mechanism of the R-PRR-PRP type. To ensure the permanent positioning of the center of a variable length segment, an additional kinematic chain has been added to the mechanism. The mechanism has been drawn in a position and some successive positions. The mechanism has leaps when the driving element approaches the ordinate, the tracer point lowering the abscissa and then climbing. The curve is of the cissoid type, the crunodal case being similar to Figure 2 [2]. By modifying the position of the fixed vertical line and of the joint in the point A, ie the distance XB and the coordinates XA and XB, similar curves are obtained, but being deformed according to these variables.

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