Sanda BODEA

THE STUDY OF PLANES TANGENT TO THE CYLINDRICAL SURFACES

Abstract: *Taking into account the practical utility of the surfaces of rotation, work proposes the study of the methods of resolution and the determination of the tangent to it, regarding the cylindrical surfaces. The planes tangent taken in the study shall be determined under certain conditions imposed, using knowledge of descriptive geometry.*

Keywords: *tangent plane, cylindrical surface, straight line and point.*

1. INTRODUCTION

To better resolve problems related to the surfaces of rotation, many times must be drawn a surface that is tangent to them.

The work proposes the determination of the plane surfaces tangent to the cylindrical surfaces, under certain conditions imposed.

In general on a curved surface, through any point can be drawn an infinite number of curves. Plane tangential at that point on the curved surface, will be the plane determined by all tangential lines drawn at these curves, through the point considered. But, in accordance with the definition, a plane is determined by the two lines concurrent to a point, therefore are enough two tangent to two curves on the curved surface.

On the unmeasured curved surfaces the tangential plane has a common point with the surface, and on the measured curved surfaces the tangential plane has an infinite number of common points, meaning a generatrix of the surface concerned.

2. TANGENTIAL PLANE TO A CYLINDRICAL SURFACE

We consider the elements that define a cylindrical surface, the direction Δ, the guiding curve Γ and two generatrices of the cylindrical surface G_1 and G_2, parallel to the direction Δ (fig.1).

The two generatrices determine a secant plane passing through the guiding curve Γ after chord A_1A_2, which belongs to the plan, points A_1 and A_2 being situated on the two generatrices [2].

If on the cylindrical surface it is considered a certain curve C, this will be intersected by the generatrix G_1 and G_2 in points B_1 and B_2. The chord B_1B_2 belongs also to the secant plane. The two chords being in the same plane are concurrent:

$A_1A_2 \cap B_1B_2 = E_1$

By rotating the secant plane around the generatrix G_1 the chords A_1A_2 and B_1B_2 are shrinking until superposition of the points, the point of concurrence E_1 also turning until the position E_2. Thus the two chords, which are two secant straight lines for the cylindrical surface, become tangent to the guiding curve Γ, in point A_1, concerned at any curve C, in point B_1.

It has been demonstrated so, that the secant plane $[A_1B_1E_1]$ has been brought in the position of tangent plane $[A_1B_1E_2]$. This tangent plane contains all the points on the generatrix G_1. It follows that a plane tangential to a cylindrical surface is defined by the generatrix of the surface at the point under consideration and by the tangent to any curve carried on the cylindrical surface, concurrent with the generatrix of the point. The curve can be considered also guiding curve of the cylindrical surface.

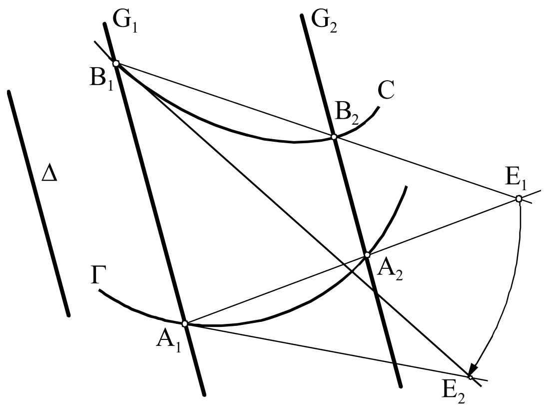

Fig. 1 Tangent plane to a cylindrical surface.

3. THE TANGENTIAL PLANE TO A CYLINDRICAL SURFACE UNDER A GIVEN ANGLE

It is considered the oblique cylinder with the base a circle in the horizontal plane, shown in figure 2 and is given the question of marking tangent planes to the surface of this, which has to make a given angle with the planes of projection.

A plane tangential to the cylinder which has to made the angle α with the horizontal plane of projection shall be parallel to the plane tangential to a straight cone, whose generatrices are inclined under the angle α from the horizontal plane of projection [4]. Either S the apex of this cone: ∠ (SA, [H]) = ∠ α.

The plane tangential to the cylinder contains a generatrix of the cylinder, so plane tangential to the cone shall be parallel to the cylinder generatrices, meaning it will contain a straight line parallel to the generatrices of the cylinder [1]. Because the plane tangential to the cone contains the cone apex S(s, s'), shall be drawn through it a straight line $\Delta(\delta, \delta')$ parallel with the cylinder generatrices: $\delta \parallel \omega\omega_1$, $s \in \delta$, $\delta' \parallel \omega'\omega_1'$, $s' \in \delta'$.

Straight line Δ (δ, δ') has the horizontal trace H (h, h') and will be contained in the plane tangential to the cone. So, may be drawn two planes $[P_1]$ and $[P_2]$, tangent to cone, drawing from the horizontal trace h the tangential lines P_1 and P_2, to the circle of the base of the cone from the horizontal plane. They represent the horizontal traces of the two planes tangential to the cone, $[P_1]$ and $[P_2]$. For drawing the vertical traces, P_1' and P_2', shall be considered a front line from each plan:

$F_1(f_1, f_1') \in [P_1]$ and $F_2(f_2, f_2') \in [P_2]$,

passing through the cone apex:

$S \in F_1$, $s \in f_1$, $f_1 \parallel Ox$, $f_1 \cap P_1 = h_1$, $f_1' = h_1' \cup s'$,

$S \in F_2$, $s \in f_2$, $f_2 \parallel Ox$, $f_2 \cap P_2 = h_2$, $f_2' = h_2' \cup s'$.

The vertical traces, P_1' and P_2', are parallel with the vertical projection of the front lines F_1 and F_2:

$P_1' \parallel f_1'$, $P_{1x} \in P_1'$, $P_2' \parallel f_2'$, $P_{2x} \in P_2'$.

The planes tangent to the cylinder will be parallel to the planes tangent to the cone, $[P_1]$ and $[P_2]$ and will contain a generator of the cylinder, meaning the horizontal trace of the plane tangential to the cylinder will contain the horizontal trace of the generatrix.

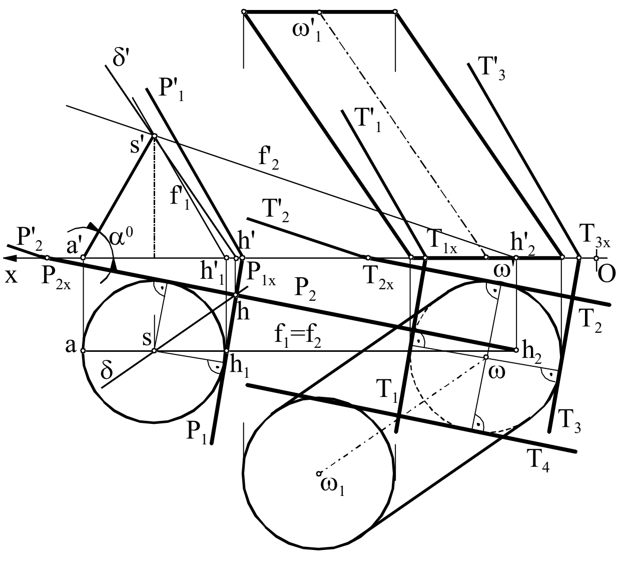

Fig. 2 Planes tangent to a cylindrical surface, inclined under the angle α, towards the horizontal plane of projection.

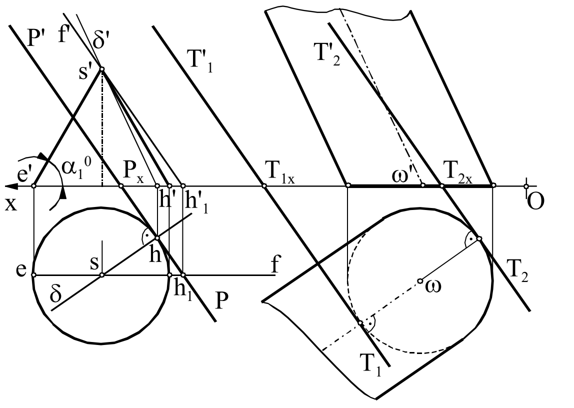

Fig. 3 Planes tangent to a cylindrical surface, inclined under the angle α_1, towards the horizontal plane of projection.

So can be found, four solutions given by the four tangent planes, which have the horizontal trace tangent of the circle of bottom basis on the horizontal plane, parallel to $[P_1]$ and $[P_2]$:

$[T_1] \parallel [P_1]$, $[T_2] \parallel [P_2]$, $[T_3] \parallel [P_1]$, $[T_4] \parallel [P_2]$.

Not the same number of solutions can be found in the case of the cylinder in figure 3, where the horizontal trace h of the straight line Δ (δ, δ'), parallel to the cylinder generatrices, is on the basic circle of the cone, from the horizontal plane. In this situation there are only two solutions for the planes tangent to the cylinder, inclined under the angle α_1 towards the horizontal plane of projection. They will be parallel to the plane tangent to the cone, [P], determined similar: $[T_1] \parallel [P]$, $[T_2] \parallel [P]$.

The cylinder in figure 4 does not accept any solution for drawing a plane tangent to the cylinder, tilted under the angle α_2 towards the horizontal plane of projection. This conclusion result from the fact that the horizontal trace h of the straight line Δ (δ, δ'), parallel to the cylinder generatrices, is inside the basic circle of the cone, from the horizontal plane.

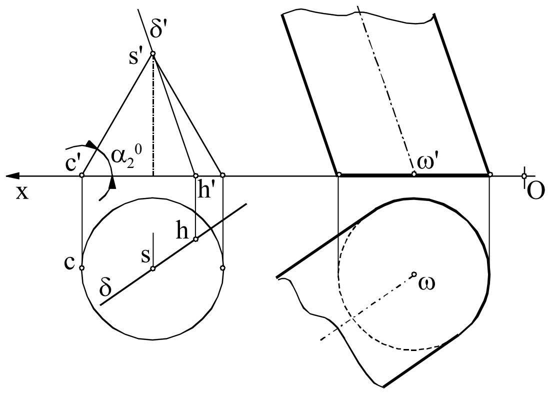

Fig. 4 Cylindrical surface without the possibility of tangent plane at an inclination α_2, towards the horizontal plane of projection.

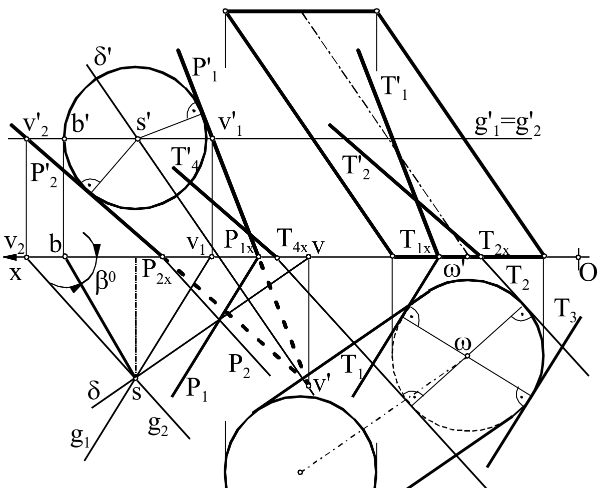

Fig. 5 Plane tangent to a cylindrical surface, inclined under the angle β, towards the vertical plane of projection.

In case of planes tangent to the cylinder, which made the angle β with the vertical plane of projection, the problem is solved alike.

The planes will be parallel to the plane tangent to a cone, whose generatrices are inclined under the angle β towards the vertical plane of projection.

Also in this case, the number of tangent planes depends on the position of the vertical trace v', of the straight line Δ (δ, δ'), parallel to the cylinder generatrices, towards the basic circle of the cone, from the vertical plane. So if the trace v' is located outside the circle, as shown in figure 5, there are four tangent planes, [T_1], [T_2], [T_3] and [T_4], if the trace v' is located on the circle there are two solutions for the tangent plane, and if the trace v' is located inside the basic circle of the cone, can not be drawn any tangent plan.

4. THE DETERMINATION OF THE PLANE TANGENT TO A CYLINDRICAL SURFACE, USING THE METHOD OF SUBSTITUTING PLANE OF PROJECTION

The same tangent plan that made the angle β with the vertical plane of projection, to the oblique cylinder in figure 6, can be determined using two successive change of plane. Thus, the oblique cylinder turns the first time in a front cylinder, through the vertical change of plane, towards the axis O_1x_1, drawn parallel to the axis of the cylinder: $O_1x_1 \parallel \omega\omega_1$.

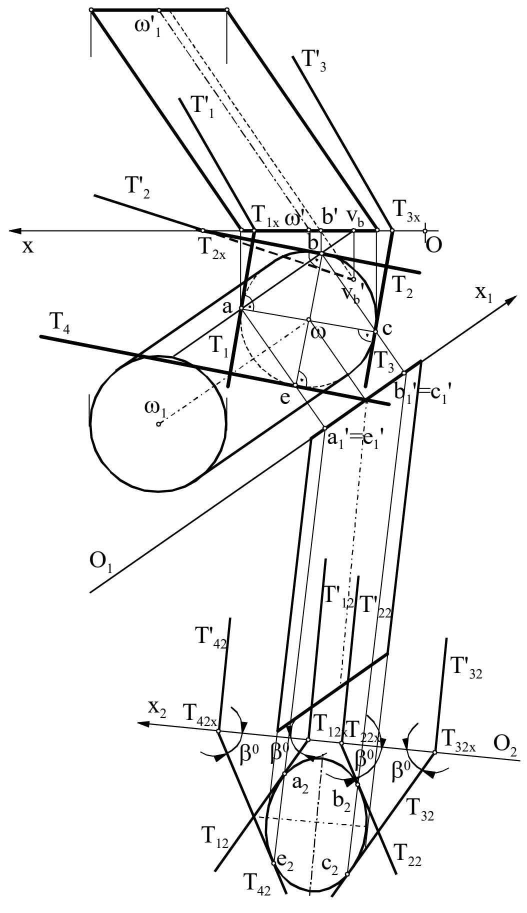

Fig. 6 Plane tangent to a cylindrical surface, determined by the double change of the plane of projection.

The second change is a horizontal change of plane, taking in consideration the axis O_2x_2 perpendicular to the axis of the front cylinder and transforming it into a straight cylinder. The projection on the new horizontal plane is the ellipse which represents the normal section in the cylinder.

The planes tangent to the straight cylinder will be four vertical edge planes, which form the angle β with the vertical plane of projection, [T_{12}], [T_{22}], [T_{32}] and [T_{42}]. The horizontal traces are drawn under the angle β towards the axis O_2x_2, tangent in points a_2, b_2, c_2 and e_2 to the ellipse of section. The vertical traces are perpendicular to the axis O_2x_2.

Considering that the tangent planes [T_{12}], [T_{22}], [T_{32}] and [T_{42}] include the generatrices of the straight cylinder from points a_2, b_2, c_2 and e_2, for drawing the planes tangent to the oblique cylinder it returns from the change of planes and determine the points of tangency on the basic circle in the horizontal plane a, b, c and e.

Horizontal traces of the four tangent planes [T_1], [T_2], [T_3] and [T_4] shall be drawn through the points a, b, c and e that are tangential to the circle, containing generatrices of the oblique cylinder in these points. The vertical traces of the tangent planes are drawn through the vertical traces of the generatrices that pass through the points a, b, c and e. Thus, in figure 6 is explained the determination of the vertical trace for the generatrix from b, (v_b, v_b') and drawing the vertical trace of the tangent plane [T_2]: $T_2' = T_{2x} \cup v_b'$, the other exceeding the drawing frame.

5. COMMON TANGENT PLANES TO TWO CYLINDRICAL SURFACES

In general any two cylindrical surfaces do not allow common tangent planes. However in certain situations this is possible, as follows:

- if the cylinders have the same guiding plane curve [3];
- if the cylinders are circumscribed to the same surface;
- if the generatrices of the cylinders are parallel with the same direction [3].

As shown in figure 7 were considered two cylindrical surfaces with the same guiding plane curve, a circle situated in the horizontal plane of projection with its center at the point Ω (ω, ω').

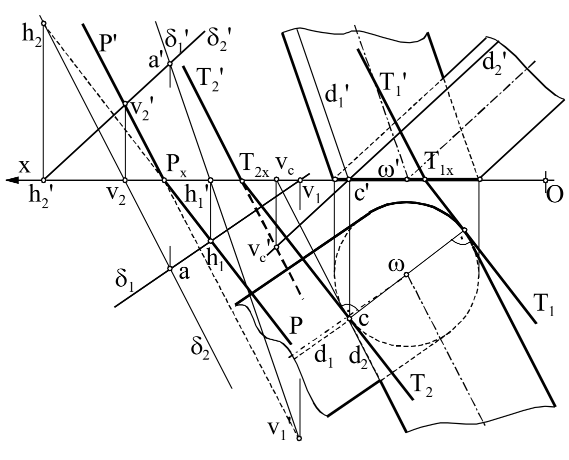

Fig. 7 Tangent planes common to two cylindrical surfaces with the same guiding curve.

The two cylinders admit two common tangent planes, which have the horizontal trace tangent to the guiding curve and contain the generatrices of the two cylinders, from the point of tangency.

The directions of the traces of the two common tangent planes are parallel with the traces of a plane [P], generated by any two straight lines, Δ_1 (δ_1, δ_1') and Δ_2(δ_2, δ_2'), drawn through the point A (a, a'), parallel to the generatrices of the cylindrical surface:

$\Delta_1 (\delta_1, \delta_1') \parallel D_1 (d_1, d_1'); \Delta_2 (\delta_2, \delta_2') \parallel D_2 (d_2, d_2')$;

$P = h_1 \cup h_2; P' = v_1' \cup v_2'$;

$[T_1] \parallel [P], [T_2] \parallel [P]$.

It can be seen that the plane [T_2], tangent in c to the guiding curve, contains the generatrices D_1 and D_2 of the two cylindrical surfaces (for the generatrix D_2, the vertical trace $v_c' \in T_2'$). Similarly can be demonstrated also, for the other tangent plane [T_1].

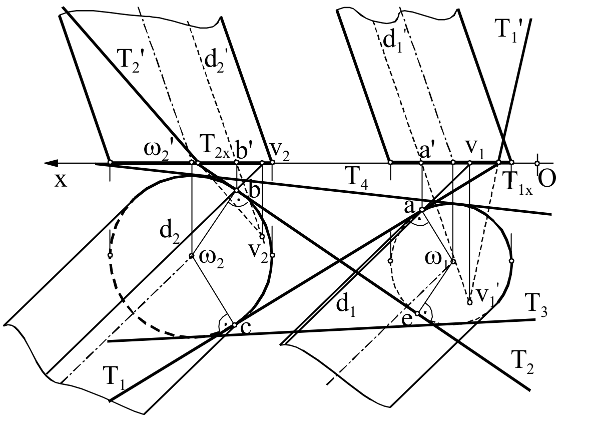

Fig. 8 Tangent planes common to two cylindrical surfaces with the base in the horizontal plane and parallel generatrices.

When the generatrices of cylinder are parallel with the same direction, as shown in figure 8, the two cylinders admit four common tangent planes, [T_1] ÷ [T_4]. Horizontal traces, T_1 ÷ T_4, are common tangent lines to the bases in the horizontal plane. The vertical traces, T_1' ÷ T_4', shall be determined from the condition of belonging of the tangency generatrix to the plane. For the tangent plane [T_1], the generatrix D_1 (d_1, d_1') has vertical trace V_1 (v_1, v_1') and determine the vertical trace T_1', $T_1' = T_{1x} \cup v_1'$. Similarly, $T_2' = T_{2x} \cup v_2'$.

The problem can be generalized (figure 9), considering two cylindrical surfaces with generatrices parallel to the direction D (d, d') and with the guiding curves two circles with centers in the points Ω_1 (ω_1, ω_1') and Ω_2 (ω_2, ω_2'), situated in any plane [P].

It is folded the plane [P], together with the two guiding curves on the vertical plane of projection. So, can be drawn, in the folded plane, the four common tangent planes, [T_{10}] ÷ [T_{40}].

As shown in figure 9 have been drawn the traces of the plane [T_1], returning from folding. Were determined the points of tangency A (a, a') and C (c, c') and has been used the tangency generatrix at point C (c, c'), Δ (δ, δ'):

$T_1 = h \cup h_1$; $T_1' = v' \cup v_1'$.

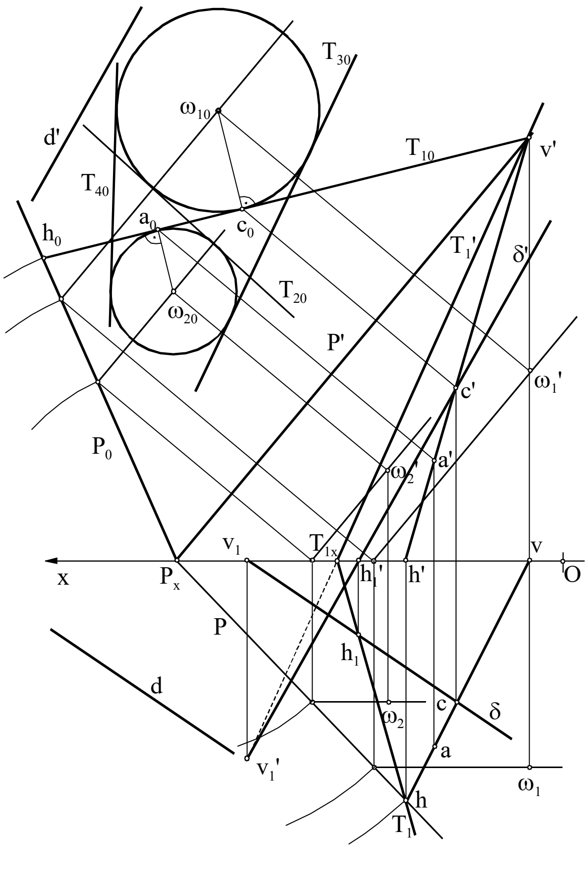

Fig. 9 Tangent planes common to two cylindrical surfaces with parallel generatrices.

6. CONCLUSIONS

The paper shall summarise the possibilities for the determination of tangent planes to cylindrical surfaces, respecting certain conditions imposed.

There are analysed the possible cases of drawing the planes tangent to the cylindrical surfaces under given angles to the planes of projection and situations when two cylindrical surfaces admit common tangent planes.

It has been noticed the use of the method of substituting plane of projection in determining the tangent plane.

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Author:

Assoc. Professor PhD. Sanda BODEA, Department of Automotive Engineering and Transports, E-mail: sanda.bodea@auto.utcluj.ro, Phone: 0264401610.