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METHODS TO DEVELOP A TOROIDAL SURFACE

Abstract: *The paper work presents two practical methods to draw the development of a surface unable to be developed applying classical methods of Descriptive Geometry, the toroidal surface, frequently met in technical practice. The described methods are approximate ones; the development is obtained with the help of points. The accuracy of the methods is given by the number of points used when drawing. As for any other approximate method, when practically manufactured the development may need to be adjusted on site.*

Key words: *toroidal surface, approximate method, development, projection, plane, section*

1. INTRODUCTION

A toroidal surface is a complex one, a fourth degree surface, generated by a circle revolving about an axis located in the same plane with the circle. The axis may or may not intersect the circle or it may be tangent to the circle. The axis will be generally assumed external to the circle, thus the surface will have a single cloth. If the axis intersects the circle, the toroid generated in such circumstances will also have an internal cloth. In this case, the two cloths of internal and external surface pass through the two double points of the surface, which are the intersection points of the axis with the circle. Their surface has a single double point if the axis is tangent to the generating circle [5].

The toroidal surface is unable to be developed just using classic methods provided by Descriptive Geometry. The paper work proposes two practical methods to draw the development of a toroidal surface combining general knowledge of plane and solid geometry with knowledge of Descriptive Geometry (projection methods, methods to obtain the true size of geometric elements plotted on draught etc.). Both methods present the developments as continuous curves drawn with the help of points; hence the necessity to assume a number of points high enough to guarantee the accuracy of the plotting.

The first method proposes to divide the toroidal surface into three bands; the second method proposes to divide the toroidal surface into two bands separated by the median circle.

2. PLOTTING OF A TOROIDAL SURFACE

The simplest toroidal surface is the torus. In figure 1 the horizontal and the vertical projections of a torus are plotted. The axis of the torus is the ending line (L), external to the generating circle of diameter (d), which is located on the level plane of the axis (meridian plane) [4].

The horizontal apparent outline is specified by the tangent lines (aa_1) and (bb_1) (parallel lines, named frame circles) and by the semicircles (aeb) and ($a_1e_1b_1$).

The vertical apparent outline consists of necklace circle of radius (r) and of equator circle of radius (R).

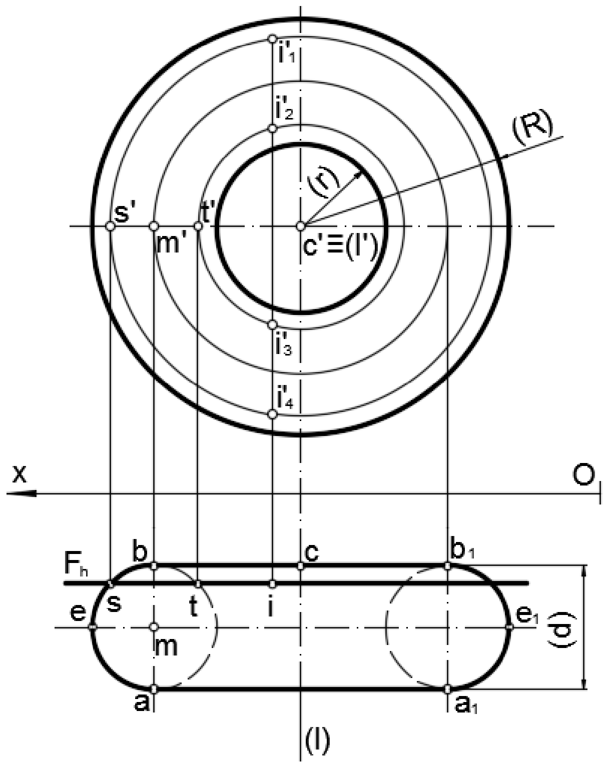

Fig. 1 Plotting of a toroidal surface.

The median circle is the circle adequate to point M (m, m').

A point I (i, i') belongs to the surface of a torus if point's projections are located on the alike projections of a parallel or a meridian circle. Assuming the horizontal projection of the point i, a front plane [F] is drawn through it, cutting the torus on the parallel circles of radii (c't') and (c's'). The line of recall drawn from point i intersects these circles in points i_1', i_2', i_3' and i_4', providing four solutions. The explanation is that a vertical line intersects a torus, having the axis an ending line in four points [2].

3. METHOD TO DEVELOP A TOROIDAL SURFACE DIVIDED INTO THREE BANDS

Given the toroidal surface of internal radius (r), external radius (R) and with the diameter (d) of the cross section, due to the symmetry of the solid, drawing the development of a quarter of the surface is enough for the whole solid.

The projection plotted on figure 2a is divided on radial direction into a convenient number (3) of equal parts.

The cross section is also divided into an even number of equal parts (6), due to the symmetry only half the circle of diameter (d) and 3 division points are plotted. Through the 3 division points arcs of circle with centres in point c are drawn.

The assumed part of the toroidal surface is thought as consisting of three solids, elements 1, 2 and 3.

The developments of the three elements 1, 2 and 3 are plotted in figure 2c and they are obtained as follows:

- Join points 0' and $1_1'$, 1' and $2_1'$, 2' and $3_1'$, thus, resulting curved triangles on each of the three bands.
- In order to draw the developments of the obtained parts, the true lengths of triangles' sides are needed.
- The true lengths of the sides of triangle $0'0_1'1_1'$ are found as follows:

- length of side ($0'0_1'$) can be measured, because its true length appears on the plotting, or it can be calculated knowing the radius of the circle the arc is part of.

$$0'0_1' = \frac{\pi R}{6} \qquad (1)$$

- length of side ($0_1'1_1'$) can be measured, because its true length appears on the plotting as equal to the length of arc of circle (01), or it can be calculated knowing the radius/diameter of the circle the arc is part of.

$$0_1'1_1' = \frac{\pi d}{6} \qquad (2)$$

- the true length of side ($0'1_1'$) is the hypotenuse of the right triangle having one cathetus the horizontal projection (01_2) of the segment, and the other cathetus the quote of point $1_1'$ (fig. 2b).

- The true lengths of the sides of triangle $1'1_1'2_1'$ are found following the stages previously explained, except for the length of side ($1'2_1'$):

- the true length of side ($1'2_1'$) appears on the plotting as the vertical projection ($1'2_1'$) of a front line whose horizontal projection is (12_2) (fig. 2a).

- The true lengths of the sides of triangle $2'3_1'3'$ are found following the stages initially explained, except for the length of side ($2'3_1'$):

- the true length of side ($2'3_1'$) is the hypotenuse of the right triangle having one cathetus the horizontal projection (23_2) of the segment, and the other cathetus the quote of point $3_1'$ (fig. 2b).

The developments of elements 1, 2 and 3 are plotted with the help of the previously obtained true lengths of

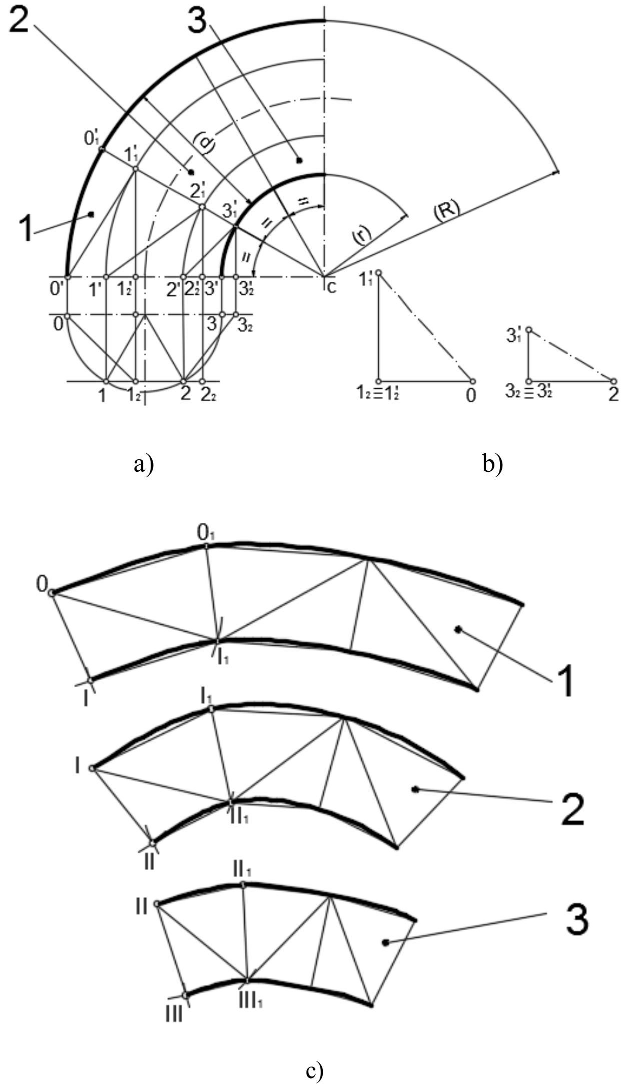

Fig. 2 Development of a toroidal surface divided into three bands.

the sides of the triangles. The method consists of drawing a triangle knowing the lengths of its sides, then multiplying the obtained triangle three times (as each element is specified by three identical triangles).

The proper development is obtained joining together the vertices of the triangles previously drawn by a continuous curve (fig. 2c).

4. METHOD TO DEVELOP A TOROIDAL SURFACE DIVIDED INTO TWO BANDS

Given the toroidal surface of internal radius (r), external radius (R) and with the diameter (d) of the cross section, due to the symmetry of the solid, drawing the development of a quarter of the surface is enough for the whole solid.

The projection plotted on figure 3a is divided on radial direction into a convenient number (3) of equal parts [2].

The cross section is also divided into an even number of equal parts (12), due to the symmetry only half the circle of diameter (d) and 6 division points are plotted. Through the 6 division points arcs of circle with centres in point c are drawn.

The assumed part of the toroidal surface is thought as consisting of two solids, elements 1 and 2, separated by the median circle (the circle adequate to point 3).

The developments of the two elements 1 and 2 are plotted in figure 3b, c and they are obtained as follows:

- Divide one of the three identical radial sections in half with the help of radius ($c6_2$).
- On horizontal direction the development of half a cross section is plotted as a line segment of length (L), which is divided into 6 equal parts of length (a). Both lengths can be calculated knowing the radius of the circle the arc is part of.

$$L = \frac{\pi d}{2} \qquad (3)$$

$$a = \frac{L}{6} \qquad (4)$$

The development of element 1 is obtained as follows (fig. 3c):

- Plotting the development of element 1 begins from the centre of the element and continues symmetrically left side and right side this point, 0_2.
- Draw the vertical line adequate to point 0_2.
- With the centre in point 0_2 and with radius equal to the length of arc of circle $00_2 = 0_20_1$ an arc of circle is drawn intersecting the vertical line previously plotted in point 0_1.
- With the centre in point 0_1 and with radius equal to (a) draw an arc of circle.
- With the centre in point I_2 and with radius equal to the length of arc of circle $11_2 = 1_21_1$ an arc of circle is drawn intersecting the arc of circle previously plotted in point I_1.
- With the centre in point I_1 and with radius equal to (a) draw an arc of circle.
- With the centre in point II_2 and with radius equal to the length of arc of circle $22_2 = 2_22_1$ an arc of circle is drawn intersecting the arc of circle previously plotted in point II_1.
- With the centre in point II_1 and with radius equal to (a) draw an arc of circle.
- With the centre in point III_2 and with radius equal to the length of arc of circle $33_2 = 3_23_1$ an arc of circle is drawn intersecting the arc of circle previously plotted in point III_1.
- The construction continues analogously to the end of all points belonging to element 1.
- The development of element 1 is obtained joining together the resulted points by a continuous curve.

The development of element 2 is obtained as follows (fig. 3b):

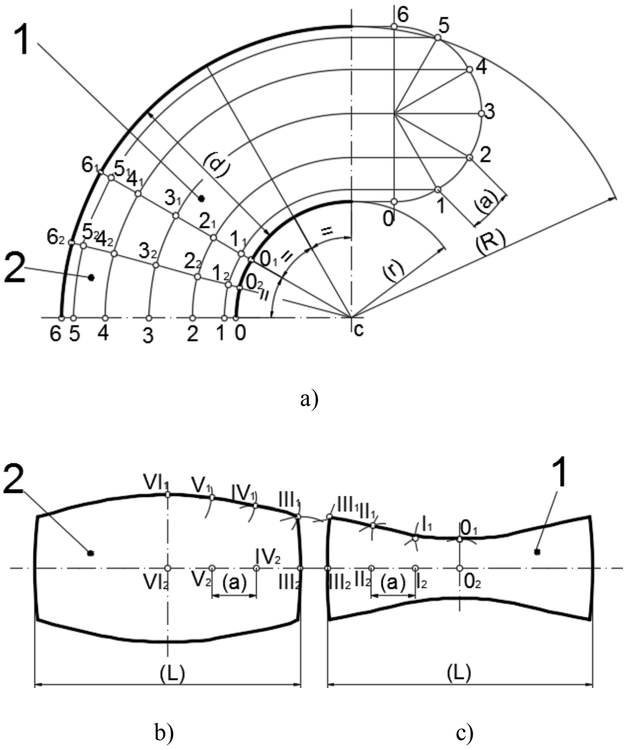

Fig. 3 Development of a toroidal surface divided into two bands.

- Plotting the development of element 2 begins from the centre of the element and continues symmetrically left side and right side this point, VI_2.
- Draw the vertical line adequate to point VI_2.
- With the centre in point VI_2 and with radius equal to the length of arc of circle $66_2 = 6_26_1$ an arc of circle is drawn intersecting the vertical line previously plotted in point VI_1.
- With the centre in point VI_1 and with radius equal to (a) draw an arc of circle.
- With the centre in point V_2 and with radius equal to the length of arc of circle $55_2 = 5_25_1$ an arc of circle is drawn intersecting the arc of circle previously plotted in point V_1.
- With the centre in point V_1 and with radius equal to (a) draw an arc of circle.
- With the centre in point V_2 and with radius equal to the length of arc of circle $55_2 = 5_25_1$ an arc of circle is drawn intersecting the arc of circle previously plotted in point IV_1.
- With the centre in point IV_1 and with radius equal to (a) draw an arc of circle.
- With the centre in point IV_2 and with radius equal to the length of arc of circle $33_2 = 3_23_1$ an arc of circle is drawn intersecting the arc of circle previously plotted in point III_1.
- The construction continues analogously to the end of all points belonging to element 2.
- The development of element 2 is obtained joining together the resulted points by a continuous curve.

5. CONCLUSIONS

The proposed paper continues authors' work in the field of finding practical methods to develop surfaces unable to be developed [1], [3], [4]. Technical literature provides few and unfinished solutions to this problem; mathematical literature only suggests a number of general ways to solve the notified problem. The problem of finding the cutting curve as the result of sectioning the toroidal surface with a certain plane is in stead widely treated in the mentioned literature.

The proposed methods are full-solving methods which describe the problem and provide the steps to plot and to find the development of a toroidal surface. The methods rely on Descriptive Geometry methods but also combine authors' technical and practical observations.

Surfaces generated by curve lines, such as the toroidal surface, are surfaces unable to be developed [1]. Approximate methods to develop such surfaces were founded for practical reasons.

The methods proposed by the present paper work, alike other practical methods, consist of dividing the surface into small geometric elements.

The first method proposes dividing the toroidal surface into curved triangles. The triangles are plotted projected on projection planes; therefore in order to draw the development it is necessary to find the true sizes of triangles' sides. Finding the lengths of these elements combines methods of Descriptive Geometry with practical observations about solid's shape.

The second method proposes dividing the toroidal surface into elements bordered by curves, which are drawn by points. The mentioned points are obtained intersecting arcs of circle. The true lengths of these arcs of circle can be measured on the projection or can be easily calculated.

The proposed methods use measuring the dimensions of various geometric elements on the plotting, therefore it is mandatory to accurately trace the drawing.

The described methods can be applied to any dimensions of the studied surface.

Practical methods have the advantage of an easy approach, the applied mathematical apparatus being an elementary one.

When practically manufactured the assembling method of the parts the development consists of is very important. The assembling method can cause deviations to be considered from the projected shape; therefore it may need adjustments on the spot [1].

According to specific cases it might be practical to work out tables with coordinates of points describing the developments of the toroidal surfaces, for ranges of characteristic dimensions of these surfaces (internal radius, external radius, diameter of the cross section) [3].

Using Computer Aided Design techniques to draw the approximate developments improves the accuracy of the curves plotted with help of points. The number of points obtained when applying the proposed methods greatly influences the quality of the curves. Computer Aided Design also has the advantage of a correct plotting and provides the instruments to accurately measure the dimensions of any geometric element.

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