Vanda Ligia DANAILA, Alina Angelica ANGHEL

GRAPHICAL AND ANALYTICAL APPROACH ON CYLINDRICAL – CONICAL INTERSECTIONS

Abstract: *The cylindrical – conical intersections are often met in technique as parts of pipes, tanks, bends etc. The paper work proposes a comparison between methods to solve the cylindrical – conical intersection problems. The problems may be solved either applying graphical, descriptive geometry methods or using analytical methods. The graphical method uses a certain number of points to find the intersection curves and the development of the solids, the analytical method uses a complex enough mathematical apparatus.*

Key words: *cylinder, cone, intersection, development, analytic, descriptive.*

1. INTRODUCTION

Cylindrical – conical intersections are frequently met in engineering practice. The intersecting cylinders and cones are usually manufactured of iron plates, therefore knowing the methods to trace out the development of the solids is compulsory. Apart the developments of the lateral surfaces of cylinders and cones an important issue is to draw the outline of the curves coming out intersections.

In order to draw the intersection curves both graphical and analytical methods can be applied. Graphical methods allow finding the developments and the curves quickly and accurately enough. Graphical methods are explicit when drawn at a conveniently large scale. Computer aided design soft provide precision both for drawing and for dimensioning the drawings. Analytical methods are based on the equations describing the intersection curves. The coefficients of the equations are trigonometrical functions of the angles between the axes of the intersecting solids and of the angles of cones' vertices. Analytical methods are laboriously enough, but the use of computers eases them and allows obtaining the required values as accurately as needed.

2. GRAPHICAL APPROACH OF CYLINDRICAL – CONICAL INTERSECTIONS

The paper work proposes three cases of intersections: the intersection of two cylinders with different radii, the intersection of two cones with different radii and the intersection of a cone with a cylinder with different radii. In all cases the cylinders and the cones are right and circular and the axes of the solids intersect under a certain angle.

2.1 Intersection of two cylinders with different radii

Given the projections of two cylinders with different radii (diameters) and having axes intersecting under a certain angle. The projections of the intersection curve of cylinders 1 and 2 are obtained applying spheres' method (Fig. 1).

In order to plot the developments of the two cylinders the lateral surfaces are first drawn as rectangles with one side equal to the circumference of the base circle and the other side equal to the length of the generating line of each cylinder (Fig. 2b, 2c).

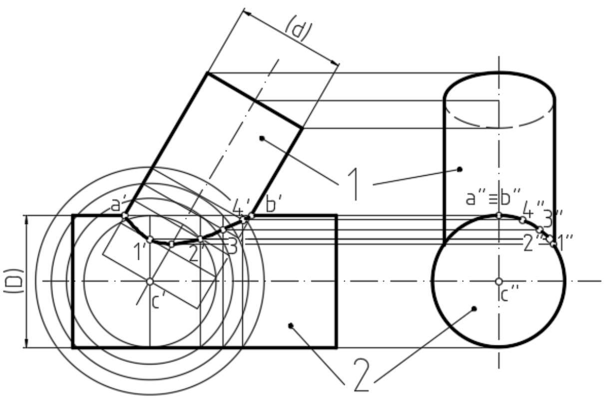


Fig. 1 The intersection curve of two cylinders.

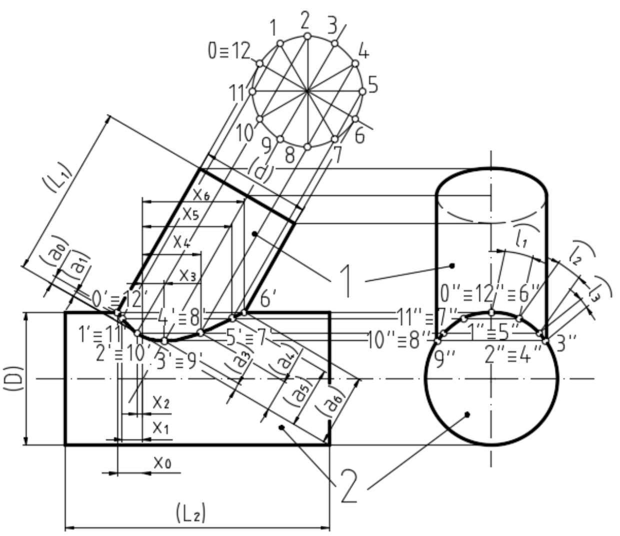
The intersection curve located on cylinder 1 is drawn dividing the base circle into 12 equal parts along with the development of the lateral surface (Fig. 2a). The distances (marked a_n) measured on the generating lines adequate to division points are transposed on the development and points are bound by a continuous curve (Fig. 2b) [2].

The intersection curve located on cylinder 2 is drawn measuring the lengths of arcs of circle (marked l_n) adequate to division points and transposing them on the development. The obtained points are bound by a continuous curve (Fig. 2c) [2].

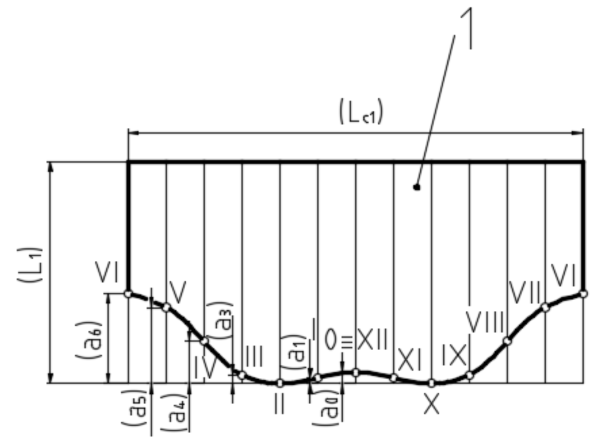
2.2 Intersection of two cones with different radii

Given the projections of two cones with different radii (diameters) and having axes intersected under a certain angle. The projection of the intersection curve of cones 1 and 2is obtained applying spheres' method (Fig. 3).

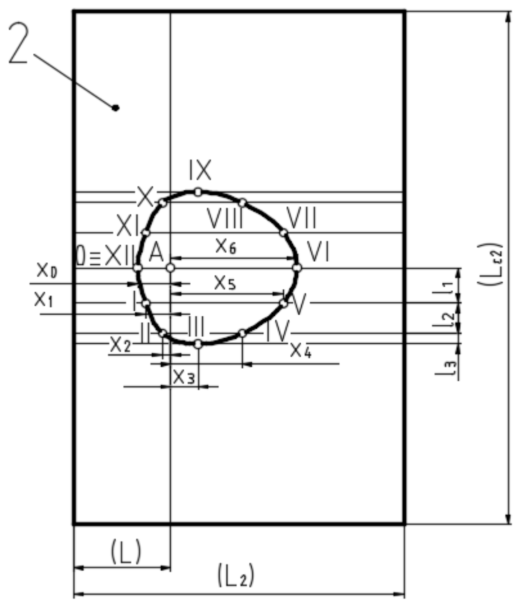
In order to plot the intersection curve located on cone 1 the development of the lateral surface is first drawn. The part of cone's base adequate to the projection of the intersection curve is divided into 6 conveniently selected parts (Fig. 4a) [4]. The true lengths of the segments of generating lines, for example ($v_1'c_0'$) and ($v_1'd_0'$), are transposed on the development. The points obtained on



a)



b)



c)

Fig. 2 The development of the lateral surfaces of the intersecting cylinders.

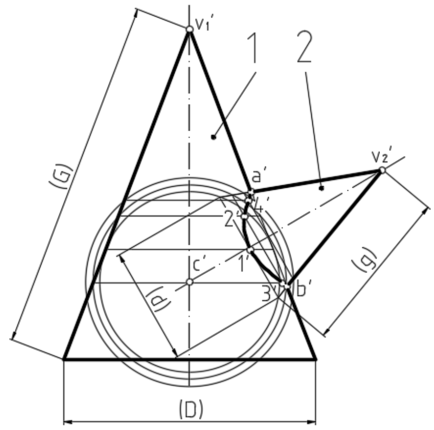
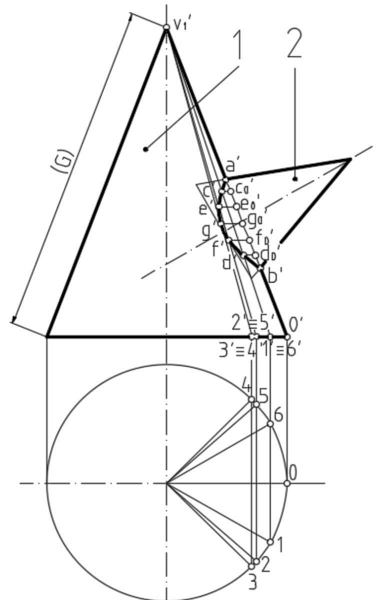
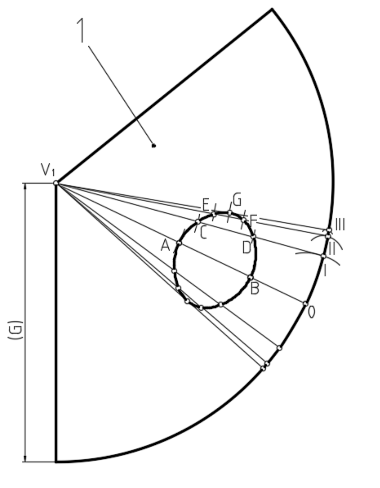


Fig. 3 The intersection curve of two cones.



a)



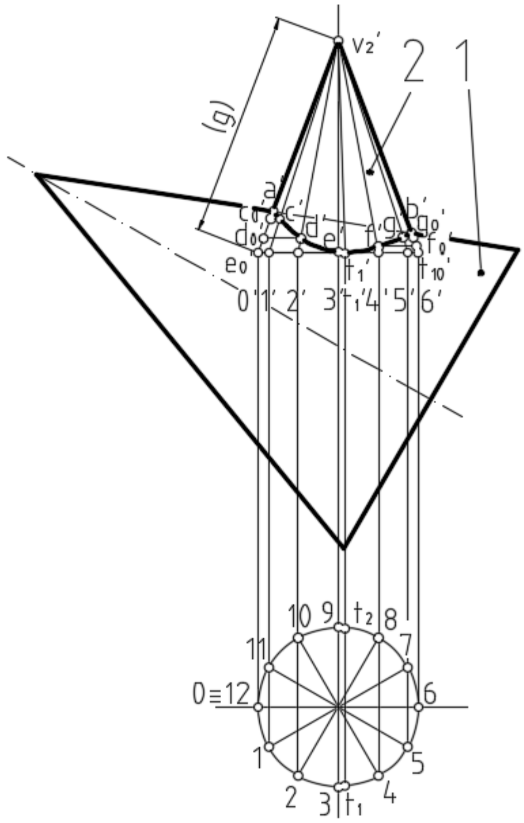
b)

Fig. 4 The development of the lateral surface of cone 1.

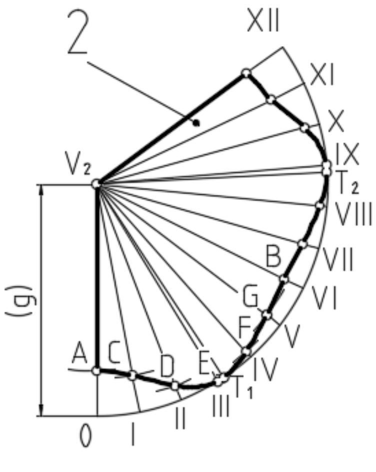
the development are bound by a continuous curve (Fig. 4b) [1].

In order to plot the intersection curve located on cone 2 the cones are rotated until cone 2 reaches a convenient position for plotting (cone's axis become vertical) (Fig. 5a).

The development of the lateral surface of the cone is first drawn and cone's base is divided into 12 equal parts along with the lateral surface (Fig. 5b). The true lengths of the segments of generating lines, for example ($v_2'c_0'$) and ($v_2'd_0'$), are transposed on the development. The points obtained on the development are bound by a continuous curve (Fig. 5b) [1].



a)



b)

Fig. 5 The development of the lateral surface of cone 2.

2.3 Intersection of a cone with a cylinder with different radii

Given the projections of a cone and of a cylinder with different radii (diameters) and having axes intersecting under a certain angle. The projections of the intersection curve of the solids are obtained applying the spheres' method (Fig. 6).

In order to plot the intersection curve located on the cone the development of the lateral surface is first drawn (Fig. 7b). Cone's base is divided into 12 equal parts along with the lateral surface (Fig. 7a, 7b). The true lengths of the segments of generating lines, ($v'1_0'$) and ($v'2_0'$), are transposed on the development. The points obtained on the development are bound by a continuous curve (Fig. 7b).

In order to plot the intersection curve located on the cylinder the development of the lateral surface is first drawn (Fig. 7c). The intersection curve is drawn measuring the lengths of arcs of circle (marked l_n) adequate to division points and transposing them on the development. The obtained points are bound by a continuous curve (Fig. 7c).

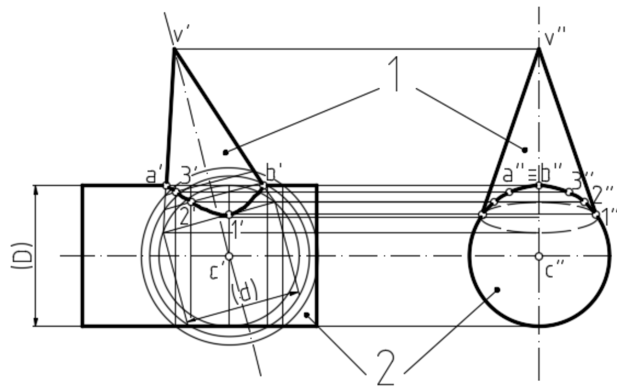


Fig. 6 The intersection curve of a cone with a cylinder.

3. ANALYTICAL APPROACH OF CYLINDRICAL – CONICAL INTERSECTIONS

The analytical approach of cylindrical – conical intersections suggests finding the profile of the intersection curves by means of mathematical analysis and of analytical geometry. Equations are associated to the intersection curves. The curves are drawn assigning an appropriate number of values to the unknowns. The solutions thus obtained for the equations represent points belonging to the intersection curves.

The paper work proposes to associate second degree equations to the closed curves located on the development of the intersecting cylinders and cones. Each of the analyzed closed curves is symmetrical in report to an axis (Fig. 8b, 9b, 10b) [3].

The coordinates of the points describing an intersection curve depend on the central angle θ_n and on the length of generating line l_n, measured from the vertex of the solid (n is the running number of the division point the base of the solid is divided into) [4].

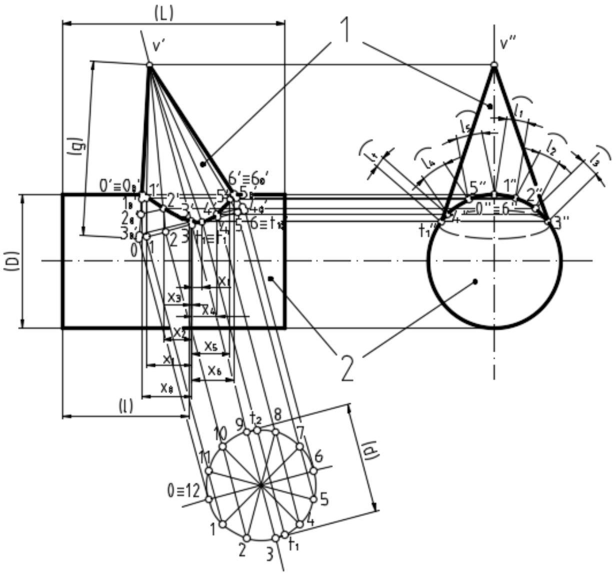
The length of the generating line is measured in report to the intersection point of the axes of the intersecting solids (Fig. 8a, 9a, 10a).

In report to the mutual positions of the two solids the lengths of the generating lines may have one or two values (l_{1n}, l_{2n}) for the same angle φ_k, respectively θ_k.

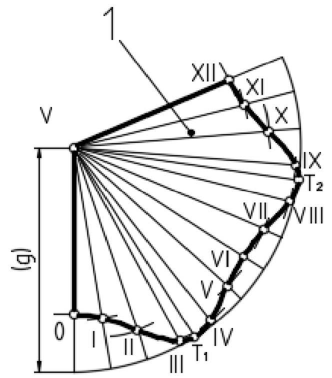
The length of the generating line is the solution of the equation:

$$l_n = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad (1)$$

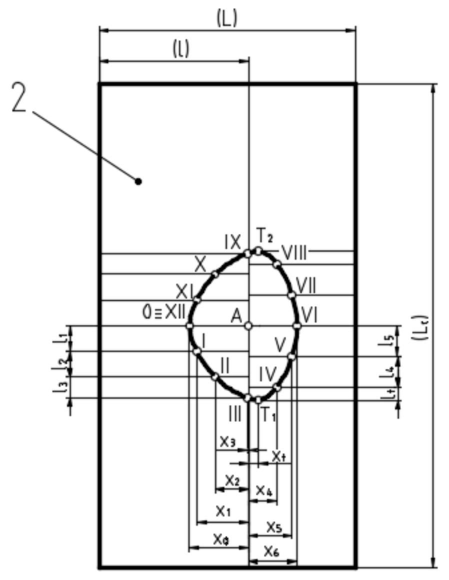
The coefficients A, B, C of the equation depend on the geometric parameters of the intersection solids: the intersection angle of the axes of the solids (α), the angles of cones' vertices (β_1, β_2), the radii of the solids on the intersection point (r_1, r_2), the heights of the cones measured from the vertices to the intersection point of the axes (a_1, a_2) [3].



a)

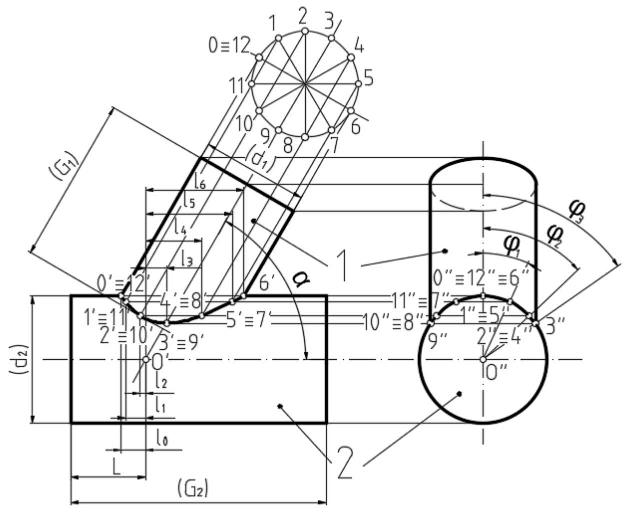


b)

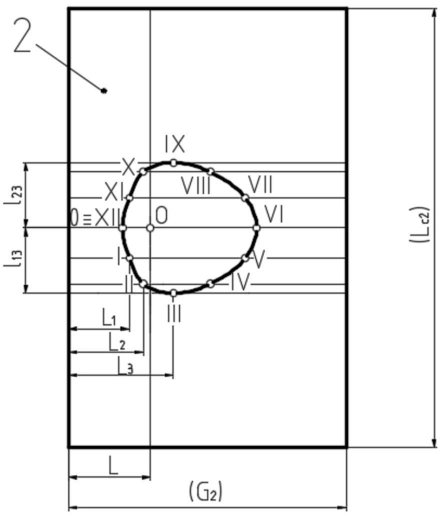


c)

Fig. 7 The development of the lateral surfaces of the intersecting cone and cylinder.



a)



b)

Fig. 8 The intersection curve of two cylinders.

The length l_n has real values in following condition:

$$B^2 - 4AC \geq 0 \quad (2)$$

The relationship between the current angle φ_k and the angle on the development θ_k is:

- for cylinders:

$$\Theta_k = \varphi_k \quad (3)$$

- for cones:

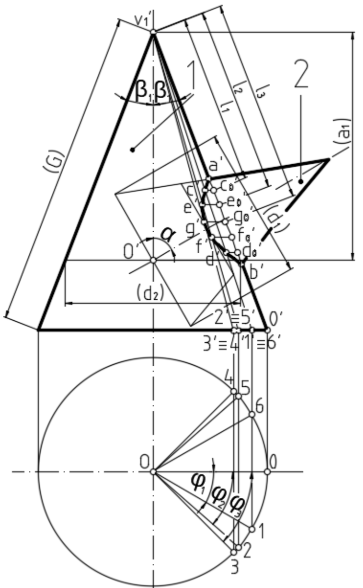
$$\Theta_k = \varphi_k \sin\beta \quad (4)$$

If the intersection solids are circumscribed to the same sphere having the centre in the intersection point of the axes, for the tangency points of the surfaces the relationship (2) becomes:

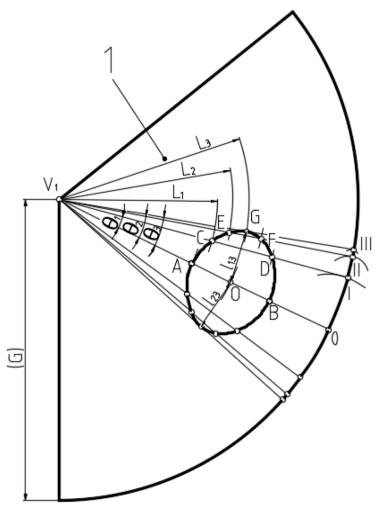
$$B_T{}^2 - 4A_T C_T = 0 \quad (5)$$

The length of the generating line in tangency point is:

$$l_T = -\frac{B_T}{2A_T} \quad (6)$$

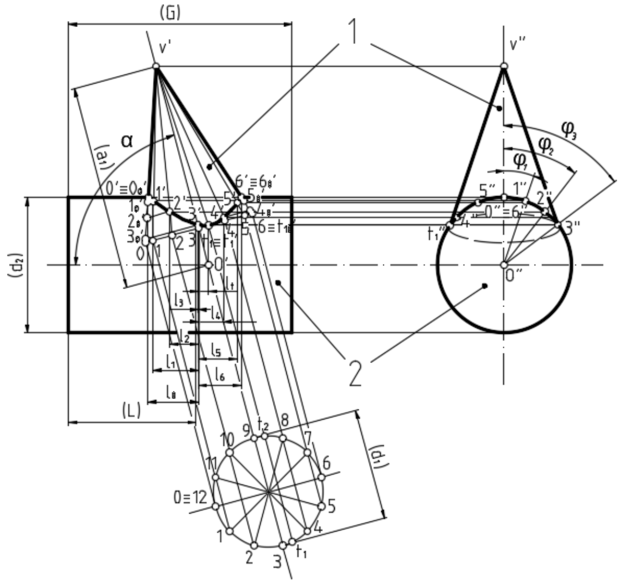


a)

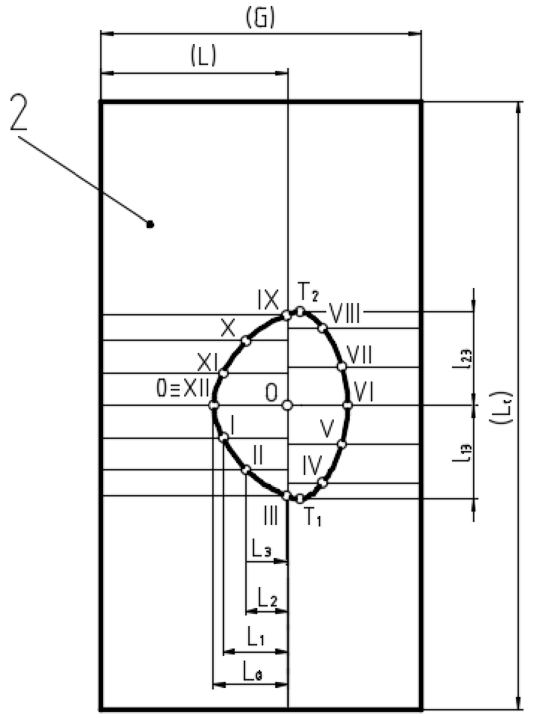


b)

Fig. 9 The intersection curve of two cones.



a)



b)

Fig. 10 The intersection curve of a cone with a cylinder.

For the two intersecting solids 1 and 2 relationship (5) becomes:

$$B^2{}_{1T} - 4A_{1T}C_{1T} = 0 \qquad (7)$$
$$B^2{}_{2T} - 4A_{2T}C_{21T} = 0 \qquad (8)$$

Relationships (7) and (8) allow finding the values of the angles φ_{1T} and φ_{2T} for the tangency points.

If the values of angles φ_{1T} and φ_{2T} along with the values of the length l_T are inserted in the previous relationship, a relationship between α, β_1, β_2, a_1, a_2 is obtained. The relationship is adequate to the case the two solids have two tangency points. In this case the intersection curve consists of two plane curves projected as two line segments on a plane parallel to the axes of the solids. The relationship proves that in case of two tangency points the values for α, β_1, β_2, a_1, a_2 can not be randomly chosen, the value of a parameter depends on the values of the other four parameters [3].

4. CONCLUSION

The paper work presents two methods to solve cylindrical – conical intersections, the graphical and the analytical method. Analyzing both methods the conclusion is that they do not exclude one another, they complete each other.

The graphical methods are simple and easy to apply; it only requires accuracy to measure the coordinates of the points and the lengths of arcs of circle. Graphical methods emphasize the importance of descriptive geometry methods and show the link between theoretical problems and the way they are used to solve practical problems.

An important advantage of the graphical methods is that a model, at a certain scale, can be manufactured from a cheap material in order to have an actual sample of the intersection before released to production.

The analytical methods have the advantage of being general ones; they can be applied to any type of cylindrical – conical intersections. An algorithm can be conceived for the coefficients of the equations describing the intersection curves, in order to contain the values for angles, radii, distances involved in intersection. The values may be written down a table in order to be used for a great number of combinations of cylinders and cones that intersect.

When analytical methods are applied it is important to settle the number of decimals for the coefficients of equations in order to avoid unnecessary reckoning.

The complex cases of intersections can be solved using both types of methods, graphical and analytical, in order to validate each other. Engineering practice has proven that there are situations when a combined, graphical and analytical, method is the best way to solve complex cases of intersection.

Both methods are general ones; they can be successfully applied to any dimensions of the intersecting solids (radii, heights, angles of cones' vertices, angles of intersecting axes etc.).

The decision to apply one or the other type of method belongs to the user.

REFERENCES

[1] Danaila, V. L., (2013). *Practical Geometry*, LAP LAMBERT Academic Publishing, ISBN 978-3-659-19362-0, Saarbrucken, Germany.

[2] Danaila, V. L., Anghel, A. A., (2006). *Descriptive Geometry*, Tehnopress Publishing, ISBN 973-702-345-5, Iasi, Romania.

[3] Danaila, V. L., *Development of Bends Consisting of Cylindrical Elements (Desfasurarea coturilor alcatuite din elemente cilindrice)*, Proceedings of Al VIII-lea Simpozion National, cu participare internationala de Grafica Tehnica si Design, "Transilvania" University, pp. 49 – 50, ISBN 973-635-195-5, 973-635-196-3, Brasov, June 2003, Transilvania University Publisher, Brasov, Romania.

[4] Enache, I., Ivanceanu, T., Buzila, V., (1982). *Descriptive Geometry and Technical Drawing. Problems and Applications (Geometrie descriptiva si desen tehnic. Probleme si aplicatii)*, Didactica si Pedagogica Publishing, Bucuresti, Romania.

Authors:

Ph. D., Lecturer Vanda – Ligia DANAILA, Technical University "Gheorghe Asachi" Iasi, Department of Graphic Communication, E-mail: wdanaila@gmail.com,
Ph. D., Lecturer Alina – Angelica ANGHEL, Technical University "Gheorghe Asachi" Iasi, Department of Graphic Communication, E-mail: a.a.anghel@gmail.com