

## ASPECTS REGARDING THE DEVELOPMENTS OF COMPOUND SURFACES

**Abstract:** The compound surfaces which form the technical parts of metal works are very diverse. Their practical production involves solving certain geometrical problems through the methods of descriptive geometry. This article wishes to emphasize the importance of knowing and applying descriptive geometry notions by presenting the surface developments of some technical parts formed of combined surfaces.

**Key words:** surface developments, Oliver's Theorem, compound surfaces, reductions

## 1. INTRODUCTION

A large variety of technical parts are made from sheets of metal. The objects are laid out, cut, formed into the required form and fastened together. The fastening is done by welding, soldering, reverting or seaming.

In the making of reservoirs, pipe fittings and metal works one can frequently see parts formed out of cylindrical, conical and polyhedral surfaces which are intersecting.

For the practical making of these compound surfaces, the unfinished goods for each surface are drawn and discharged separately. This is followed by the individual processing of these surfaces and their assembling.

For the drawing and discharging operations of unfinished goods, it is necessary to know the unrolling manner of the surfaces, to determine their intersection curve and its transposition on the surface development. During these operations, patterns are built, taking into account the fact that to a curve drawn on the lateral side of a cylinder, a cone or a polyhedron, another curve of the same length has to correspond on the surface development, called modified curve through the unrolling of the given curve.

The construction of the surface developments of technical parts comprises a big diversity of cases which can be solved through the methods of descriptive geometry. Thus, revolving, rotation and changing of plans are applied in order to find out the real sizes of the segments, angles and surfaces which form the proper surfaces.

This article wishes to exemplify the above mentioned in order to unroll certain technical parts made of combined surfaces.

## 2. ASPECTS REGARDING THE CONSTRUCTION OF SURFACE DEVELOPMENTS MADE OF SECOND DEGREE QUADRICS

Generally, when intersecting surfaces are second degree quadrics, the intersection curve is a fourth degree curve which can have one or two inflection points, as surfaces have one or two common tangent planes.

In the following applications, the cylinders are considered second degree quadrics, due to their highest technical applicability. We can see the surface developments of two cylinders with perpendicular, respectively concurrent axes at a given angle.

In the first case there are two right circular cylinders, with perpendicular axes and equal diameters, (Figure 1).

The first cylinder is vertical, having as base the  $C_1 (c_1, c'_1, c''_1)$  centre circle, and the second cylinder is frontal horizontal, having as base the  $C_2 (c_2, c'_2, c''_2)$  centre circle.

In order to determine the common curve of the two cylinders, the intersection points of the coplanar generatrices are determined. For this, the two cylinders are sectioned with frontal planes. Thus, the frontal plane  $F_1$  sections the  $C_1$  cylinder after the generatrices from points  $A (a, a', a'')$  and  $B (b, b', b'')$  and the  $C_2$  cylinder after the generatrices from points 2 and 3. The needed curve intersection points are found at the intersection of the segments:

$$AA_1 \cap 24 = A_2 \quad ; \quad a'a'_1 \cap 2'4' = a_2' \quad (1)$$

$$AA_1 \cap 35 = A_3 \quad ; \quad a'a'_1 \cap 3'5' = a_3' \quad (2)$$

$$BB_1 \cap 24 = B_2 \quad ; \quad b'b'_1 \cap 2'4' = b_2' \quad (3)$$

$$BB_1 \cap 35 = B_3 \quad ; \quad b'b'_1 \cap 3'5' = b_3' \quad (4)$$

The other intersection points are analogically determined.

The analysed intersection curve is a continuous, breaking, simple tangent curve and it divides in two identical branches. This curve degenerates into two plane curves, called ellipses, situated on end planes. In the horizontal and lateral projections, the ellipses' projections coincide with the bases of the projecting cylinders.

The surface developments of cylinders  $C_1$  and  $C_2$  are built and the modified curves through unrolling of the intersection curve are transposed.

According to Oliver's Theorem, the modified curve through unrolling of the section made by a plane into a cylinder presents inflections in the points where the tangent plane to the cylinder is perpendicular to the secant plane. If one of the cylinder's generatrices is perpendicular on the secant plane, then the modified curve through unrolling of the section has no inflection point. Also, the angle of two curves of the surface is equal to the angle of their modified curves through unrolling.

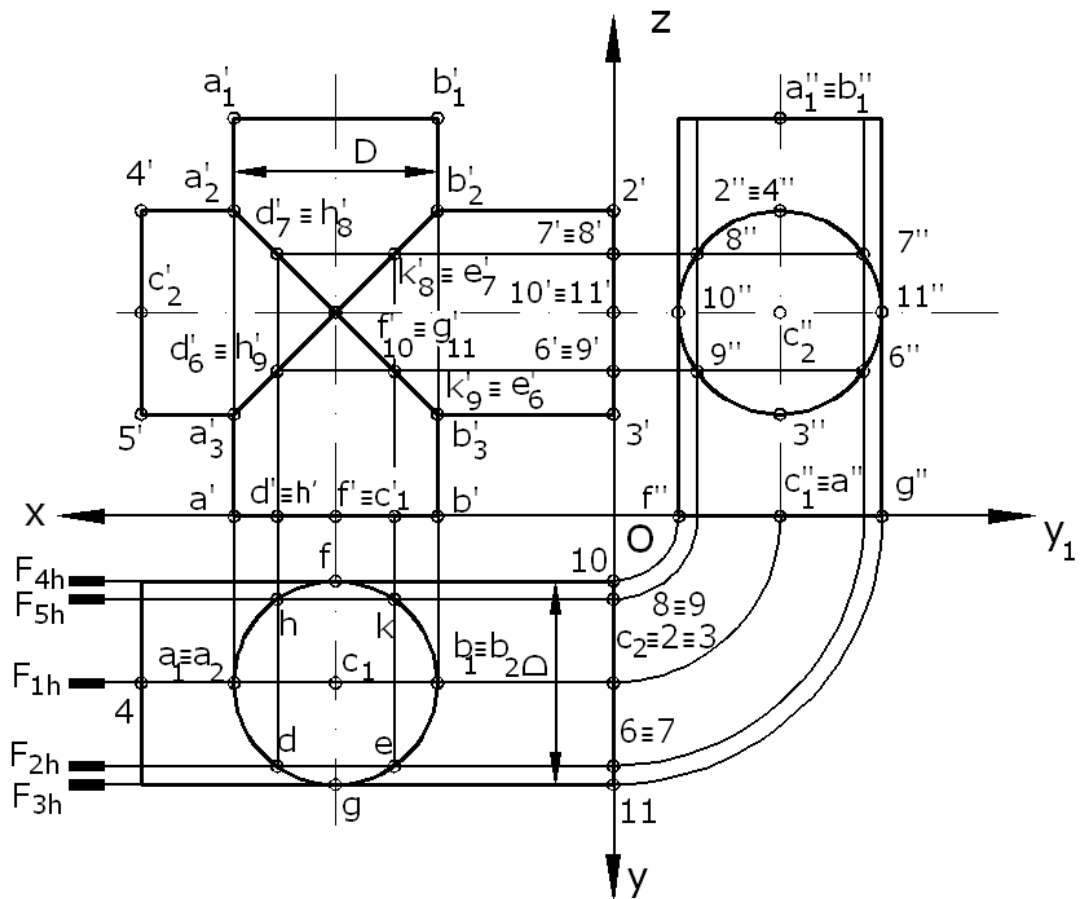


Fig.1 The intersection of two right circular cylinders, with perpendicular axes and equal diameters.

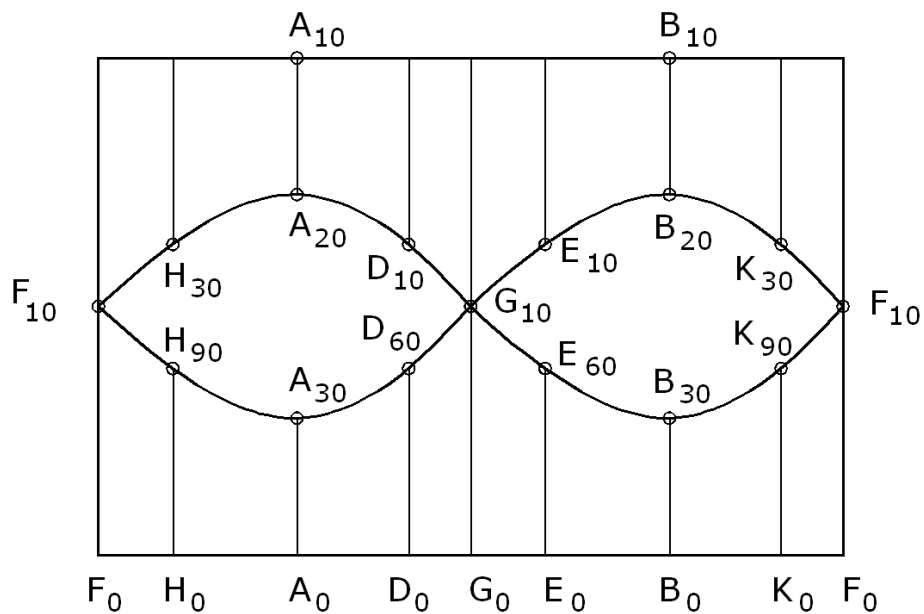


Fig.2 The surface development of the cylinder with a vertical axis.

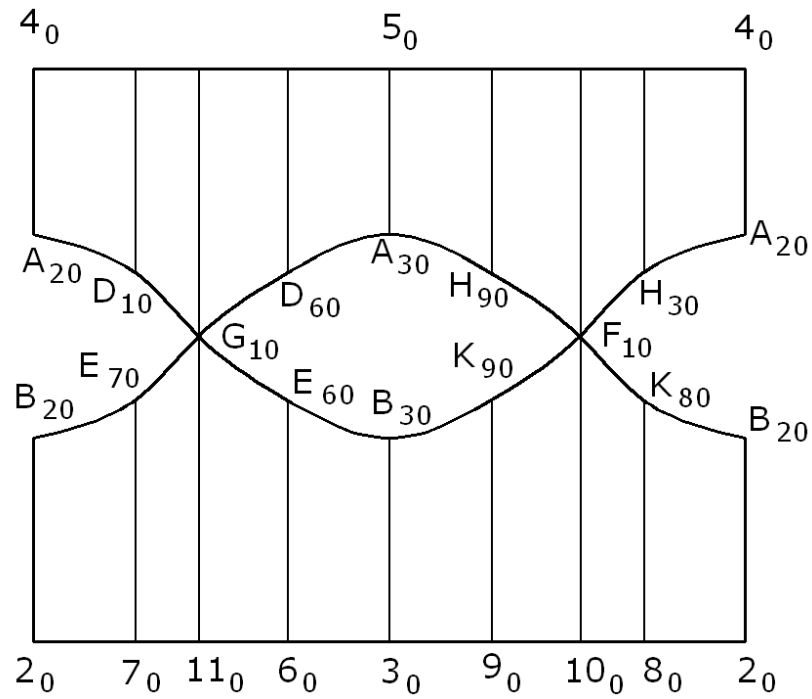


Fig.3 The surface development of the cylinder with a horizontal axis.

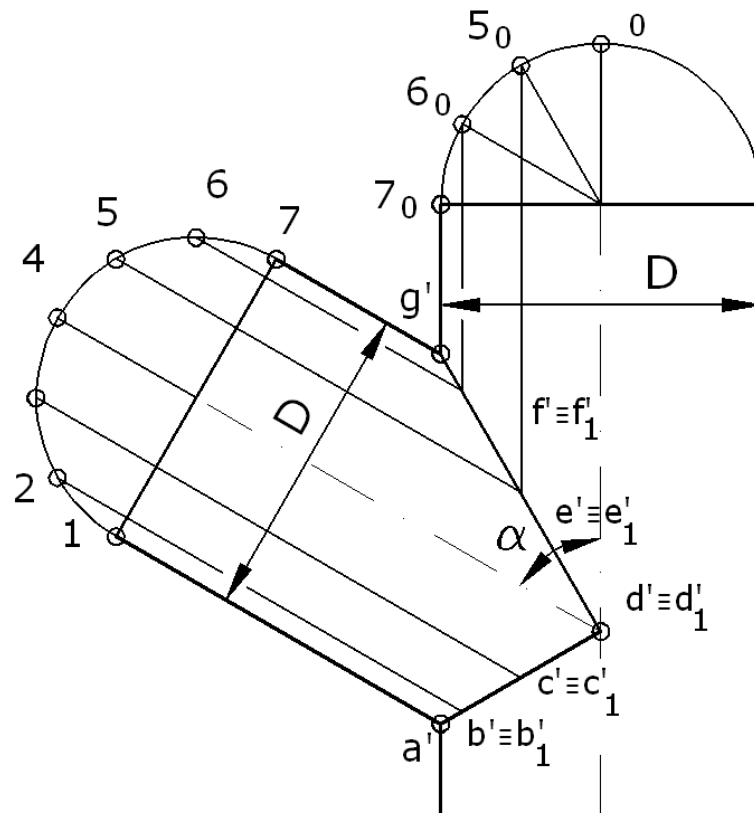
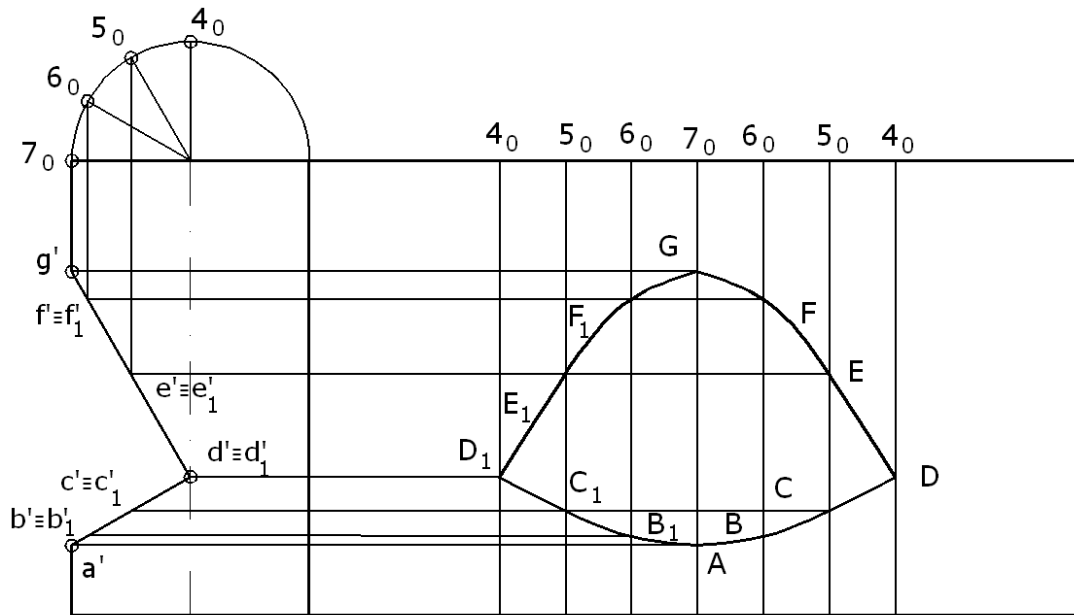
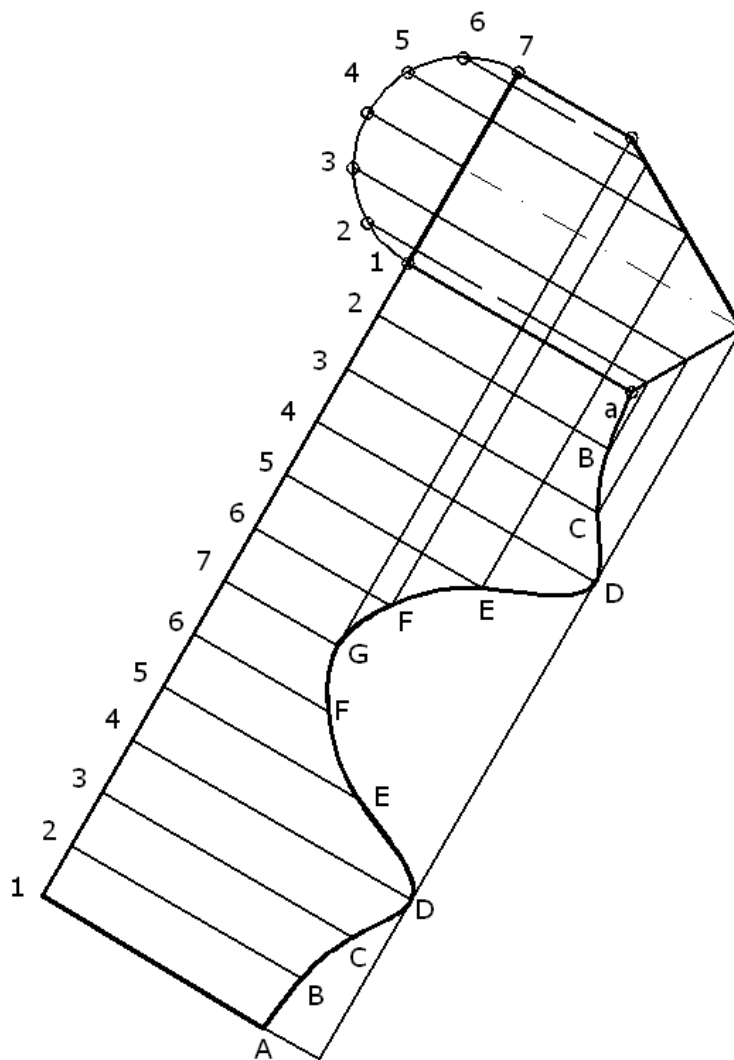


Fig.4 Example of metal part made of two circular right cylinders, with equal diameters and concurrent axes at an angle  $\alpha$ .



**Fig.5** The surface development of the cylinder with a vertical axis.



**Fig.6** The surface development of the oblique cylinder.

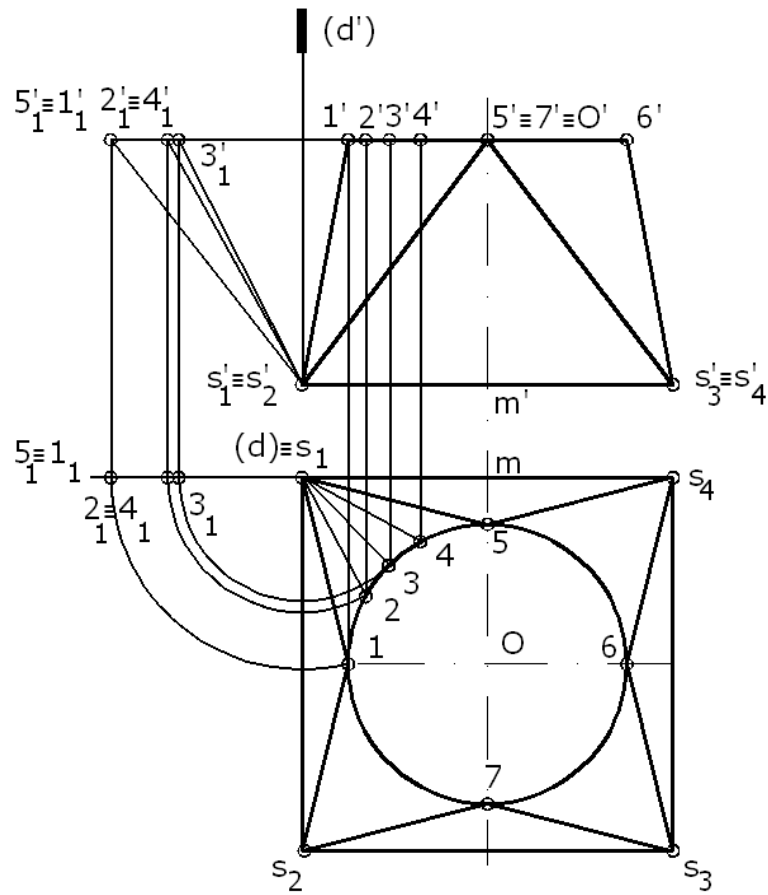


Fig.7 Reduction made by a prismatic surface, with a square base and a right circular cylinder.

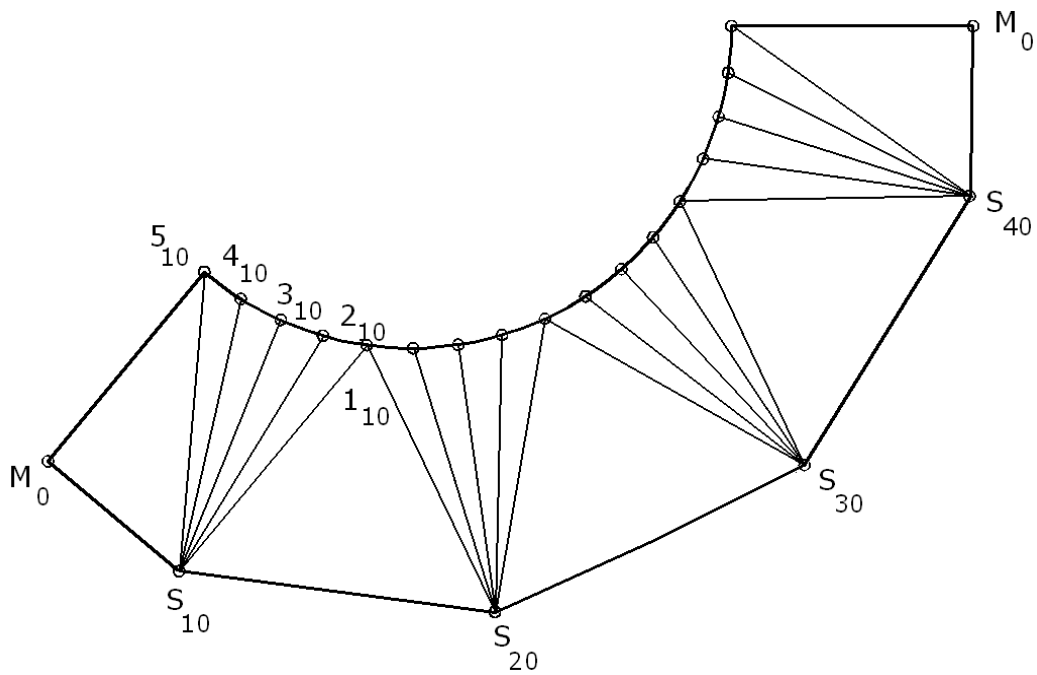


Fig.8 Surface development of the reduction.

In Figure 2 we can see the surface development of the cylinder with a vertical axis and in Figure 3, of the one with a horizontal axis [1].

To apply Oliver's Theorem to the analysed modified curves through unrolling leads to the  $F_{10}$  and  $G_{10}$  inflection points, common to both geometrical solids. In these points, the tangent plane to the  $C_1$  and  $C_2$  cylinders is perpendicular on the secant plane.

Another frequently encountered technical case is the one of the metal parts made of two circular right cylinders, with equal diameters and concurrent axes at an angle  $\alpha$ , Figure 4, [2].

For the drawing and discharging operations of unfinished goods, it is necessary to build certain patterns. Flat patterns are designed, printed, cut, creased, folded on machines made for these purpose. The patterns contain the surface developments of the cylinders and the transposition of their intersection curve through unrolling.

The bases of the two cylinders are revolved on the vertical plane (Figures 5, 6). Fourteen generatrices are built on the surface of the cylinders, with a uniform distribution and noted  $a'1, b'2$ , etc. The two cylinders are sectioned with frontal planes which contain these generatrices and it is analogically made with the previous case. The normal sections in the cylinders transform into segments with the length equal to their circumference.

The intersection curve is a penetration with a double tangent.

### 3. ASPECTS REGARDING THE CONSTRUCTION OF SURFACE DEVELOPMENTS MADE OF SECOND DEGREE QUADRICS AND POLYHEDRAL SURFACES

The connection of second degree quadrics with the polyhedral surfaces can be made through link surfaces, called reductions [3].

Reductions are very frequently used in technique for funnels, ducts, elbows, pipes, parts of chimneys, tanks, boilers, a.s.o.

In Figure 7 we can see the reduction which represents the link between a prismatic surface, with a square base and a right circular cylinder [4].

The reduction's surface is composed of four conic surfaces and four triangular surfaces.

The tips of the cones are the points  $S_1, S_2, S_3$  and  $S_4$  and their bases represent the circle of the superior base. The lengths of the generatrices of the cone with the tip in  $S_1$  are determined through the rotation around the  $D(d')$  axis, which contains the  $S_1$  tip.

In Figure 8 the surface development of the reduction connection surface is drawn.

The surface development of the cone with the centre in  $S_1$  is completed with the one of the  $S_{10}I_{10}S_{20}$  triangle and the result repeats itself for four times, because on the connection surface there are four cones and four triangles.

### 4. CONCLUSIONS

The surface developments of compound surfaces which form metal works are frequently encountered in technique. Also, in the construction of pipes there are used non-standardised fittings and which need to be produced in workshops, such as: leg-pipes made of segments, wrinkling leg-pipes, ramifications made by fittings production, etc.

This subject is very interesting due to the fact that the accuracy and precision of geometrical constructions are determined in the production of parts which have to fully respect the geometrical and functional parameters.

These actual topics represent the source of future research based on the drawing and discharging operations of metal parts, in order to increase precision and to reduce production costs.

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