Kevin NOACK, Martin Friedrich EICHENAUER, Daniel LORDICK

COMPLEX HEAT EXCHANGERS – THE COMBINATION OF FRACTAL GEOMETRY AND ADDITIVE MANUFACTURING

Abstract: *Complex structures for heat exchangers in high-temperature applications are developed in the research project* ***instaf****. The driving force behind these developments is the new design freedom given by the additive manufacturing (AM) of ceramics. With this technology, the complexity of form is no longer an obstacle for profitable solutions. The application of fractals in this context is very promising, since it takes advantage of the new freedom while on the other hand the algorithms for the generation of such forms are relatively easy to handle. This paper presents some proposals for heat exchangers or mixing structures, which are inspired by natural formations. The underlying assumption is: the efficiency of a heat exchanger increases with the surface area of the wall between the fluids. The surfaces here are derived from a series of fractal curves within a given outer shape. The algorithms for the curves use the Lindemayer system (L-system), which allows for steering a large variety of curves systematically.*

Keywords: *Heat Exchanger, Biomimetics, Fractal Geometry, Space-Filling Curves, Lindenmayer-System, Form-Finding Methods, Computer-Aided Design, Additive Manufacturing*

1. INTRODUCTION

The **instaf** project – integrated structures for additive manufacturing – is a joint research project of research facilities and companies in Austria and Germany. It focuses on the development and design of new structures for mixers and heat exchangers. The resulting structures take advantage of the new design freedom within additive manufacturing (AM) technologies. In contrast to the established technologies, where the design to a high degree needs to follow fabrication constraints, the new paradigm may be at last: form follows function.

Our working group focuses on the development and design of the inner part of mixers and heat exchangers. Other partners within **instaf** are responsible for material research or the comparison of different AM technologies. One partner is evaluating some of the structures with computational fluid dynamics (CFD). The fabrication of selected prototypes for experimental testing is also an ongoing part of the research. Therefore, in cooperation with all partners, the complete sequence of designing, simulation, fabrication and physical testing is available to improve the structures in an iterative process.

One motivation for the research is the insufficiency of common metal heat exchangers at high temperatures, since they are composed of very many components and assembled with solder. This is an issue especially in applications with fuel cells.

One solution for this problem lies in the additive manufacturing of heat exchangers from ceramics. AM expands the spectrum of materials to ceramics and polymers, and, with special ceramics, allows high-temperature applications in the range up to 1,900 °C. The new materials additionally have a good thermal and electric conductivity, a good chemical resistance and a lower weight than the metal counterparts. Nevertheless, the major advantage of AM is the new freedom of forms with material thicknesses down to a few micrometers and the fabrication as a whole in one production step. Now, heat exchangers can be designed and optimized with the focus set on their usage and function in the working environment. The implementation of the more efficient countercurrent flow principle in opposition to the current standard with fuel cells, the cross flow, can be obtained.

The geometrical part of **instaf** concentrates on the generation of a large heat exchange area. With a large exchange area (corresponding to a long profile curve), the fluids can transfer more thermal energy from the warmer fluid to the colder one. This shall improve the heat exchanging performance.

The outer shape is given externally; the inner structure is subject to this research. For the transfer surface, fractal geometry gives an encouraging starting point. Inspired by the forms and shapes in the living nature, self-similar structures like the space-filling curves are a key for first results. Their implementation is rather simple but enables complex structures for mixers and heat exchangers. First examples are given in [1], [2].

On top of the fractal design, the performance may be improved by putting microstructures onto the macrostructures. Such microstructures can result from stochastic processes like the partial Brownian motion. A first approach is given below (7.). Besides geometric considerations, the limiting parameters of AM, like the smallest achievable wall thickness, are a constraint in the form-finding phase.

The first evaluation criterion is the temperature gradient between fluid in and fluid out. Other criteria depending on the flow behavior like the average flow velocity, the dispersion of the fluids or inducing turbulences are not part of this work. Further CFD simulations are subject to future investigations.

2. FRACTAL GEOMETRY AS A BIOMIMETIC MODEL

In the living nature, many examples for large functional surfaces can be found [3]. Their purpose lies in the raise of a molecular or thermal exchange. The exchange of oxygen with carbon dioxide or nutrients and water need that principle in breathing organs or intestinal villus for digestion. However, structures without any material transfer are of higher relevance here. In a heat exchanger, the fluids should intertwine especially homogenously without blending them.

Another approach used here exploits branching structures. They split a starting path into a multitude of paths. A meaningful condition is a constant profile area. A famous split structure is the Romanesco (see figure 1). With the help of branching, the surface can be multiplied by a large factor. A zoom into the structure reveals approximatively the same structure as the starting point, the whole Romanesco. This property is known as self-similarity. Fractals are self-similar sets and can be found stochastically in nature or can be computed with recursive algorithms.

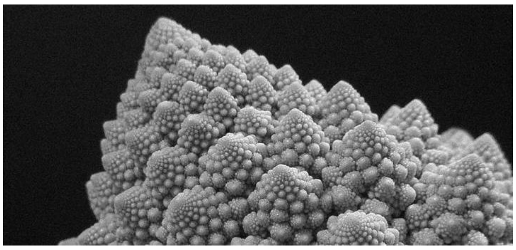

Fig. 1 The Romanesco is a natural fractal [10].

In nature, all examples are self-similar only down to a certain point. This clearly is because of the desired function of the structure and was evolved during many generations of the species. Approximatively self-similar structures with a cooling function can be found for the feet of birds. Penguins or ducks always have cold feet due to their environment. A structure in their legs allows them to save energy in cooling the blood in the arteries and simultaneously warming the vein's blood. Also in the fin of the common porpoise with the large surface of capillaries the blood can reach any distant blood vessel with low temperature (see figure 2, left). In the transition of fin and torso, the capillaries and veins are placed efficiently next to each other. Thin-walled venous channels embrace the great artery (see figure 2, right). Such, the warm blood from the torso gets cooled while the counter-current flowing cold venous blood gets warmed. The advantage is, an energy-consuming heat transfer in the torso is not required.

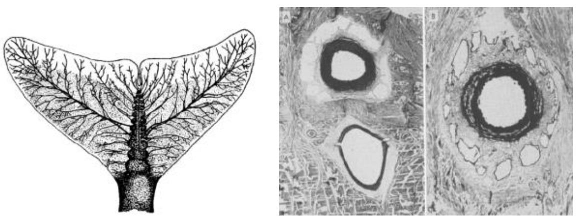

Fig. 2 Arterial system in the fin of the Phocoena phocoena. Left: The dispersion of the capillaries leads to an energy-saving way of temperature regulation [7]. Right: The artery of the Tursiops truncalus is surrounded by veins, visible as thin-walled channels [8].

3. FRACTAL GEOMETRY AND SPACE-FILLING CURVES

One approach to model biological structures as given in the previous chapter is fractal geometry. A fractal is the border case of a certain recursive algorithm. It can be read as a multiple reduction copy machine (MRCM) (see [9]), which maps one starting geometry into N smaller copies of it in each iteration step. The downsizing factor is denoted by ε.

Fractal structures are self-similar and invariant to scaling. In addition they can be characterized by the so-called fractal or self-similarity dimension, which is an application of the Hausdorff dimension [11].

A fractal is an object (curve, surface, etc.) with an in general non-integer dimension D. Its fractal dimension can be calculated with

$$D = \frac{log(N)}{log(\varepsilon)}. \tag{1}$$

Here, fractal curves have a dimension $D \in]1;2]$. The Koch curve is a good example to start with. The process is beginning with a straight line, which is split into three parts of equal length. The center part is then replaced with two reduced lines (see figure 3).

Fig. 3 The first four iteration steps of the Koch curve, a well-known fractal.

Such, you get $N = 4$ straight lines with a third of the original length each ($\varepsilon = 3$). The fractal dimension of the Koch kurve is

$$D_{Koch} = \frac{\log(4)}{\log(3)} \approx 1.26. \tag{2}$$

The fact that value differs from the dimension of the initial object (the straight line), it is an indication for a mathematical fractal. Random numbers can be added to the iterative process, so that the final appearance differs a little (see figure 4). Such structures are called statistical self-similar and still fulfill the definition of the fractal dimension.

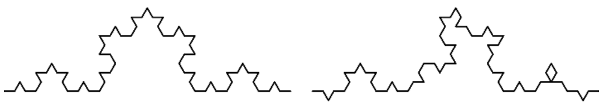

Fig. 4 The standard Koch curve on the left and the generation with a parameter as random number on the right side [6][12].

The Koch curve can be extended to the Koch snowflake. This example can be seen in figure 7, right.

A very interesting subset of the fractals, are the space-filling curves. The more fractal curves approach $D \to 2$, the more they are space-filling in the plane. Space-filling curves play the key role in the strategy of designing heat exchangers for additive manufacturing as presented in this paper. A space-filling curve is a one-dimensional curve that covers a two-dimensional surface or a n-dimensional space completely. This is done in a regular grid of knots with a certain path. Such a curve is self-similar, self-avoiding and simple (FASS – space-**f**illing, self-**a**voiding, **s**imple, self-**s**imilar) [4].

Following the formal definition, a curve is the image of a continuous mapping within the domain $[0,1]$. A FASS curve needs a lower (topological) dimension than the space it covers up (as fractal dimension D, which is an integer). As an example the Hilbert curve starts in the unit square with its center. The unit square is divided into four equal but smaller squares. The central points are connected with straight lines (see figure 5 left). In the next iteration step, the resulting four curve parts are connected with additional lines to become a continuous curve (see figure 5 right).

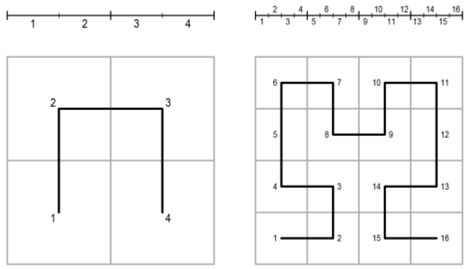

Fig. 5 The generation of the Hilbert curve, starting in the inital unit square.

The Hilbert curve obviously has topological dimension 1. Its fractal dimension is

$$D_{Hilbert} = \frac{\log(4)}{\log(2)} = 2 \qquad (3)$$

because the initial square is mapped into $N = 4$ squares with each side of halfed length ($\varepsilon = 2$). The Hilbert curve has a lower topological dimension than D, so it fulfills the conditions to be a FASS curve. This means, the Hilbert curve can cover up the unit square (in the border case of infinite iterations) and, after scaling, the whole $\mathbb{R}^2$ completely.

The implementation of FASS curves can be managed with the Lindenmayer system (L-system) [5]. Originally, Lindenmayer developed his recursive system to model the growth processes of plant development. But, the underlying rewriting technique also corresponds to the self-similarity of fractal curves.

The algorithmic generation of the curve is described as the movement of a point with F (draw a straight line), - (turn left with the angle α, for a FASS curve $90°$) and + (turn right with the angle α). This kind of procedure is known as turtle graphics. F, +, and - belong to the alphabet of the L-system.

An L-system needs an initial axiom. With a given production rule, the starting curve is mapped into the next-level curve. The result serves as input for the next iteration step. This leads to a scale-invariant geometry because the same production rule is used in every iteration step. The application of such a scheme to the Hilbert curve can be seen in figure 6. The variables for drawing a line and turning left or right are part of the production rule as well as in the curve itself. The production rules depend on the chosen curve. With this scheme a certain variety of FASS curves can easily obtained.

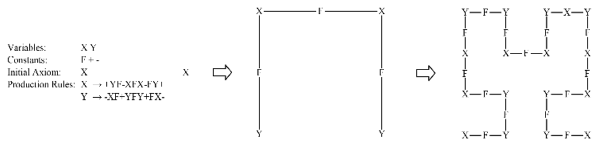

Fig. 6 The Hilbert curve and its generation with the L-system [2]. The recursive scheme takes the result of an iteration step as input geometry for the next step.

On a given grid with n^2 squares, the behaviour of different curves can be modeled. This has been done with a curve generator in [2]. In [2], all the possible motions to describe a FASS curve are classified and evaluated with respect to the usability for the **instaf** project. Such, the variety of feasible curves was reduced to a healthy amount.

Since this approach corresponds to turtle graphics, it was tempting to present it during the Long Night of the Sciences 2018 and other public relations activities at the TU Dresden. A LEGO® Mindstorms robot was constructed and programmed by a gifted student to draw FASS and fractal curves in different iteration steps onto paper. The code was implemented again with the L-system. Two versions have been realized: a stationary plotter, which writes on small paper reels (see figure 7, left) and a mobile robot, which navigates over large sheets of paper (see figure 7, right).

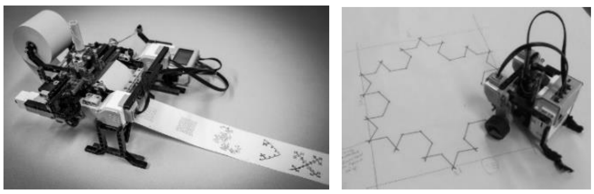

Fig. 7 A LEGO® Mindstorms robot drawing FASS curves onto paper. The left version is stationary. The right version draws while driving autonomously.

4. DESIGN OF HEAT-EXCHANGING OR MIXING STRUCTURES WITH TWO FLUIDS

The first assumption for the following example is, that two fluids shall pass a box. Starting with two simple inlet shapes (triangles) the outlet shall be of a complex intertwined shape. The oultlet could be used for mixing the fluids or for passing them on to a similar but extruded structure for heat exchange, either cocurrent or countercurrent. In both applications the seperating profile between the fluids at the outlet should be of maximum length in relation to a minimal wall thickness. At the same time the cross-section area of the fluid channels was choosen to be constant from the inlet to the outlet. For the seperating wall this results in a simple but rather thick profile at the beginning and a rather long but thin profile at the end. For the creation of such a separating wall, single iteration steps of a FASS curve were combined to one NURBS surface by a so-called lofting process. The CAD software used for this process is Rhinoceros 3D with the plug-in Grasshopper. The latter is a visual programming environment for parametric modeling.

The first example is based on the Peano curve (Fig. 8). The Peano curve itself serves as the control polygon of a NURBS curve, which results in a curve similar to the Wunderlich curve. In contrary to the Peano curve, this smoothed curve avoids self-intersections. The first four iteration steps are selected and stacked with a predefined distance. With the application AM in mind, the stacking can be arranged corresponding to the layers in the AM process. This guaranties that the structure is processible and none of the layers has unconnected parts (islands).

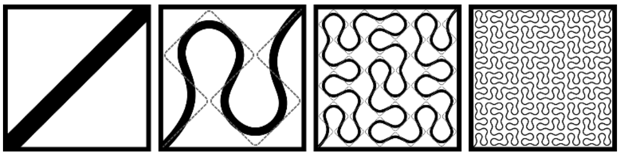

Fig. 8 Mixing structure based on the Peano curve (red). The second picture shows the smoothing of the edges to avoid self-intersections. The layers are combined with a NURBS surface to create the three-dimensional mixing structure.

Next, four iteration steps of the Peano curve become the profile curves of a NURBS surface. The NURBS surface is thickened according to the previous assumption of constant cross-section areas. Such the separating wall's thickness decreases from 4.5 mm to 0.2 mm. The factor 22.5 of decreasing implicates that the length of the separating curve grows by the same factor 22.5, leading to a large transfer surface at the outlet.

The whole structure was arranged within a cubic box (Fig. 9). The structure was built in different materials and with different AM technologies at the Fraunhofer IKTS (Dresden) and by FOTEC (Vienna). The tested materials are ceramics, silicon carbide, polymers and aluminum. The tested AM processes are Fused Filament Fabrication (FFF) and Lithography-based Ceramic Manufacturing (LCM) [1].

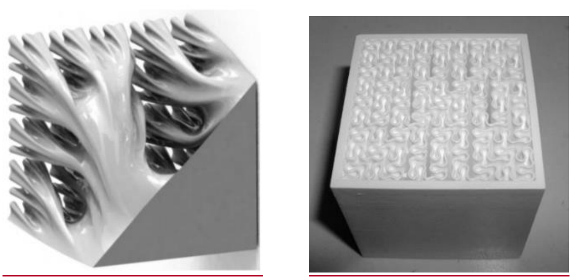

Fig. 9 Dispersion of two fluids with a mixer (left: rendering of one fluid channel, right: sintered component). The design is based on the curves presented in figure 8. The right component was fabricated at the Fraunhofer IKTS in Dresden by using Lithography-based Ceramic Manufacturing (LCM) with Al_2O_3 suspension.

5. OTHER EXAMPLE STRUCTURES

The cubical form is not very useful for some of the desired technical applications. The next example applies for two concentric channels. In this case the transfer wall has to be derived from a closed fractal curve (Fig. 10). Such a curve can be based on the Hilbert curve (see fig. 5 and 6), when the curve's orientation is altered [2]. This example also applies for the case, where one fluid flows through the structure, while the other one, like air, is just on the outside with no additional tubular structure. This is an often-used principle to cool down a hot fluid with surrounding air. The structure with a height of about 20 mm was fabricated at the Fraunhofer IKTS (Dresden).

Fig. 10 Another way of designing a transfer surface for a heat exchanger with a FASS curve, here the Hilbert curve. The models show a 5/8 cutout.

6. DEALING WITH DIFFERENT OUTER SHAPES

The previous idea can be expanded to curved tubular shapes like a U-shape for an immersion heater. Still the Hilbert curve from the previous example is a good choice for the profile. But, in order to adapt to the curvature, it is meaningful to treat the inner and the outer side of the U-shape differently. In this example, the fractal curve is split in two halves and the inner side of the bow is equipped with one iteration step lower than the outer side. Like in the previous examples, the cross-section area for the fluid channel remains constant.

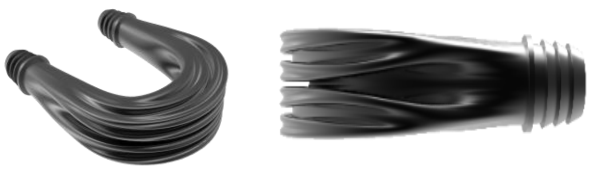

Fig. 11 A curved heat exchanger (U-shape) as immersion heater. The curves have a different number of iteration steps in the inner and outer side of the curve.

In addition to the other examples, in figure 11 a possible connection geometry for a hose has been modeled.

The last example in figure 12 shows an S-shape. This is beneficial in many technical application like gas turbines, because the connections for the fluids need to lie on opposite sides of the component. An extension to a meander is obviously possible. One drawback can be that in some AM processes the inside of the shape can hardly be cleaned from residues like powder or resin.

Fig. 12 The S-shape of a curved heat exchanger enables connections on opposite sides of the component.

For the S shape, flow simulations with ANSYS are in preparation. Also, after fabricating the structure, physical experiments will complement the simulation results.

7. ADDITIONAL MICROSTRUCTURES

Additionally to the fractal macrostructures, some microstructures can be added. Barriers in the flow or a rough surface, generated for example with fractional Brownian motions (fBM), raise the surface area even more. A fBM can be defined with the variable $H \in [0,1]$, as Hurst parameter to measure the roughness with

$$E[B_H(t)B_H(s)] = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t-s|^{2H}) \qquad (4)$$

as a covariance function for two time steps s and t. The character of the roughness depends on the choice of H, of the standard deviation σ and the number of grid points N. For $H \to 1$, the surface gets smoother. A simple example for $N = 2048$ shows the difference in figure 14.

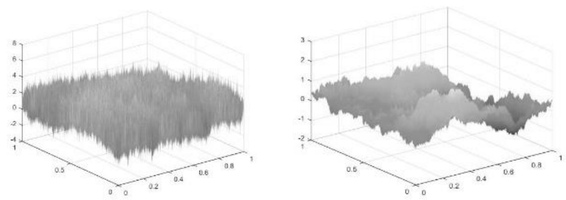

Fig. 13 Two fractional Brownian motions as rough surfaces for $N = 2048$ and σ=0.4 are displayed. The Hurst parameter is $H = 0.05$ on the left and is $H = 0.5$ on the right side.

But, more important, the rough surfaces on the macrostructures disturb the fluid's flow. This induces turbulences which may lead to a better performance. The difference between a laminar flow and a turbulent one is illustrated in figure 14.

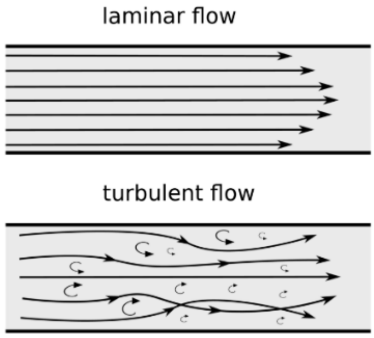

Fig. 14 The laminar or straight flow of a medium through a tube can be seen in the upper figure. The lower one shows the turbulent flow with some whirls [13].

An evaluation of the quality needs to be done. The resistance near the wall could be a problem. The Reynolds number is the criteria to determine the way of the flow (turbulent or laminar). It depends on the characteristic length, the fluid's velocity and its kinematic viscosity. The local difference in the wall thickness must be at least the minimum of the printable layer thickness. Else it would not be fabricated due to the limits of the AM technology. There might also be some issues with offset areas and surfaces which can result from relatively large prongs. These often cannot be fabricated with the desired mechanical stability.

8. CONCLUSION

It was shown, how heat exchangers and mixing structures can be designed based on self-similar curves. Inspired by fractal and branching structures in nature, space-filling curves are adapted for a functional approach. These curves are the starting point for a large transfer surface, which is beneficial for an outstanding heat exchanging performance. Their implementation with the Lindenmayer system enables a large variety of curves and forms with a relatively simple algorithm. A selection of feasible curves can be extracted with respect to the performance parameters of the respective heat exchanger.

In combination with the possibilities of additive manufacturing, a new way of building complex forms in one production step is available. This should be regarded as a promising design path for the near future.

In the range of possible forms, only the producible versions are considered. They have to avoid self-intersections and, for mechanical stability, the wall thickness may not be less than at least a few layers in the used AM process.

9. FURTHER RESEARCH

More structures will be modeled in the future. They will be evaluated with simulations and experimental tests. One evaluation parameter is the temperature at inlet and outlet of one or two fluids. Additionally, the flow behaviour will be valued and improved. Furthermore, the connections to the adjacent components need to be modeled.

A collection of all the parameters of AM in cooperation with the **instaf** partners should lead to a software which evaluates a geometry on-time (live). This software tool should include among other especially the various techniques, the materials with their physical properties and the flow behavior. The criteria for the evaluation are the possibilities of AM (and with which technique) and a rough overview about the heat exchanging ratio, including a finite element analysis and Computational Fluid Dynamics.

The search for the best curve in specific components will be continued. This includes computational fluid dynamics for simulations and experimental tests with prototypes within the **instaf** project.

In addition to the design with fractal curves, the fluid's straight flow should be interfered by turbulences. They can be induced by roughness, morphed onto the transfer surface. This first leads to an even larger surface area and second, to a lower flow velocity. First studies of rough surfaces with the help of Gaussian distributed random variables and partial Brownian motions have already been explored in [1]. Still, the limitations of the additive manufacturing processes have to be considered.

The selection of the best form also greatly depends on the working environment and other external parameters (used fluids, initial temperature, pressure, material, average working time per day, etc.). A fitting connection geometry to adjacent components is another crucial point. This and the material combinations have to be explored further. Thermal short circuits at the connection geometry have to be avoided.

Other microstructures will be developed and tested. Starting point can be some noise functions like Perlin noise or a variation of the Weierstrass function.

Still, there is a limiting factor given by the maximum file size of some software products within the AM production process. Therefore, very detailed structures are still not achievable in larger components.

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Authors:

Kevin NOACK, Research Assistant and PhD student, Technische Universitat Dresden, Department of Mathematics, Institute of Geometry, E-mail: Kevin.Noack@tu-dresden.de

Martin Friedrich EICHENAUER, Research Assistant and PhD student, Technische Universitat Dresden, Department of Mathematics, Institute of Geometry, E-mail : Martin_Friedrich.Eichenauer@tu-dresden.de

Prof. Dr.Ing. Daniel LORDICK, Head of Geometric Modeling and Visualization, Technische Universitat Dresden, Department of Mathematics, Institute of Geometry, E-mail : Daniel.Lordick@tu-dresden.de