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SPECIFYING A PLANE BY USING OF THE DIHEDRAL ANGLE

Abstract: *Sometimes, in the drawing's process of a part, it is necessary to be determine a plane by using the value of the dihedral angle between this and another plane, which is already known. The authors did not find in the literature solutions for this problem, but they observed that can be easily found only methods for measuring of the dihedral angle. Starting from these considerations, in this paper, the authors propose two solutions for specifying a plane by using the dihedral angle, then compares them and recommend one of the two. Finally, the authors present the solutions offered for this problem by the computer programs when 3D model is used.*

Key words: *Descriptive geometry, dihedral angles, Inventor, AutoCAD*

1. INTRODUCTION

When a part is drawing or designed, in some circumstance it may be necessary for the engineer to specifies a plane, knowing another plane which intersect the desired plane and the dihedral angle between these two planes. If the engineer does this traditionally, using pencil, paper, ruler and so on, or if he uses a computer program like AutoCAD but works in the 2D model, he must have strong knowledge of Descriptive Geometry in order to solve this problem. In the literature there is not solutions for this problem so, the engineer must find a way to solve it.

The authors' aim was that to find at least one solution that would give the possibility to the engineers to specifies a plane using the dihedral angle. They understood that, in order to accomplish this task, the first two objectives are to present and analyse the methods from the literature that provides the possibility of determining the value of a dihedral angle. The authors considered if that several solutions will be found, a third objective will be to compare the solutions found to determine which of them could be recommended for use. Finally, they agreed that it necessary to analyse which tools provide software such AutoCAD or Inventor to solve this problem when a part is designed, from the start, using 3D model.

A previously analysed and tested solution that offers the possibility of specifying a plane using the dihedral angle gives to the engineers the chance to save time in the design process of a part and reduces the effort that they need to do.

Known methods for determining the dihedral angle can be divided in two categories. From the first category are part the methods used in case that the intersection line is given and from the second category are part the methods used in case in that the intersection line is not given [1]. The authors decided that the case in where the intersection line is not given is not of interest from the point of the design process and have remove this category of methods from their study.

Taking to account only the case in that the intersection line is given, they have identified only two methods which offer the possibility to measure the dihedral angle. First one is based on the method of substituting planes of projection, and the second is based on the folding method of the planes on a level plane [1], [2], [3].

The use of the substituting planes of projection method has as objective to transform the given planes into ending planes [2]. For example, in fig.1, planes [P] and [Q] and their intersection line are given. By making two successive substitutions of the projection planes these two planes [P] and [Q], will be transformed in vertical planes, thus finally having the possibility to measure the dihedral angle in the new horizontal plane. Substituting of the projection planes will be made in relation to the intersection line of the two given planes, line (D)(d, d’) because when the planes [P] and [Q] will be in the position of some vertical planes, their intersection line will be a vertical line. Thus, in the first step, vertical plane [V] is substituted with another vertical plane [V_1] in such way in that the intersection line becomes a frontal line. The new projection axis (O_1X_1) is parallel with the projection (d) of the line (D)(d, d'). Now the intersection line (D) has the projections (d) and (d'_1). At this stage since the intersection line is a frontal line and at the same time lies both in plane [P] and [Q], the traces P_v and Q_v are parallel with the (d'_1) projection. In the next step the intersection line will become a vertical line.

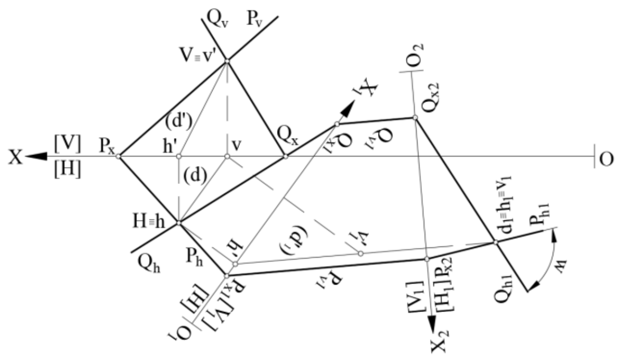

Fig. 1 The method based on the substitution of the projection planes

Therefore, it can be observed that the new projection axis (O_2X_2) is perpendicular on (d'_1) projection and, in the same time on the P_{v1} and Q_{v1} traces. Now, in the new horizontal projection plane [H_1] the dihedral angle can be read. The value of dihedral angle is *w*.

Using the second method involves, actually to measuring the angle between two intersecting lines. To determine the value of this angle, the plan that includes the two intersecting line is folded over a level plane.

In fig.2 can be seen that the planes [P] and [Q] and their intersection line are given. From a point called N(n, n'), who does not lies in plane [P] or [Q], two straight lines are drawn. The line (D_1)(d_1, d'_1) is orthogonal on [P] plane and the line (D_2)(d_2, d'_2) is orthogonal on [Q] plane. These two straight lines will be folded on the [H_1] plane which is a level plane. Its trace is called [H_{1v}].

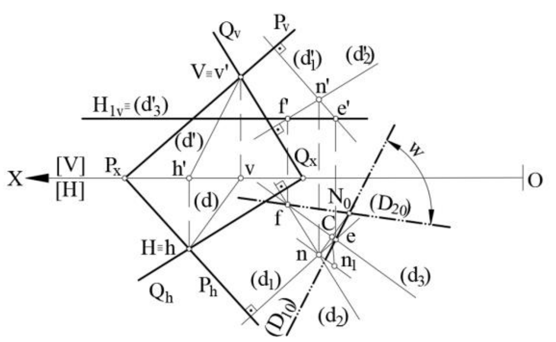

Fig. 2 The method based on the folding method of the planes on a level plane

The folding axis is the straight line (D_3)(d_3, d'_3) which is given by the points F(f, f') and E(e, e'). Since the points F(f, f') and E(e, e') lie right on the folding axis, it is necessary to be found only the folded position of the point N(n, n') on the level plane [H_1]. For this it is necessary to be found the position triangle of the N(n, n') point in relation to the level plane [H_1]. The position triangle of the N(n, n') is the triangle nn_1C, and the folded position of the point N(n, n') is N_0. The line (D_1)(d_1, d'_1) is folded as line (D_{10}) and was obtained joining together N_0 and e. Finally, the line (D_2)(d_2, d'_2) is folded as line (D_{20}) and was obtained joining together N_0 and f. The value of dihedral angle is *w*.

Having as starting point these two methods briefly described above, the authors proposed two solutions which give the possibility to specifying a plane by using the dihedral angle.

2. AUTHORS' SOLUTIONS

The first solution proposed by the authors is based on the method which provides the possibility of determining the value of the dihedral angle by using the method of substituting planes of projection. Analysing this method, the authors noticed that the idea around which it was built is the transformation of the intersection line into a vertical straight line by successively substituting the projection planes. Also, they noticed that, in the same time, the planes are transformed in vertical planes. They concluded that if one of the planes is given, in parallel with the transformation of the intersection line into a vertical line, the given plane can be converted in a vertical plane and the traces of the second plane can be found reversing the steps of this method.

In fig.3 can be observed that an ordinary plane [P] and the intersection line (D)(d, d') are given. The problem to find the traces of a plane [Q] that intersect the plane [P] along the line (D)(d, d') with the required dihedral angle, may have the solution described below.

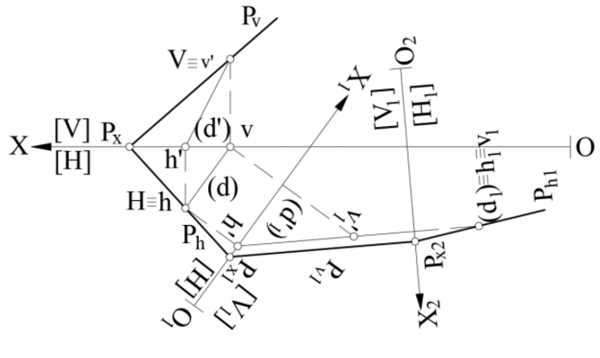

Fig. 3 Transforming the intersection line into a vertical line and the given plane into a vertical plane.

First the vertical plane [V] is substituted with a new one, called [V_1]. This new plane was choose in such a way that, in the new projection system (O_1X_1)[V_1][H], the intersection line (D), having the projections (d) and (d'_1), to be a front line. In the same time, the new vertical trace - P_{v1}, of the plane [P], is drawn. It is parallel with the (d'_1) projection because the line (D)(d, d'_1) is a front line which is lying in plane [P] represented in the new projection system (O_1X_1)[V_1][H]. Then a new projection plane replacement is made. The old horizontal plane [H] is substituted by a new horizontal plane [H_1]. The new projection axis (O_2X_2) is choose in such way in that, in the new projection system O_2X_2[V_1][H_1], the intersection line is a vertical line. Thus, the projection line O_2X_2 is perpendicular on the (d'_1) projection, and the intersection line (D) has, in the projection system (O_2X_2)[V_1][H_1], the projections (d'_1) and (d_1). It can be seen that the (d_1) projection is just a point. The new horizontal trace of the plane [P], P_{h1} results by joining together P_{x2} and (d_1).

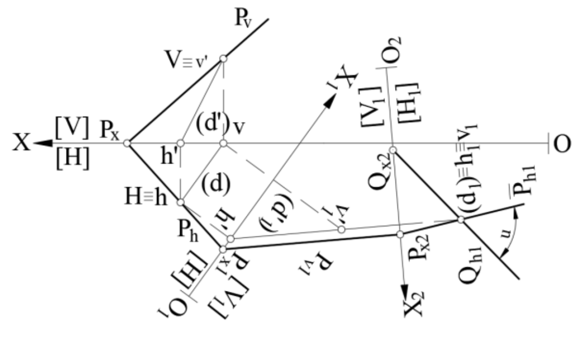

Fig. 4 How to obtain the horizontal trace of the [Q] plane in the projection system (O_2X_2)[V_1][H_1]

Now it is possible to use the known value of the dihedral angle. In the (d_1) projection a straight line is drawing, see figure 4. This one makes angle *u* with the horizontal trace Ph_1. The last line drawn represents in fact the horizontal trace Q_{h1}, in the projection system $(O_2X_2)[V_1][H_1]$, of the plane [Q] which makes with the plane [P] the dihedral angle *u*. Lengthening the Q_{h1} horizontal trace until it intersects the (O_2X_2) axis, the Qx_2 point is obtained.

Next, the Q_{v1} vertical trace, of the plane [Q], in the projection system $O_1X_1[V_1][H]$ must be drawn, see figure 5. So, the vertical trace Q_{v1} starts in the Q_{x2} point and it is parallel with the (d'_1) projection. That because the section line lies, in the same time not only in [P] plane but in [Q] plane and, in addition, in the projection system $(O_1X_1)[V_1][H]$ it is a front line.

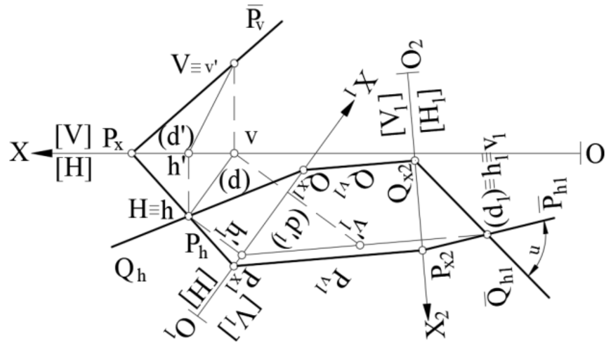

Fig. 5 How to obtain the vertical trace of the [Q] plane in the projection system $(O_1X_1)[V_1][H]$

Lengthening the Q_{v1} vertical trace until it intersects the (O_1X_1) axis, the Q_{x1} point is obtained. Joining together the points Q_{x1} and H results, in the projection system (OX)[V][H], the horizontal trace Q_h, see figure 5.

Next, lengthening the horizontal trace Q_h until it intersects the (OX) axis, the Q_x point is obtained, see figure 6.

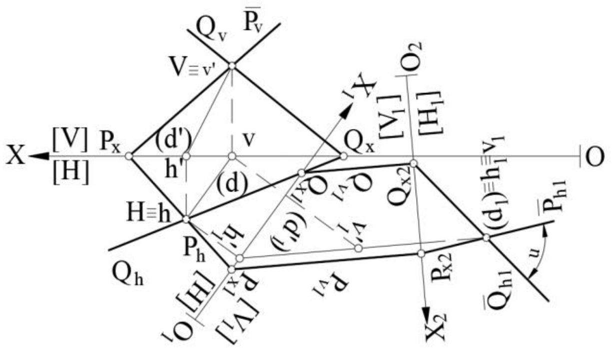

Fig. 6 How to obtain the vertical trace of the [Q] plane in the projection system (OX)[V][H]

Finally, joining together the points Q_x and V results, in the projection system (QX)[V][H], the vertical trace Q_v, see figure 6.

So, the traces of the plane [Q] were obtained starting from the traces of the plane [P], projections of the section line and the value of the dihedral angle.

The second solution proposed by the authors is based on the method which provides the possibility of determining the value of the dihedral angle by using the folding method of the planes on a level plane.

Once again an ordinary plane [P] and the section line (D)(d, d') are given, see figure 7. From a point N situated on the line (D) a perpendicular line on the plane [P] - $(D_1)(d_1, d'_1)$, is drawn.

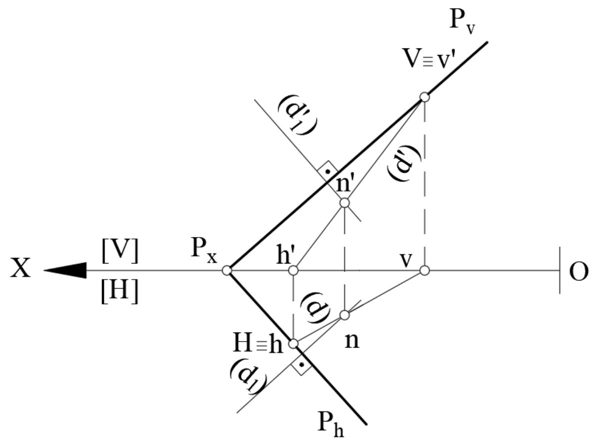

Fig. 7 The start point for the second solution

The main challenge of this solution is to drawn, by point N, another line that together with the right (D_1) determines a plane perpendicular to the section line. To do this the plane [P] is folded on the horizontal projection plane [H], see figure 8. Then a perpendicular line on the (D_0) line is drawn, and is called (D_{40}). In the next step, the projections (d_4) and (d'_4) of this line, are found. In this point may be noticed the fact that the section line (D) is perpendicular both (D_1) line and (D_4) line.

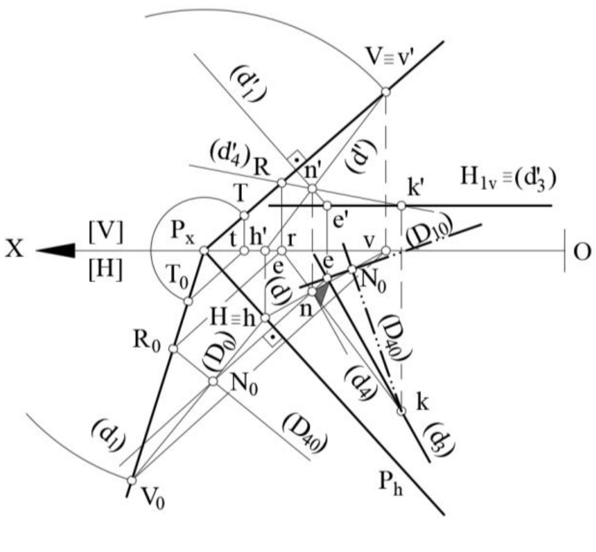

Fig. 8 Finding the correct folding axis

Thus, the line (D) is perpendicular on the plane specified by the lines (D_1) and (D_4). Having this plane and choosing a level plane [H_1], it is possible to determine the correct folded axis which is the $(D_3)(d_3, d'_3)$ line. The $(D_3)(d_3, d'_3)$ line is determined by the points

E(e, e') and K(k, k'). The folded projection of the point N(n, n'), which is N_0, is easily found by using the position triangle of the point N(n, n') in relation to the level plane [H_1], see figure 8. Joining together the projection k and the point N_0 and then the projection e and the point N_0 the folded projections (D_{40}) and (D_{10}) are obtained.

In the next step the (D_{20}) line, which makes with the (D_{10}) line the dihedral angle imposed, is drawn, see figure 9. Then the vertical projection d'_2 of this line is determined.

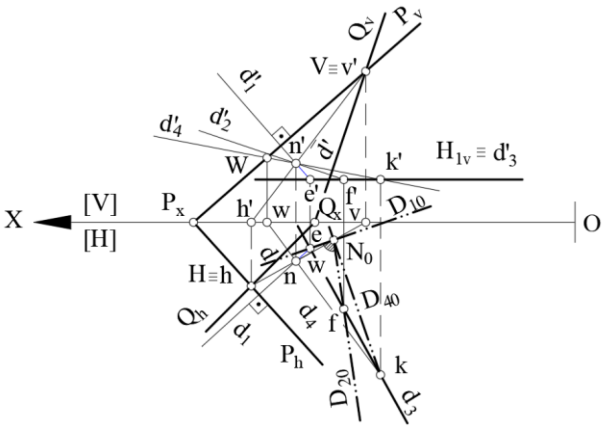

Fig. 9 Finding the traces of the plane [Q]

The vertical trace Q_v is obtained if from the V point is drawing a perpendicular line to the d'_2 projection. Lengthening the trace Q_v until it intersects the (OX) axis, the Q_x point is obtained. Joining together points Q_x and H the horizontal trace Q_h is obtained.

3. TOOLS OFFERED BY THE COMPUTER PROGRAMS

If the engineer chooses to work in AutoCAD 3D the tool which solves this problem is the UCS command. First, a new UCS is set to be aligned with the given plane. Then, the UCS is rotated around to the correct axis, with the desired angle.

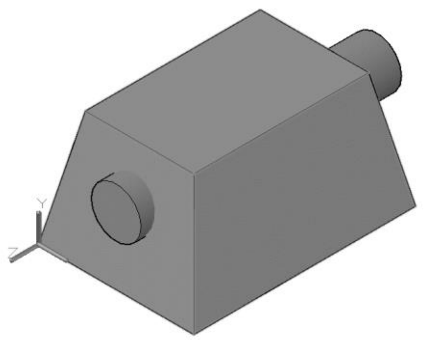

Fig. 10 Specifying a plane in AutoCAD knowing the dihedral angle

In figure 10 can be seen that the UCS was aligned with the base of the truncated pyramid then it was rotated around OX axis. In this case the dihedral angle between the base of the truncated pyramid and the plane of the base of the cylinder is 90 degrees.

If Autodesk Inventor is chosen for work is even easier that in AutoCAD. The user must specify only the given plane and the intersection line, see figure 11.

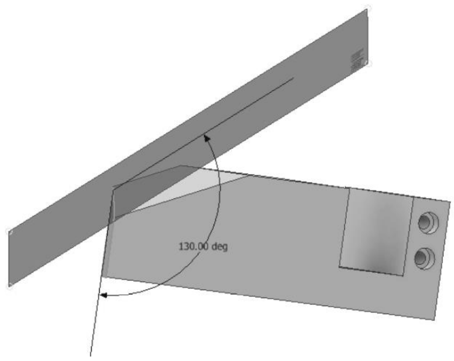

Fig. 11 Specifying a plane in Autodesk Inventor knowing the dihedral angle

4. CONCLUSION

The authors found two solutions to establish a plane when another plane, the intersection line and the dihedral angle between these two planes, are given. These solutions were designed for the case in that the drawing is done in 2D, on a piece of paper or on the computer.

The authors recommend for use the solution based on substitution of the projection planes. This solution is easier to use than the solution based on the folding method of the plane because it is simpler.

If drawing is done in 3D model the computer programs such as Autodesk Inventor or AutoCAD provide powerful tools, easy to understand and use.

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