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APPLYING THE THEORY OF THE PLANE IN THE CIVIL ENGINEERING FIELD

Abstract: *To avoid the routine of buildings composed of classical geometric bodies, some methods used in contemporary architecture consist in: the use of intersected solids, the cutting with planes in different positions, as well as the recomposing of volumes.*
In this paper, after a theoretical introduction regarding the intersections of planes, from both analytically and graphically point of view, the authors approach applications of these in the achievement of some elements in the civil engineering field.

Key words: *architectural styles, intersections of planes, facades, awnings, roofs, geodesic dome.*

1. INTRODUCTION

Starting with the XXth century the trends in architecture have undergone a real metamorphosis caused by the necessity to build in a fast, ratioanal and efficiently rate. Modern architecture is characterized by vastness and diversity - Fig. 1.

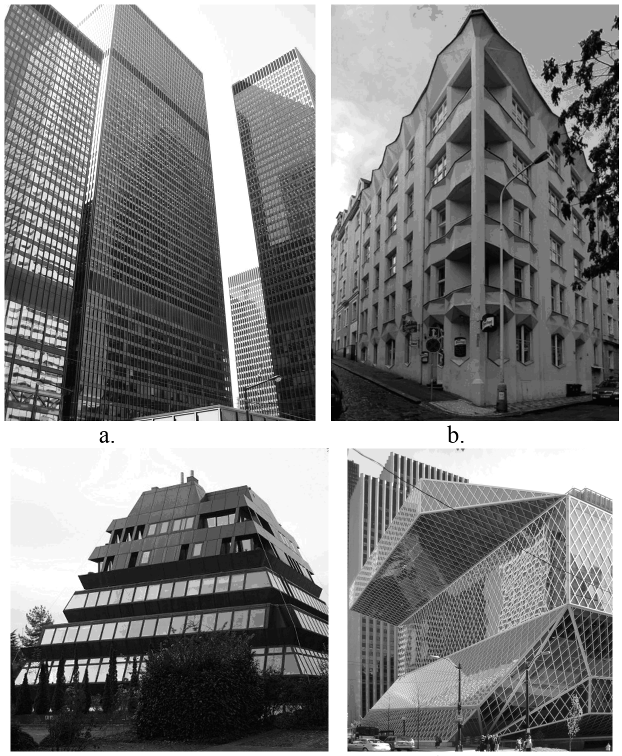
Thus, in a first stage the internationalist style was contoured (Fig. 1a [5]) which was characterized by the construction of tall buildings (skyscrapers) of parallelepipedic form with flat and bright surfaces, suggesting order and regularity. One of the prominent architects of this style was Le Corbusier who established as a priority the importance of the functionality features and has combined in an efficent way the available materials, respectively steel, glass and concrete.

Then it followed the Dadaism and Cubism movements (Fig. 1b [4]) which represented attempts to rebuild the form through decomposition and recomposition of the geometric solids.

In parallel, the futurism current was contoured (Fig. 1c [5]). It manifested as a rejection of the historical currents that, one can admit, are veritable architectural jewels, but no longer corresponded with the idea of speed, movement, dynamism in the most general way. It was inspired by the military architecture.

Once with the unprecedented technological development in the last decades of the XXth century, it appeared a post modern style, the so called high tech architecture named also structured or technological expressionism, characterized by simplicity and elegance. Thus, by applying various methods to the classical geometric volumes as the using of cuts (intersections of planes) or some simple curved surfaces, result buildings with their own personality, specific to sophisticated urban areas, denoting the courage and valorizing the imagination of architects. Taken to the edge of the possible, from a structural viewpoint, this current turns into deconstructivism (Fig. 1d [5]), which by displacement and distortion of solids suggests a controlled chaos, ie a non-Euclidean volumetry (composition).

The achievement of these types of construction was made possible by the discovery of advanced materials - as high-strength glass, carbon fiber and other composite materials, of the high performance technologies, and also due to the possibility of computer modeling of the structures.



a. b. c. d.

Fig. 1 Modern and postmodern style buildings [4, 5].

In this paper the authors aim an analysis regarding the applications of the intersections of planes in the civil engineering field, as a way out method from the conformism of simplistic volumes specific to modern and postmodern architecture trends.

2. BASIC ABOUT THE PLANE AND INTERSECTION OF THE PLANES

As it is known from geometry, the plane can be defined by its elements - three non-collinear points, a line and an exterior point, two parallel or intersecting lines, respectively by (two) of its traces. The most direct applications in civil engineering result from the definition of the plane by its elements, namely defining the plane as a plate shape. If the plate is a different polygon than a triangle, will put the condition that the other vertices to belong to a straight line of the plane that was defined by three explicit vertices/points.

The intersection of two planes is a line, obtained either if certain conditions are fulfilled - when the plane is analytically defined, either using the points of intersection of the edges of a plate with the other plate - when the plane is graphically defined.

2.1. Analytical determination of the intersection of planes

A plane can be defined mathematically in several ways. For the current applications in civil engineering are of interest the forms that will be further presented.

- Cartesian implicit equation of the plane [1]:

$$[P]: Ax + By + Cz + D = 0 \qquad (1)$$

respecting the condition $A^2 + B^2 + C^2 > 0$
where,
A, B, C = directors parameters of the plane (components of a normal vector to the plane)
D = parameter depending on the position of the plane relative to the axes origin.

- The equation of plane defined by three non-collinear points $M_1(x_1,y_1, z_1)$, $M_2(x_2,y_2, z_2)$, $M_3(x_3,y_3, z_3)$, is expressed in the form [1]:

$$[P]: \begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \qquad (2)$$

where (x_i, y_i, z_i), i=1,2,3 represent the points coordinates through which the plane passes.

- If the points through which the plane will pass represent exactly the intersections with the coordinate axes, respectively A(a,0,0), B(0,b,0), and C(0,0,c) - called in descriptive geometry vanishing points, the equations (2) became:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0 \qquad (3)$$

The coordinates of the previous vanishing points can be expressed as:

$$a = -\frac{D}{A};\ b = -\frac{D}{B};\ c = -\frac{D}{C}; \qquad (4)$$

if $D \neq 0$.

Being two planes $[P_1]$ and $[P_2]$ defined by the implicit form:

$$[P_1]: A_1x + B_1y + C_1z + D_1 = 0 \qquad (5)$$

$$[P_2]: A_2x + B_2y + C_2z + D_2 = 0 \qquad (6)$$

these are intersecting (secant) about a straight line, if is satisfied the condition that the rank of the matrix to be two [1]:

$$rang\begin{pmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{pmatrix} = 2 \qquad (7)$$

or, expressed in other way, if the following condition is accomplish:

$$\frac{A_1}{A_2} \neq \frac{B_1}{B_2} \neq \frac{C_1}{C_2} \neq \frac{D_1}{D_2} \qquad (8)$$

2.2. The graphical/descriptive determination of the intersection of planes

As specified previously, in civil engineering applications, the representation of planes in the form of plates are frequent used. In Fig.2 it is represented the intersection between two triangular plates [ABC] and [MNP].

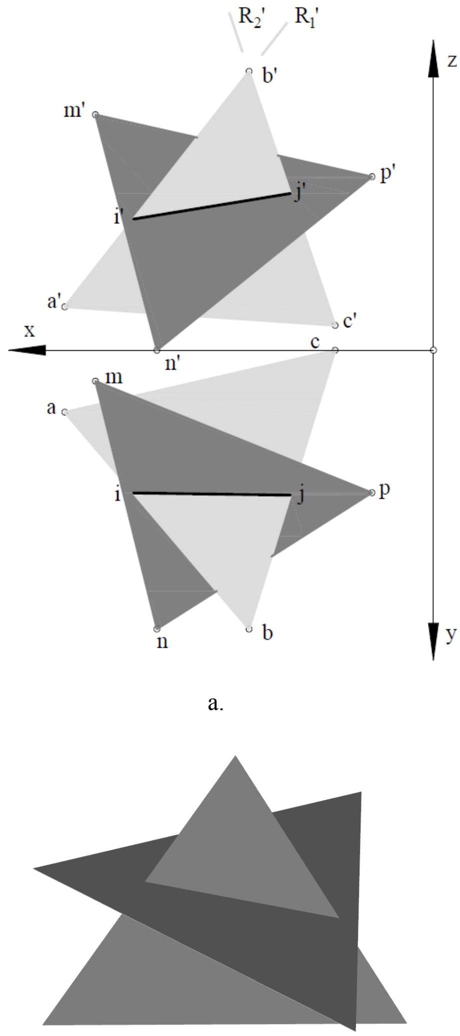


Fig. 2 Intersection of two oblique triangular plates.

The intersection line (IJ) is obtained in this case using the auxiliary vertical projecting planes [R_1] and [R_2], passing through the sides (AB) and (BC) of the plate [ABC] [3]. One established the visibility of the plates in each projection. In Fig. 2b is the 3D representation of the intersection of the plates. But in applications to various elements of construction - such as cuts on the facades, awnings, decorations, are kept only the visible part of the plates bounded by the intersection line.

Similarly, in Fig.3 it is represented the intersection between two quadrilateral oblique plates. One observe that for establish the projections of the forth point of the plate it is necessary to impose the condition that the last point to be lying in the plane defined by the three known vertices. This can be solved with the help of diagonals of the quadrilateral from the known projection, respecting the affinity condition [3].

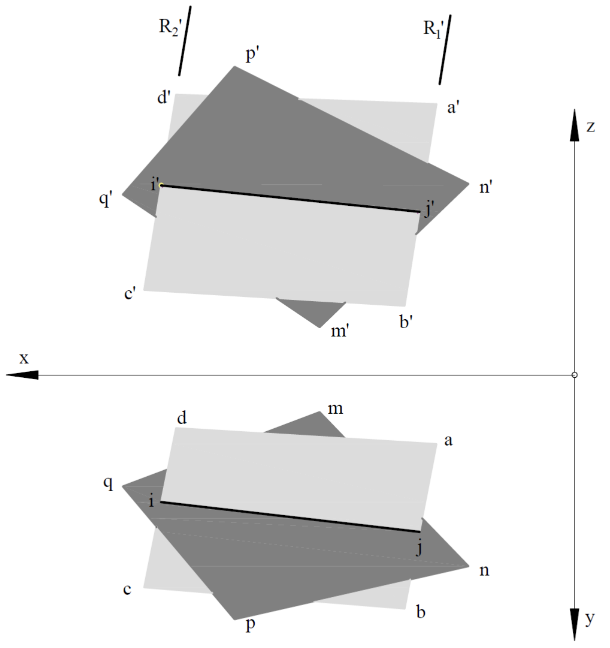


Fig. 3 Intersection of oblique quadrilateral plates.

In many applications, at least one of the plates is in particular position relative to the projection planes. In this case, the intersetion yields directly. In Fig. 4a is represented the intersection between an oblique plate and a horizontal plate, respectively in Fig. 4b is represented the intersection between an oblique plate and a frontal plate.

3. APPLICATIONS OF THE INTERSECTION OF THE PLANES IN CIVIL ENGINEERING FIELD

Even if from analytical or descriptive point of view the intersection of planes do not raise problems, we meet frequently in construction applications of this subject, which is the reason why the authors have focused to detail some specific aspects of this field. It must be mentioned that in terms of structure, they generate, in some cases, additional calculus, not always easy.

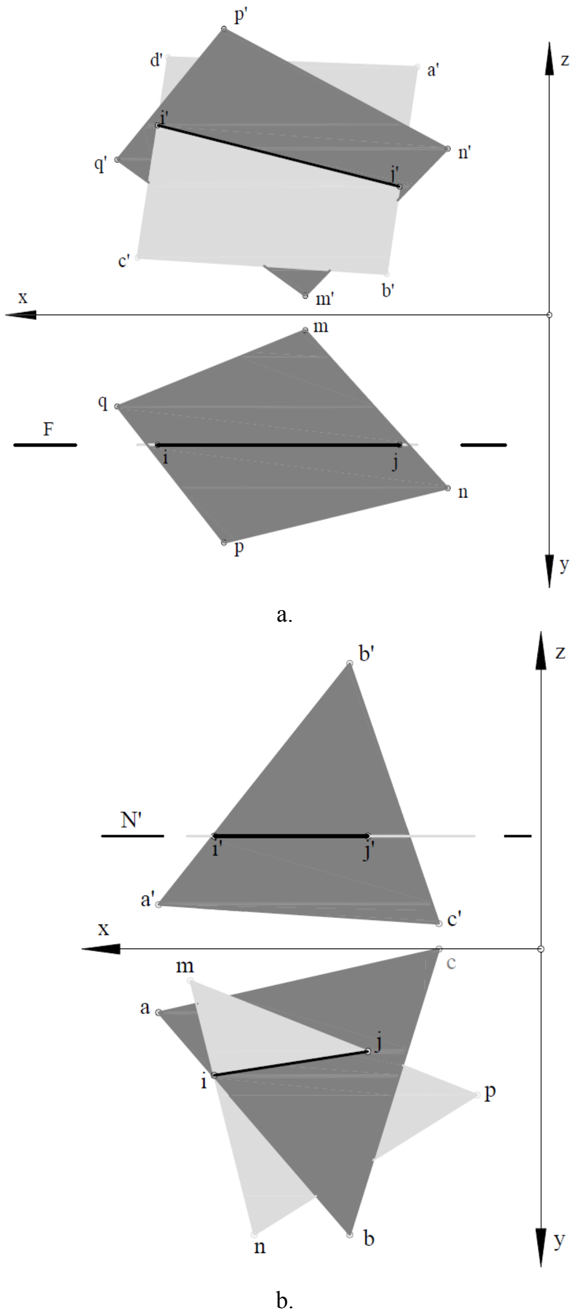
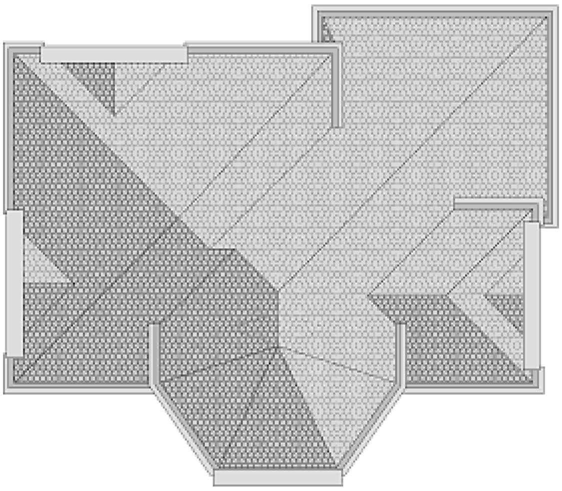


Fig. 4 Intersection between an oblique plate and plates in particular positions.

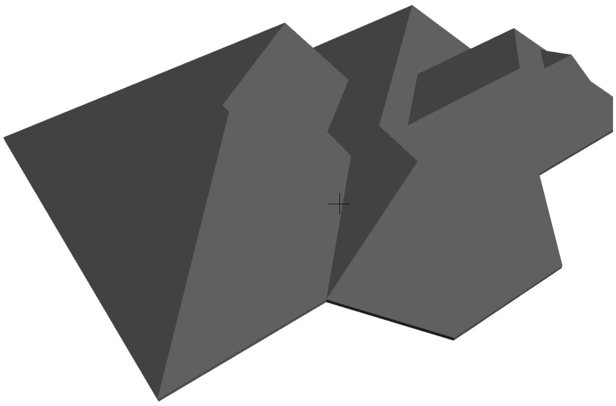
As was specified in the introduction, in modern architecture - which responds to the needs of contemporary people to be pragmatic and efficient, one of the solutions for avoiding the ordinary building appearance, is to use intersections of planes or polyhedra cutting by planes.

Among the most known applications, are the buildings having truss roofs, which, depending on the complexity of the outline shape, consist of a number of plane faces called slopes with equal or unequal inclination that intersect after rafters, valleys or ridges [2]. In Fig. 5a was represented the solving of a roof having equal slopes and a polygonal contour as horizontal projection, using projections with elevations. In Fig. 5b is given the 3D representation of the roof.

As it is known, the intersection edges of equal inclination slopes are projected on the horizontal plane about the bisecting lines of the interior angles of the slopes traces on the eaves plane, which in particular are represented by the contour line.



a.



b.

Fig. 5 Solving a roof with slopes of equal inclinations.

In the case of the roofs having slopes with unequal inclination, it is necessary to use the steepest line (the line of maximum inclination) of the sides, which is perpendicular to the horizontal trace of the plane, respectively on each of the horizontal line of the given plane. For determining the intersection edges of the sides, one draws slope planes having the steepest line perpendicular to each side of the polygon contour, that is the perimeter projection. Using the horizontal lines of the planes one obtained the intersectios of the slopes.

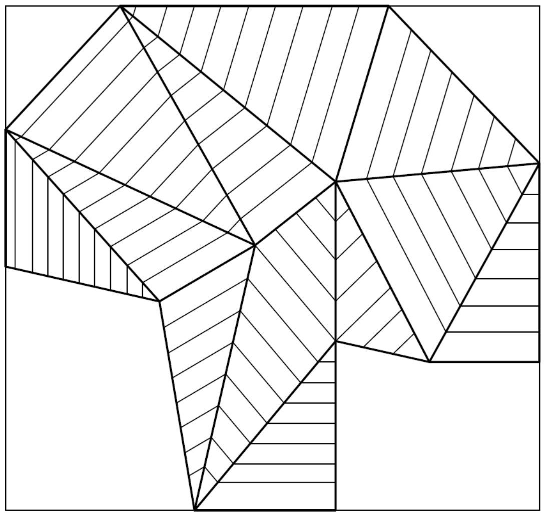
There are situations when a nonconformist approach is desired in order to achieve the covering of a building – see Fig. 6. Thus, in Fig. 6a it is represented the top view of the roof respectively, in Fig. 6b a picture of the roof composed of a joining of planes having different inclinations.

Building envelopes have an important aesthetic role. In some situations one meet different plane joints to decorate windows, loggias and balconies or their intrados, and often to the achivement of the facades (Fig.7). The special form of the facade is in some cases related to the interior distribution of useful space indoors, but in other cases represent merely curtain walls or false glass surfaces.

In all cases, the aim is to obtain aesthetic and visual effects, to avoid the monotony and regularity of flat surfaces.

Fig. 7 shows the folowing representative buildings:

- Fig. 7a and 7d – commercial buildings on Orchard Road in Singapore;
- Fig.7b – Polyvalent Hall in Cluj Napoca;
- Fig.7c – China Financial Information Center in Shanghai;
- Fig. 7e - Health Department building in Bilbao, Spain;
- Fig. 7f - production and office building in the industrial zone of Munich, Germany.



a.

b.

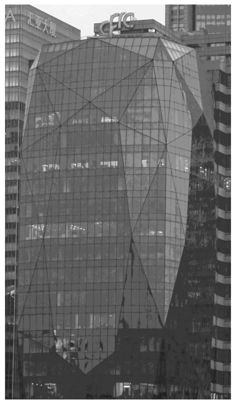
Fig. 6 Roof with slopes having arbitrary inclination [8].

There are frequent situations where the space achieved by applying these types of facades are used to provide natural ventilation, respectively for directing air flow, depending on the season.

Another frequently application is that of the awnings composed of intersecting planes used to cover or closed open spaces - see Fig. 8 as it follows:

- Fig. 8a – Park Pavilion, Cuerca, Spain [9];
- Fig. 8b - Compass Stadium, Huston, Texas, USA [6].





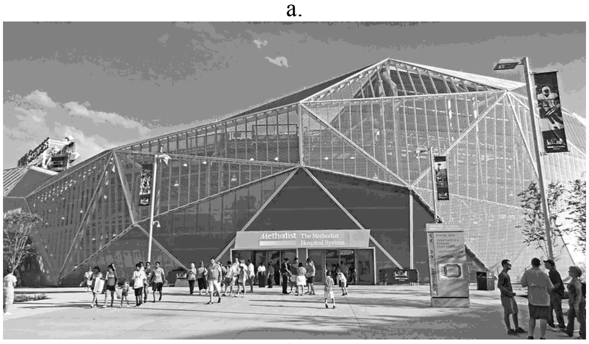
a. b. c.



d. e. f.

Fig. 7 Facade details [9,10].

a.



b.

Fig. 8 Types of awnings [6,9].

4. REASERCH DIRECTIONS

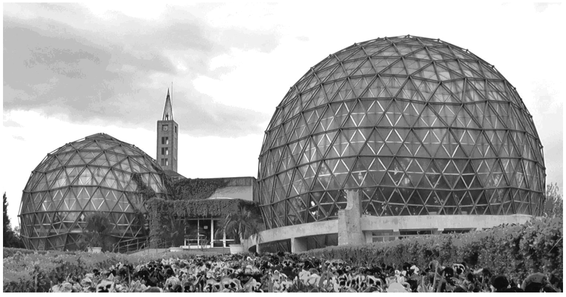
In cases where there are special aesthetic requirements, the covering can be designed in the form of geodesic domes or parts of them.

What are and how are obtained these domes? We know that any surface can be approximated by a polygonal spatial network, taking on that surface indirect points and resulting vertices, edges and triangular or polygonal faces. The approximation of a surface with a polyhedron is more accurate as the polyhedron has smaller edges. Otherwise the two-dimensional equipartition in space lead, as particular cases, to the five regular or Platon’s polyhedra - tetrahedron, hexahedron, octahedron, icosahedron and dodecahedron, respectively those 13 semiregular or Arhimede’s polyhedra inscribed in a sphere.

To achieve large opening reticular domes, it is necessary to multiply the faces and vertices of these polyhedra, such as to result as many equal edges (struts) and identical solid angles (knodes) [2]. This multiplication, which is performed according to certain laws, involve that the polyhedron to follow a sphere through a series of quasi-regular polyhedra having increasingly smaller edges and faces, closer in size to the surface of the sphere or, in generally to the curved surface which supports. In the context of the topic of the paper, the reticular surfaces can be treated as a succession of joined planes (polygons), lying on a surface.

The most stable geodesic surfaces are the omni-triangular structures that provides a solid structure with outstanding resistance to the natural factors: earthquakes up to 8.9 degrees on the Richter scale, wind speeds below 200 km/h, due to the aerodynamic shape [7]. But according to the load to which the building is subjected one can also find domes having pentagons or hexagons regular faces.

Geodesic domes are successfully used in the construction of recreational facilities, exhibition pavilions, greenhouses, and recently even in the construction of houses. Besides the previously described advantages, others should be mention, that in the context of sustainable development of constructions, makes them attractive: their shape maximizes the light effect, allows optimum distribution of sound and heat, providing thermal comfort. This types of constructions require approximately 30% lower energy consumption than a conventional parallelipipedic building [7].



a.

b.

Fig. 13 Geodesic dome constructions [7].

In Fig. 13a are shown the green houses from Jibou's Botanical Garden and Fig. 13b shows an ecological house in the form of geodesic dome.

5. CONCLUSIONS

Contemporary architecture reflects the dynamics that characterizes modern society. Thus, spectacular shapes and meticulous decorative details of styles preceding the XXth century were replaced with simple volumes derived from classical geometric solids.

One of the methods used to achieve a particular appearance consists in the intersection with other plane or spatial geometric solids.

Present and future buildings are and will be designed in order to pursue other two challenges of the contemporary world (society):
- energy efficiency;
- responsibility towards the environment.

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