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THE USE OF AXONOMETRIC AND PERSPECTIVE PROJECTIONS FOR THE REPRESENTATION OF STAIRS IN ARCHITECTURAL CONSTRUCTIONS

Abstract: *The functional and aesthetic composition of the technical concepts requires the use of specific methods of representation of geometric shapes through perspective images and axonometric representations. Applying these methods, the paper presents several exercises for representing some stairs within architectural constructions, as support in the acquisition of the geometric reasoning based on the symmetries of rotation and translation of the shapes that make up these volumes. The geometric and constructive examples analyzed in the paper help to form and develop the ability to see in space, this study being of particular theoretical and practical importance for design practitioners.*

Key words: *axonometric projection, perspective image, picture plane, vanishing point, horizon line.*

1. INTRODUCTION

The representation in double or triple orthogonal projections (the descriptive representation) of the spatial objects allows the precise determination of their dimensions, if their faces are parallel to one of the projection planes, situation in which these faces are projected in true size and therefore can be measured [1].

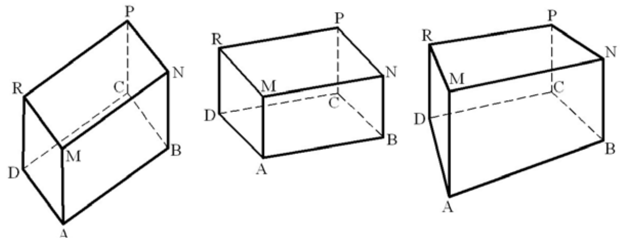
The views thus obtained have the disadvantage that they do not suddenly suggest the images of these objects as they are seen, especially for an observer without a particular practice and attention, because for the intuition of the form it is necessary to combine these projections in the imagination in order to realize the spatial image.

Thus, if a rectangular parallelepiped is represented in double orthogonal projection placed with two faces parallel to the projection planes, its orthogonal projections are reduced to two rectangles. Looking at the two projections, the impression of volume in the form of a parallelepiped is realized only after the observer has managed to combine the faces of the polyhedron in imagination, according to the rules that served to make his projections. This disadvantage can be eliminated by projecting the object on a single plane, so that the number of its visible faces in the projection is as large as possible and none of its faces appear, if possible, with the projection reduced to a straight segment. The plane representation thus obtained becomes intuitive, close to the visual one and therefore easy to interpret even by an inexperienced observer.

The realization of such an image can be obtained by:

- use of the oblique parallel projection (Figure 1, a) or orthogonal (Figure 1, b), but respecting the condition that none of the faces of the object should not be parallel to the direction of projection (parallel axonometric projection);
- using the central (conical) projection and making the condition that none of the faces of the object do not pass through the projection center (central or perspective projection - Figure 1, c).

In the axonometric representation, orthogonal or oblique, one can obtain images of the objects as close as possible to the real ones on which direct measurements can be made, both from angles and distances.



a b c

Figure 1 Axonometric and perspective projections.

2. AXONOMETRIC REPRESENTATIONS

Axonometric projection allows the representation of an object on a certain plane using the Cartesian coordinates of the different points of the object and admits the parallelism of the visual rays, that is, the infinite positioning of the station point. The graphical constructions are simple, with uniform deformations in certain directions, which allow to appreciate the shape and dimensions of the object.

The axonometric representation is the parallel projection of an object on a plane inclined to the axes of the three principal planes and which renders the image of the object in perspective. This representation gives a suggestive and uplifting image of the spatial form of the object in question.

The main advantage of the axonometric representation is that the object is represented and identified completely by a single projection.

In the perspective representation, the image of the object appears as recorded by the eye, but it is not possible to determine the exact dimensions of the objects (lengths and angles) and the parallelism of the lines is not preserved [1], [2].

2.1 Representation of a cylindrical helical stairs

It is considered the basic square in which is inscribed the horizontal circle whose axonometric image is constructed as shown in Figure 3. This circle is divided into 8 equal parts and the corresponding heights of the steps are transposed on the respective verticals. The vertical divisions are equal to *h/n*. Thus the axonometric image of the cylindrical spiral is obtained which contains the extremities of the edges of the steps and the counter steps (Figure 2 - principle of the method, Figure 3 - axonometric image).

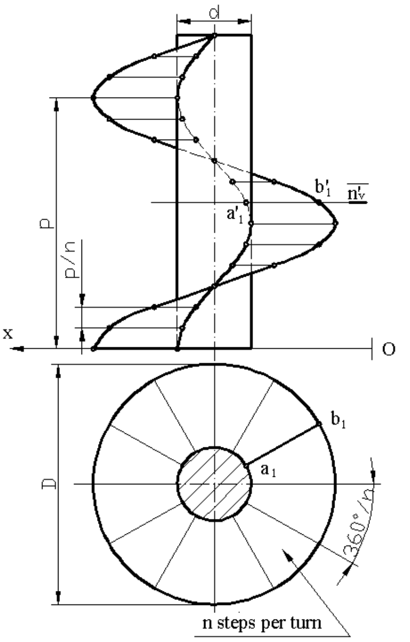


Figure 2 Cylindrical helical stairs employing descriptive geometry principles.

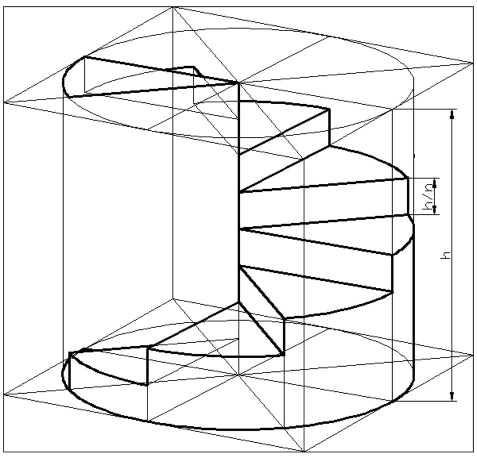


Figure 3 Axonometric projection.

2.2 Representation of a stairs with two ramps in isometric axonometric projection

In Figure 4 we consider at scale 1: 100, the plane and elevation of a stairs, and in Figure 5, the image axis system $O_0x_0y_0z_0$ in isometric orthogonal representation, corresponding to the systems *Oxyz* and *O'x'y'z'* to which the stairs was reported. In order to construct the axonometric image, the following steps have been taken:

- the "plane" is first represented with the help of line networks parallel to the two image axes *Ox* and *Oy* and the projections of the floors are obtained in axonometric representation after $a_0e_0h_0d_0$ and $f_0b_0c_0g_0$ and of ramps without steps after $e_0f_0j_0i_0$ and $m_0n_0g_oh_o$;
- the floors are raised and lowered, with the respective thickness, to the levels corresponding to the elevations, after which the ramps with the respective thickness are represented (Figure 5);
- then the detail elements (the steps of the stairs) are represented using the divisions given by the points i_0, *1, 2, ..., 8*, j_0 and m_0, *9, 10, ...,* n_0 on the line segments i_0j_0, respectively m_0n_0 (Figure 6).

The verticals taken through the points i_0, *1, 2, ..., 8*, j_0 at the intersection with the lines i_58_1 and i_0j_2 give points that determine the section in steps. Analogously, the section on the second ramp is also obtained, using the points on the lines $9n_5$ and m_0n_1 given by the verticals carried through m_0, *9, 10, ...,* n_0.

The lines e_58_2 and h_0g_i will delimit the edges of the steps on the other side.

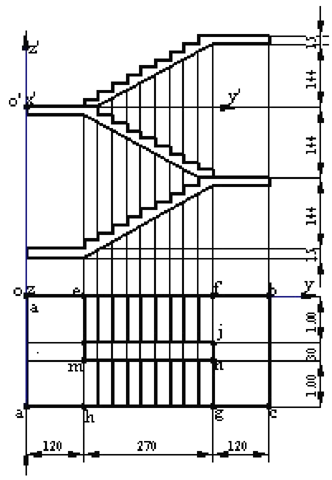


Figure 4 Plane and elevation of a stairs.

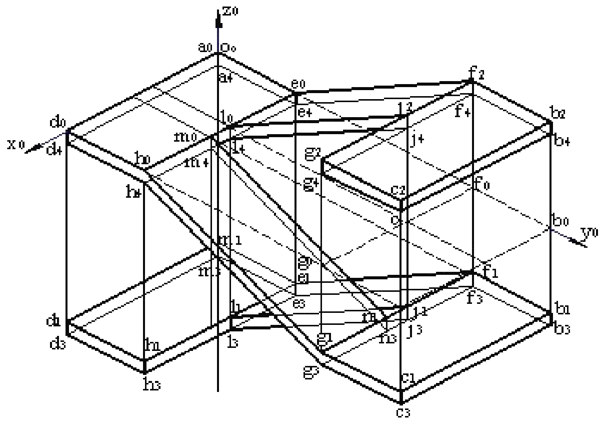


Figure 5 Projections of the floors and ramps.

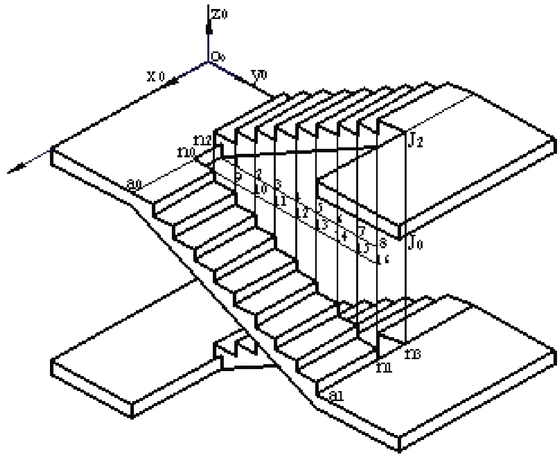


Figure 6 Axonometric image of the stairs.

3. REPRESENTATIONS IN PERSPECTIVE PROJECTION

In order to obtain the best image of reality, that is, the rendering of the three dimensional space and of the elements that are in it, it is necessary to take into account the values of depth, determined by: elements of orientation, distance, direction, light, color etc., all being influenced by visual perception [3].

There are two broad conceptions of space representation: Cartesian space and perspective. If the Cartesian space has as its fundamental element a coordinate system (so that any point has a precise positioning to a system of axes), on the other hand, the perspective implies a more natural vision that corresponds to the image that the human eye realizes when it regards an object, respecting both the position and metric relationships that characterize the object from a geometric point of view, as well as the laws of visual perception. The eye records the images in the central (conical) projection system, the projection center being even its optical center.

If we imagine that a transparent picture plane is placed between the eye of the observer and the observed object, then the rays of light that leave from all points of the object towards the eye of the observer form at their intersection with this transparent picture plane a multitude of points which, united by lines, render the visual image of the object on the picture plane, which is the same if the object is removed.

3.1 Construction of the perspective of a stairs with steps consisting of horizontal and profile planes

The slope of the stairs is known determined by the angle *ABC* and the height of each step, noted *Bm*. To determine the depth of the step, is drawn *mn* || *AB* until it intersects *BC* at point *n*. Then measure along the base *AB* this length equal to *BD*, so that a stairs results consisting of four horizontal steps and 4 vertical counter steps (Figure 7).

These points are joined to the point of sight *P* and from *F* the frontal *EF* is drawn parallel to *BC* (the *BF* segment determines the width of the stairs ramp). With these lines the accuracy of the construction is ensured.

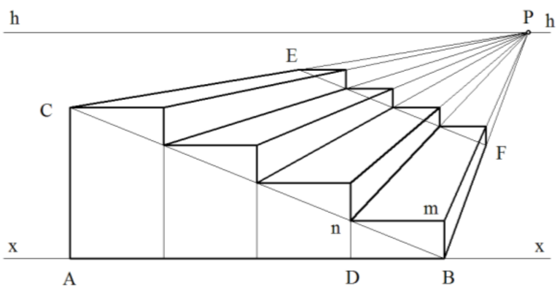


Figure 7 Stairs in perspective projection.

3.2 The perspective of a stairs in frontal view

In Figure 8 the height of each step is measured on the vertical line *AB* (also called the line of heights) and their depth is determined using the diagonals drawn to the distance point D^-. The rest of the graphical constructions results from the figure.

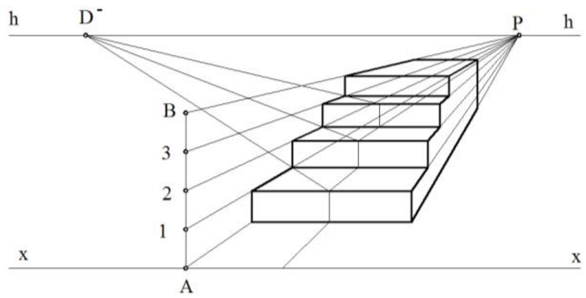


Figure 8 Perspective of a stairs in frontal view.

3.3 Construction of the perspective of a stairs with steps consisting of horizontal and frontal planes

P is considered the point of sight, D^- the distance point, *ab* the width of the ramp, *a1* the height of the step and $a1_0$ the depth of the counter step (Figure 9). The heights of the steps are chosen directly on the vertical from point *a*. The depths of the counter steps are transposed from the frontal-horizontal line *ab* to the main line *aP* by the help of distance point D^-.

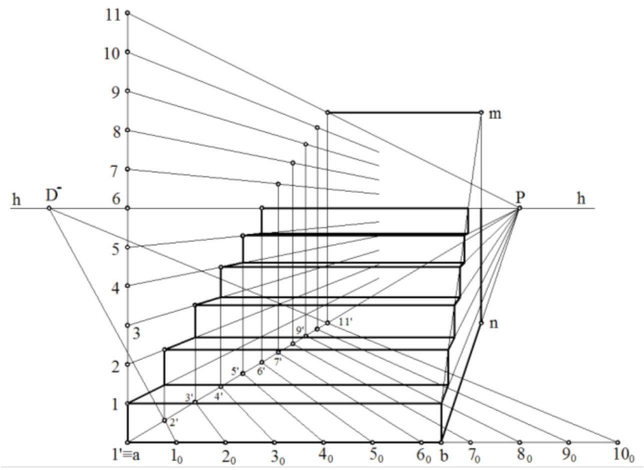


Figure 9 Perspective of a stairs with steps consisting of horizontal and frontal planes.

3.4 The perspective of a square stairs

Knowing the side *AB* of the first step and the point of sight *P*, the depth of this step is determined taking a fourth of the base and a fourth of the distance and drawing a line from *1/4* base to *1/4* distance. Then we draw the two diagonals of this square (Figure 10).

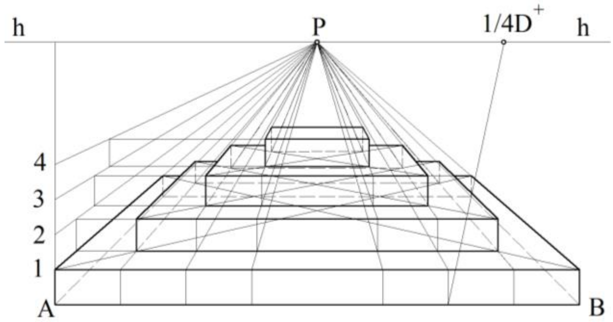


Figure 10 Perspective of a staircase with square steps.

On the vertical raised in point *A*, the height of each step is measured, obtaining points *1, 2, 3* and *4*, and then the vanishing scale is formed, by joining these points with the point of sight. The second step is smaller than the first by *1/8* base. This step is drawn using the diagonals of the first step and taking its height from the vanishing scale as shown in Figure 10. Then the third step and the fourth step are drawn in the same way. The rest of the figure explains this simple procedure.

3.5 The perspective of a winding stairs in a square

Consider the square *ABCD* in parallel perspective whose sides are divided into *4* and raise verticals from each division. These verticals will mark the positions of the steps on each side, four in number. The steps are to wind around the vertical raised in *O*. Let *AF* be the height of each step. Draw the scale *AB* on which the height of each step is shown. The segment *mn* determines the height of the step at the central line of *O*.

This measurement is repeated upwards, so that we have the height of each step on the vertical line. Draw the rectangle *af1D* with the height of *AF* and complete the first step. On the raised vertical in *D*, measure heights equal to *D1=AF*, then complete the second step. The figure illustrates the determination of the height of each ascending step on the wall to the right, then on the wall opposite to us, and on the wall to the left (Figure 11).

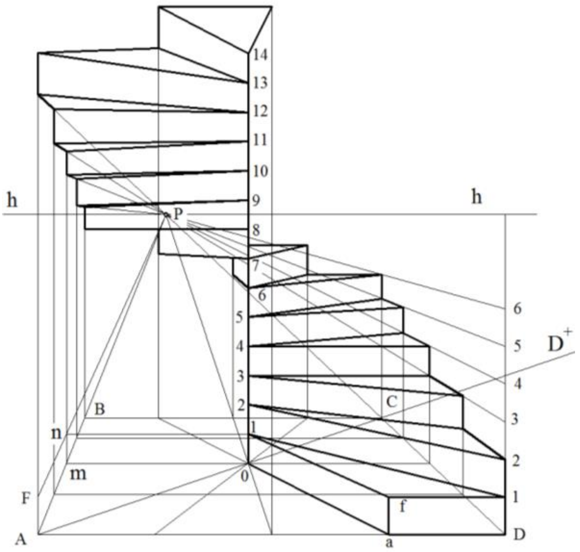


Figure 11 Perspective of a winding stairs in a square.

3.6 The perspective of a cylindrical helical stairs with a core cylinder

The stairs will have *12* steps corresponding to one pitch. Consider *h-h* the horizon line and *P* the point of sight towards which converge two of the sides of the square in which the two concentric circles are inscribed. Divide the circular base of diameter *CD* into *12* equal parts and measure the corresponding step heights on the respective verticals. The divisions on the cylinder axis are equal. For the construction, a vertical plane is considered in perspective, which has a horizontal trace *AP* on which the *APF* scale is formed and the larger *APB* scale, on which the perspective measurements of the steps are shown, in accordance with their positions. The heights of steps *1, 2, 3...* appear on the center line according to the numbers in the large scale *APB*, to the left (Figure 12).

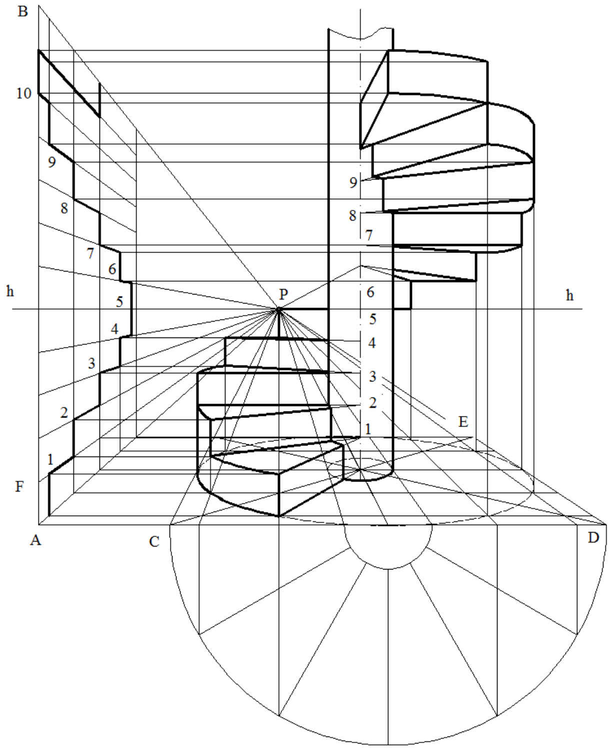


Figure 12 Perspective of a cylindrical helical stairs.

4. CONCLUSIONS

This paper aims to explain the usefulness of knowing the principles of axonometric representation and the rules of perspective in approaching some subjects, from simple to complex, that students need to solve so that they finally get the ability to find solutions to any new problem for themselves, but also for their future professional career in design.

The representation of complex surfaces, such as winding stairs, is based on the rules and conditions of a good perspective, so that their image is in harmony and proportion and has a more natural appearance.

Descriptive geometry and perspective help design practitioners to give appropriate dimensions and proportions to the objects in accordance with the observer's distance from the picture plane, both for functional requirements and for aesthetic needs.

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