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SOME CONSIDERATIONS ABOUT SURFACES' INTERSECTION

Abstract: *This paper wants to systematically and generally present the surfaces' intersection, with special interest for the method of auxiliary surfaces. The intersections types presented are plane-cylinder, plane-cone, and cylinder-cylinder. Like auxiliary surfaces will be used, particular planes (level, frontal, profile planes) and sphere surfaces, which every one determine as intersections with the given surfaces, easy to draw straight lines and curves (circles).*

Key words*: auxiliary surfaces, intersection, straight lines, plane, plane curves, sphere surfaces.*

1. INTRODUCTION

The surfaces' intersections represent one of the most important chapters of the Descriptive Geometry. A correct and precisely intersection of two surfaces, allows to determine the functional aspects of theme and of the assemblies in which they exist. In same time, the surfaces' intersection has an important place in order to determine the development of the surfaces, especially in the case of thin parts or laminates.

2. GENERAL ASPECTS

The intersection of two surfaces represents all their common points. It can be obtaining point by point, being necessary an important number of linking points to define the intersection line. To solve the intersection problem Descriptive Geometry used the auxiliary surfaces method. This method involves the next steps to be done: the choosing of the auxiliary surfaces, than the intersection between the auxiliary surfaces and every given surfaces. The result is intersection lines, straight or curve. These lines have to be used to obtain their common points, representing intersection points, than joining them in a logical order, the intersection line can be drawn and the visibility can be determined. Fig. 1 presents the intersection between the conical surface $\{S_1\}$ and the cylindrical one $\{S_2\}$. First is choose the auxiliary surface $[S]$ which intersects both $\{S_1\}$ and $\{S_2\}$ surfaces in long of the (C_1) and (C_2) circles; I_1 and I_2 are the intersection point and many similar pair of points can be obtained beginning with an other auxiliary surface:

$$\{S_1\} \cap [S] \Rightarrow (C_1)\ ;\ \{S_2\} \cap [S] \Rightarrow (C_2)\ ;\quad (C_1) \cap (C_2) \Rightarrow I_1\, , I_2 \qquad (1)$$

With many auxiliary surfaces, parallel each other and to $[S]$, intersection points will result, and joining them, the intersection curve (L), will be obtain. This procedure can be used to all types of intersections.

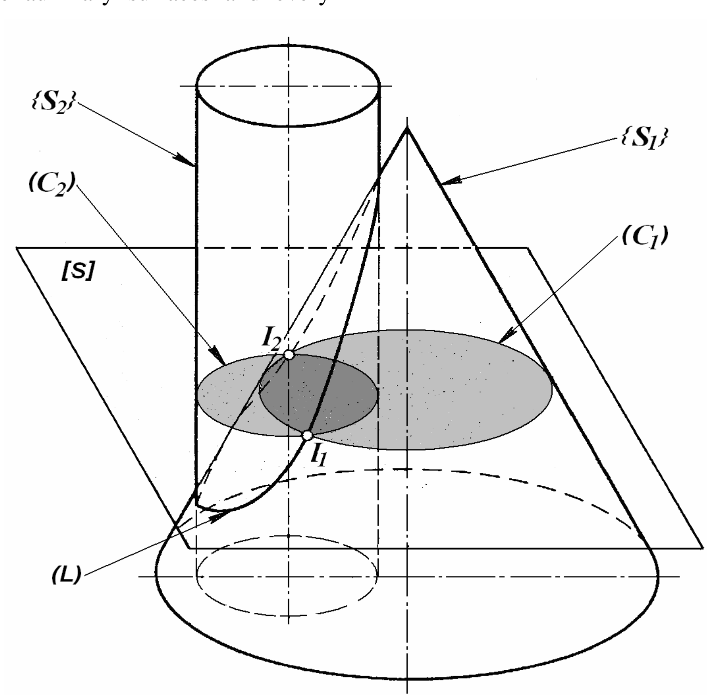


Fig. 1 Auxiliary surfaces method**.**

3. CLASSIFICATION OF THE SURFACES' INTERSECTIONS

There are four groups of intersections, in Descriptive Geometry: surface – straight line, surface - plane, plane – plane, surface – surface. For different particular surfaces, many different types of intersections can be obtained. Table 1 presents the main cases of intersections, obtained by methods of Descriptive Geometry and the auxiliary surfaces used in every case:

Table 1

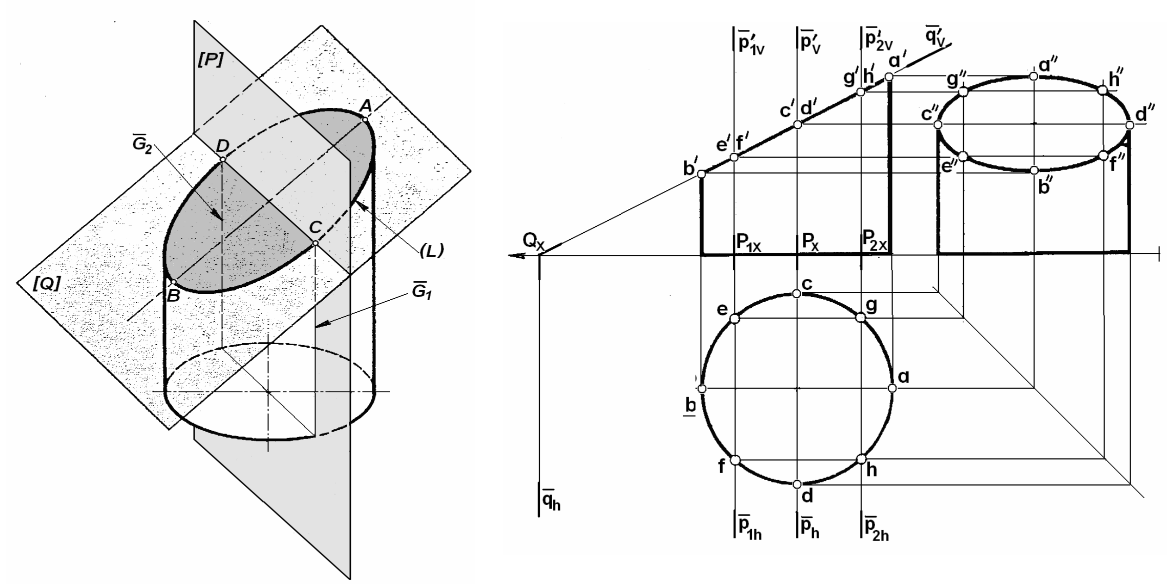
Classification of surfaces' intersections.

No.	Geometrical elements to be intersected		Auxiliary surfaces used ***[S]***	Resulting intersection
	element - S_1	element - S_2,		
1	Plane	Straight line	Projecting plane through the line	A point
2	Some surface	Straight line	Plane through the line	A point / Two points
3	Plane	Plane	Two particular planes	A straight line
4	Polyhedral surface	Plane	Particular planes	A plane polygon
5	Curved surface	Plane	Particular planes	A plane curve
6	Polyhedral surface	Polyhedral surface	Particular planes	One or two plane or spatial polygons
7	Polyhedral surface	Curved surface	Particular planes	Multi curved arcs line
8	Curved surface	Curved surface	Particular planes / Curved surface	Spatial curves

4. APPLICATIONS

4.1 Intersection plane – cylindre

An intersection between a vertical cylinder and a perpendicular plane to the vertical projection plane [Q], is represented in Fig. 2. To obtain a point of the intersection curve, a profile auxiliary plane [P], is chosen. Its intersection with the cylinder surface are two generetrix $\overline{G}_1$, $\overline{G}_2$, and with the plane [P], is the line $\overline{CD}$. The two generetrix intersection with the line $\overline{CD}$ is represented by the points C and D. Using many similarly auxiliary planes, many points of the intersection line (L) will be obtained. This intersection line is an ellipse, available in all cases in which the cutting plane is not a perpendicular one on the cylinder axes and cuts all the cylinder generetrix. Constructing the intersection line, there is very important to determine the principal and particular points: the maximum or highest point A the minimum or lowest point B and the two points belong the extreme generetrix from the lateral projection, C and D. For a good precision, more are the points determinates, better is the precision of the intersection curve. Epura of the intersection between a circular right cylinder and a projecting plane to the vertical plane, is presented in Fig. 3. Three auxiliary profile planes were used to obtain six points of the intersection line, and other two points, which define the big axes of the ellipse intersection, are obtained cutting the extreme generetrix of the cylinder in the vertical projection, by the cutting plane [Q].



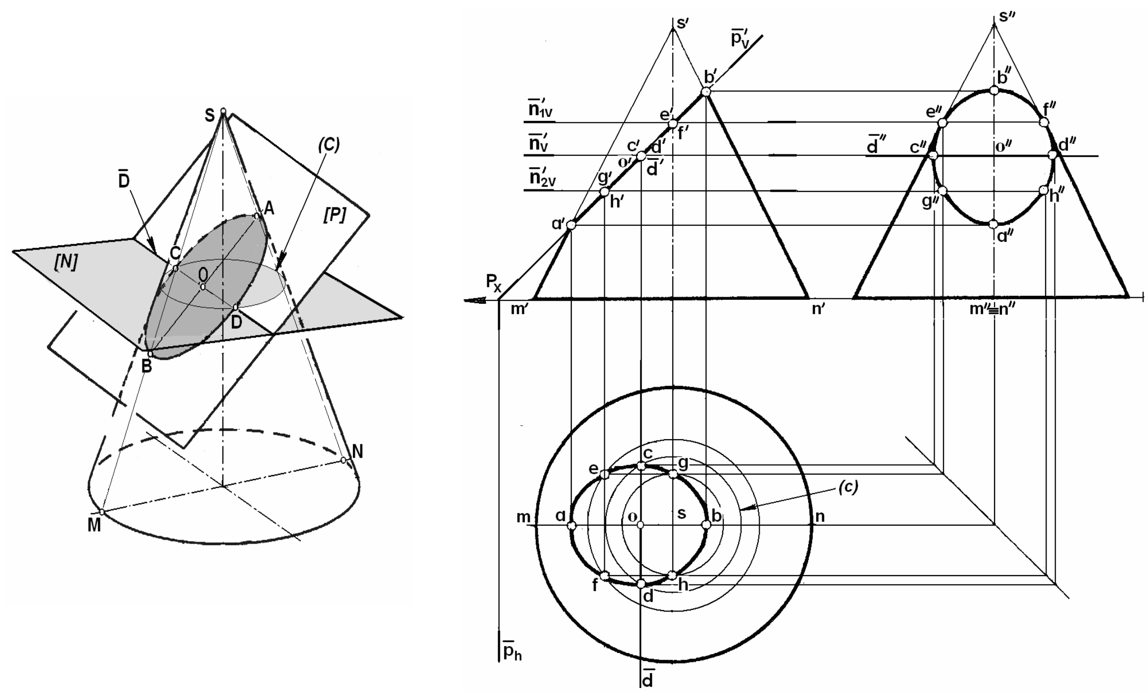
a b

Fig. 2 Intersection plane – cylinder.

4.2 Intersection plane – circular right cone

This second study case presents an intersection between a circular right cone, which base is located in the horizontal case which cuts all the generetrix of the cone. (Fig. 3). plane of projection and a projecting plane *[P]*, perpendicular to the vertical plane of projections According to the Dandelin theorem, the intersection curve will be an ellipse which big axes is the front segment $\overline{AB}$ and the small axes is the segment $\overline{CD}$, perpendicular on the vertical plane of projections. In this case, the auxiliary surfaces used for a precisely intersection to be obtained, are two types of particular planes. Level planes which intersections with the cone are circles, and with the projecting plane *[P]* are lines perpendicular on the vertical plane ('end' lines). A pair circle (*C*) – end segment ($\overline{CD}$), belong in the same level plane *[N]*, determines two points *C* and *D* of the intersection curve (Fig. 3a). The second type of auxiliary surfaces to be use in this case are the axial planes which cat the cone in long of two generetrix, and the plane *[P]*, in long of an end line. The intersection between the two generetrix and the end line, are points of the intersection curve.

In the Fig. 3b example, auxiliary surfaces used are level planes. The vertical projection of the ellipse intersection is complete distorted on the vertical trace of the plane *[P]*. The front segment $\overline{AB}\,(\overline{ab}, \overline{a'b'}, \overline{a''b''})$, defined by the extreme generetrix $\overline{SM}$ and $\overline{SN}$ with the given plane, represents the big axes of the ellipse. The small axes $\overline{CD}\,(\overline{cd},\ \overline{c'd'},\ \overline{c''d''})$ of the ellipse is determined using a level plane *[N]*, through the middle *O*(*o, o´, o˝*),of the big axes. Its intersection with the cone is the circle *(C)* and with the plane *[P],* the end line $\overline{D}$. In the horizontal projection $(c) \cap \overline{d} \Rightarrow c, d$. Other two level planes *[N₁]* and *[N₂]* are used to obtain the intersection points *E(e, e´, e˝)*, *F(f ,f´, f˝)*, *G(g, g´, g˝)* and *H(h, h´, h˝)*.



a **b**

Fig. 3 Intersection plane – circular right cone.

4.3 Intersection cylinder – cylinder

An intersection between two curved surfaces is presented in Fig. 4, curved surfaces being two concurrent axis cylinders. There are two solutions for this problem: using auxiliary surfaces planes or spheres. In the first case (Fig. 4a), auxiliary surfaces are frontal planes *[P]*, parallel to both cylinders' generetrix, which cut the cylinder *{C₁}* in long of two generetrix – fronto-horizontal segments, and the cylinder *{C₂}* in long of two generetrix – vertical segments. In every case, the pairs of generetrix from the same determine points of the intersection curve. Five auxiliary planes are used in this case. The intersection is penetration type one and the dark zone of the fronto-horizontal cylinder do not participate to the intersection. The second method, also named *the sphere method,* is based on the intersection between a sphere and a rotation surface, with the same axes. This intersection is formed by two circles, located in perpendicular planes on the common axes. The projections of these circles, on evry parallel plane with the axes, are complete distorted. In Fig. 4b, the auxiliary spheres used have the center in Ω, the intersection point of the two axes of the cylinders. They can be used only the auxiliary spheres which cut the generetrix of both cylinders. The maximum (the biggest) sphere, contains the points a', b', c' and d', like extreme points on the vertical projection. The minimum (the smallest) sphere is tangent to the cylinder $\{C_1\}$, on the circle with $\overline{1'2'}$ diameter, and intersects the cylinder $\{C_2\}$ in two circles of $\overline{3'4'}$ and $\overline{5'6'}$ diameter. These three circles intersect in the points $m' \equiv n'$ and $p' \equiv q'$, which are the maximum of the intersection curves. To obtain others points of the intersection, many auxiliary spheres can be used, all of them between the maximum and the minimum one.

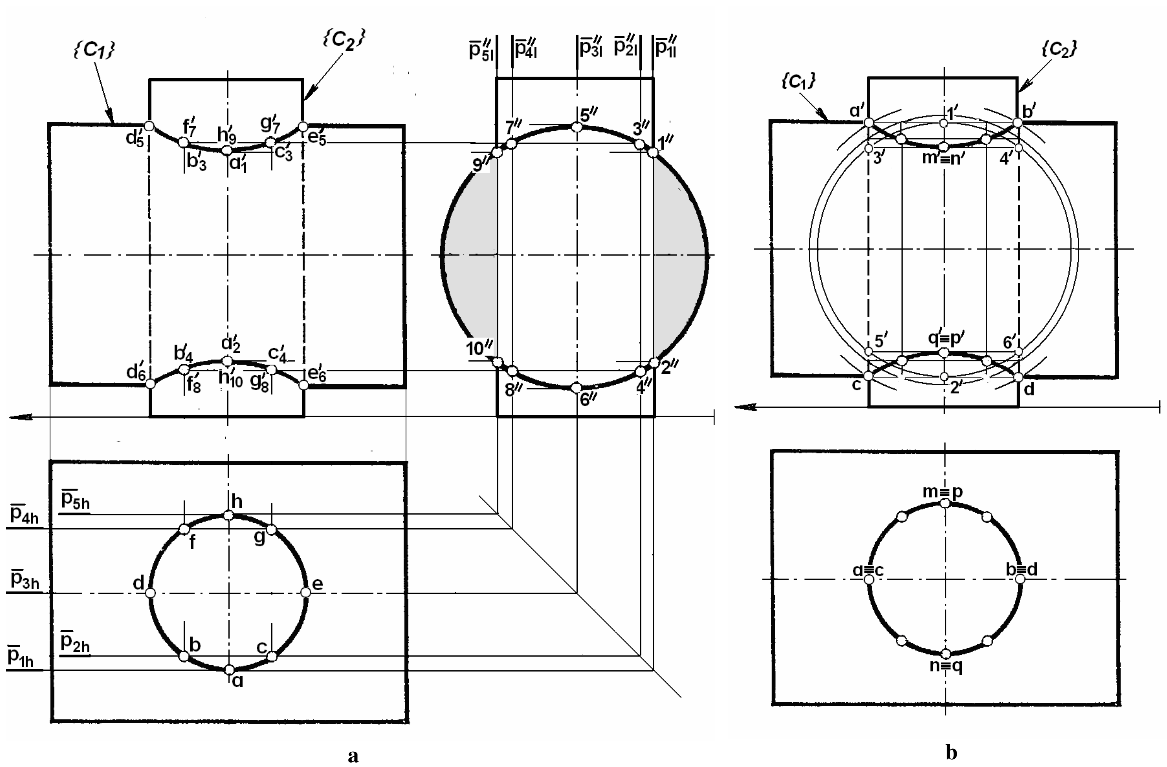


Fig. 4 Intersection cylinder – cylinder.

5. CONCLUSION

1. The auxiliary planes are chosen in such a way that the remarkable points can be determine easy, and then the logical way to join them permits to obtain the intersection curve.

2. Generally, the auxiliary surfaces used to determine surfaces' intersections are particular planes. (level, frontal, profile planes).

3. When two rotation surfaces, with concurrent axes have to be intersected, is recommended the sphere method that means the auxiliary surfaces are spheres. The method is easier and the problem can be solved in one projection.

6. REFERENCES

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