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THE STUDY OF TANGENCY TO CONICAL SURFACES

Abstract: *This paper aims to study the problem of tangency of straight lines and planes, with application at conical surfaces. With the help of descriptive geometry knowledge, the plane and straight lines tangent to the conical surfaces are being determined, respecting the special conditions imposed.*

Key words: *tangent plane, tangent straight line and conical surface.*

1. INTRODUCTION

Starting from the problem of tangency to random surface, the paper studies the problem of tangency to a conical surface that has its base in the horizontal plane, vertical plane or random plane.

The problem of tangency can be related to planes or straight lines, that are tangent to single or double conical surfaces.

2. PLAN TANGENT TO RANDOM SURFACE

The general problem of a plane tangent at a point A(a,a') on a curved surface, such as the one in Figure 1, can be handled by drawing the tangents to two curved lines, concurrent at that point [4]. These tangents will determine the tangent plane.

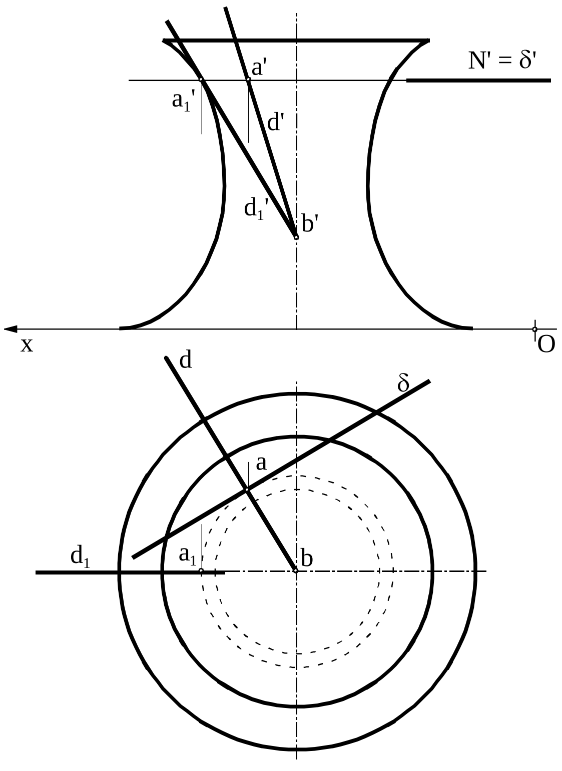

Fig. 1 Plane tangent to a random surface.

One of the curves was considered the circle determined by sectioning the surface with the level plane [N], through point A(a,a'). The tangent is the straight line Δ(δ,δ'), a horizontal one.

The second curve is the meridian of the surface through point A(a,a'). It is applied the level rotation method, point A(a,a') rotating in point $A_1(a_1,a_1')$, on the main meridian. The tangent to it in point $A_1(a_1,a_1')$ is the front line $D_1(d_1,d_1')$, which intersects the axis of rotation in point B(b,b'). The tangent at point A(a,a') is determined by returning from rotation and will be given by point A(a,a') and point B(b,b'), which is its own rotated; D(d, d'): $d = a \cup c$, $d' = a' \cup c'$.

3. PLANE TANGENT TO A CONE, WHICH FORM A GIVEN ANGLE WITH THE HORIZONTAL PLANE

To determine the plane tangent to a cone, which form an angle α with the horizontal plane, it starts from the fact that, all the planes passing through a fixed point and making an angle α with the horizontal plane, wrap a cone, having the apex at the fixed point.

There will be made a case study of possible tangent planes, depending on the angle α value [2].

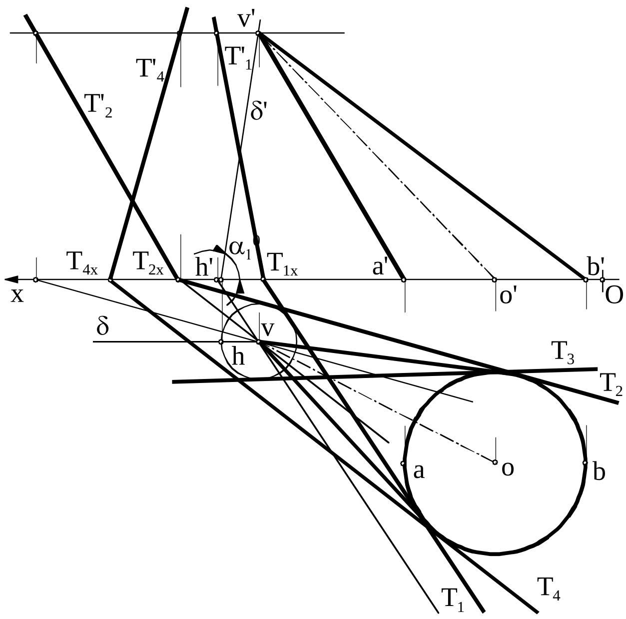

Fig. 2 Plane tangent to the cone that makes angle α_1 with the horizontal plane.

The guiding curve of this cone is the circle tangent to the horizontal traces of these planes. So the plane looked for will be the common plane tangent to the given cone and to the cone wrapped by these planes.

In Figure 2 the apex V(v,v') of the given cone is considered to be also the apex of the cone wrapped by the planes that make the angle α_1 with the horizontal plane. This cone has one of the generators the segment of the straight line VH(vh,v'h'), located on the front straight line Δ(δ,δ'), which makes the angle α_1 with the horizontal plane and has the guiding curve the circle with the radius vh.

If the two cones have the common apex V, they admit a common tangent plane. Its horizontal trace is the common tangent of the guiding curves. The refore there are four solutions; two solutions when the horizontal trace of the tangent plane is tangent on the outside to the two base circles [T_2] and [T_4]; other two when the horizontal trace of the tangent plane is tangent on the inside of the two base circles [T_1] and [T_3].

The determination of the vertical traces of the tangent planes was done using some horizontal straight lines, from the tangent planes, which pass through the apex V of the two cones.

Depending on the angle α, the following cases may be encountered:

a – the circle with the radius vh is tangent to the outside of the cone base circle (Figure 3), in which case three tangent planes can be drawn, two when the horizontal trace of the tangent plane is tangent to the outside of the two base circles [T_1] and [T_2] and one through the point of tangency of the two base circles, [T_3];

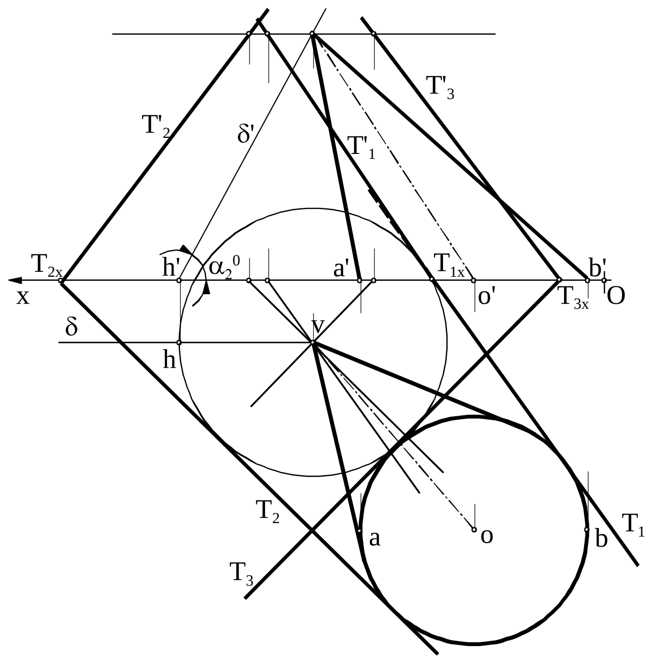

Fig. 3 Plane tangent to the cone that makes the angle α_2 with the horizontal plane.

b – the circle with the radius vh intersects the cone base circle (Figure 4), in which case two tangent planes can be drawn, whose horizontal trace is tangent to the outside of the two base circles [T_1] and [T_2];

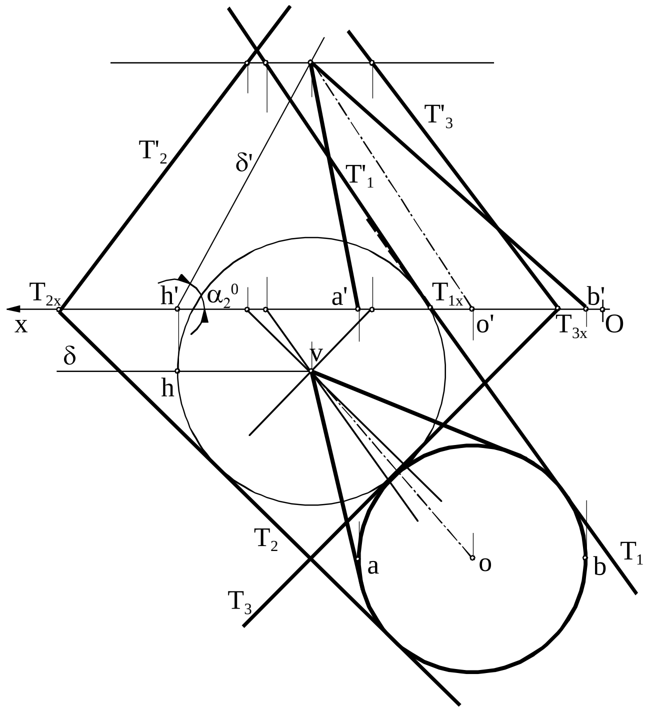

Fig. 4 Plane tangent to a cone that makes the angle α_3 with the horizontal plane.

c – the circle with the radius vh is tangent on the inside to the cone base circle (Figure 5), in which case there is only one tangent plane [T_1], that has the horizontal trace tangent to the circles at the common point of tangency;

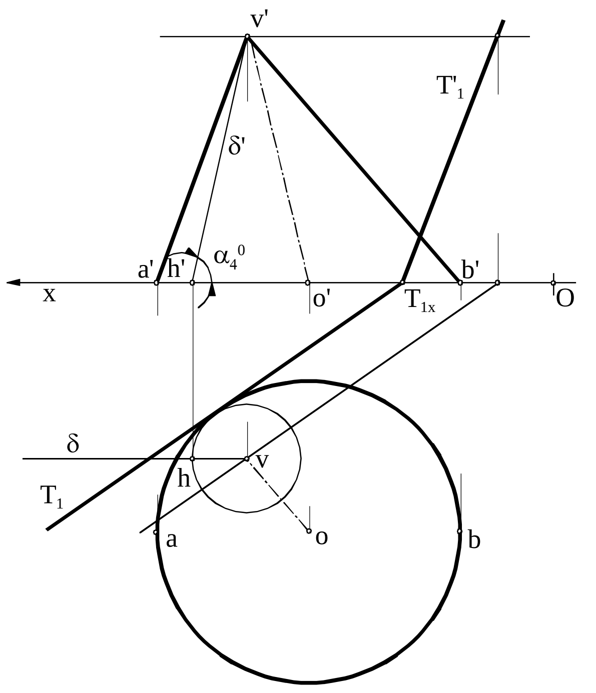

Fig. 5 Plane tangent to a cone that makes angle α_4 with the horizontal plane.

d – the circle with the radius vh is inside the base circle of the cone (Figure 6), in which case there is no tangent plane.

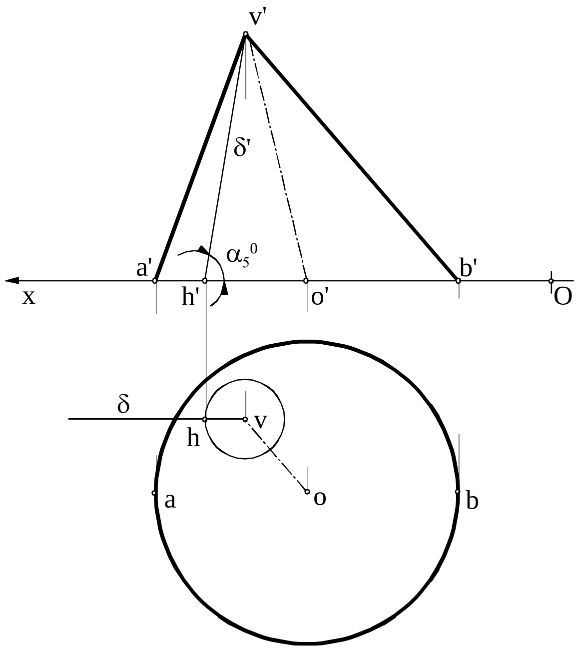

Fig. 6 Cone which does not admit tangent plane that makes angle α_5 with the horizontal plane.

4. PLANES AND STRAIGHT LINES TANGENT TO CONICAL SURFACES, IN IMPOSED CONDITIONS

In Figure 7, has been determined the plane tangent to a cone with apex V(v,v'), placed on the Ox axis, which contains the straight line D(d,d'), which passes through the cone apex. The guiding curve of the cone is located in the random plane [P], with its center in point A.

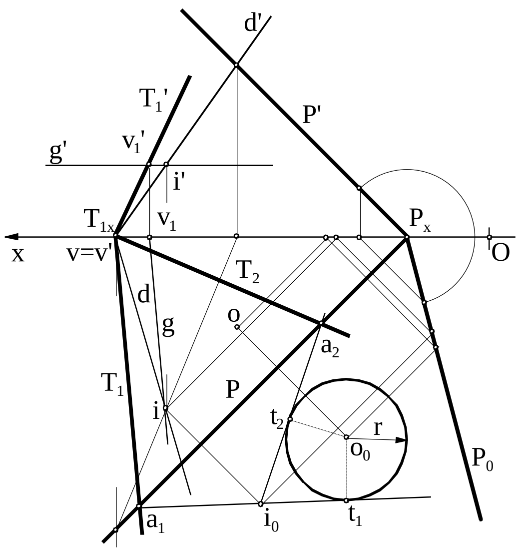

Fig. 7 Plane tangent to a con having the apex on Ox.

The guiding curve is rabattated on the horizontal plane as the circle of radius R, with the center in point o_0 (Figure 7). The straight line D(d,d') intersects plane [P], in point I(i,i'), whose tilt is point i_0. Drawing the tangents i_0t_1 and i_0t_2 to the circle, points a_1 and a_2 are obtained on the horizontal trace of plane [P]. These are points of the horizontal traces of the tangent planes at the cone. It results that two tangent planes [T_1] and [T_2] can be drawn.

In the drawing area was drawn the tangent plane [T_1], using the horizontal line G(g,g'), taken through the point I(i,i'): $T_1 = a_1 \cup T_x$, $T_1' = v_1' \cup T_x$.

It is known how to construct a plane tangent to two conical surfaces having the guiding line in the same plane. An interesting case is when one of the two cones has the base in the horizontal plane and the other one in the vertical plane.

If it is known a cone that has the base in a circle with the center at point $O_1(o_1, o_1')$, placed in the horizontal plane and the apex at point $S_1(s_1, s_1')$, another cone can be determined, having the base a circle with the center at point $O_2(o_2, o_2')$, placed in the vertical plane and the apex at point $S_2(s_2, s_2')$, so that the two cones admit a common tangent plane (Figure 8).

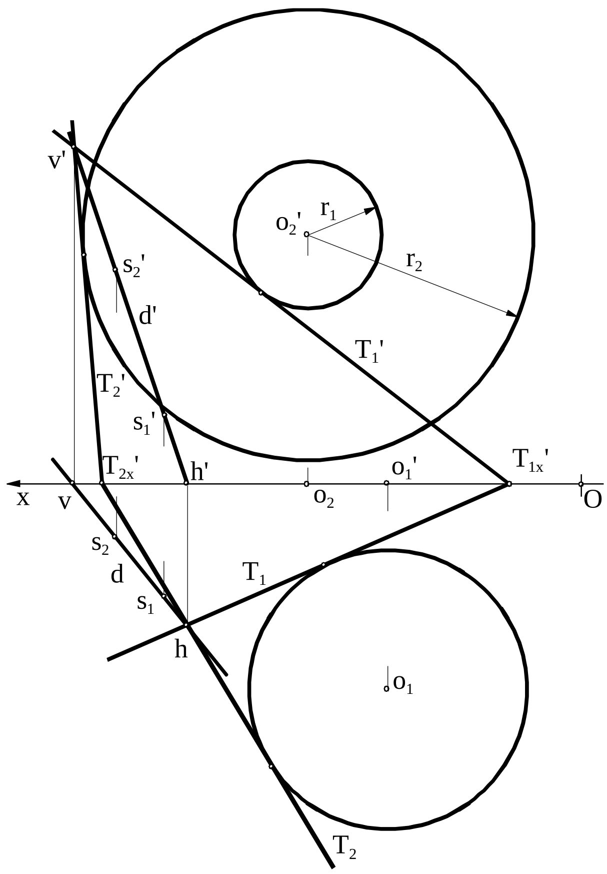

Fig. 8 Plane tangent to two cones.

The common tangent plane of the two cones will contain the straight line connecting their apexes: $D = S_1 \cup S_2$. So, the horizontal trace T_1 of the tangent plane is tangent to the circle having center at point o_1, passing through the horizontal trace of the straight line D, h. The vertical trace T_1' of the tangent plane passes through the vertical trace of the straight line D, v': $T_1' = T_x \cup v'$. The base circle of the second cone will have the radius r_1, being tangent to the vertical trace T_1' of the tangent plane.

Similarly, a second tangent plane [T_2] and another cone having base a circle of radius r_2, also tangent to the vertical trace T_2' can be determined.

There are applications that involve the construction of a straight line tangent to a conical surface, under certain imposed conditions.

This type of straight line $\Delta(\delta,\delta')$ is determined in Figure 9, by imposing that the tangent straight line is concurrent with a given straight line D(d,d').

Initially the plane tangent to the cone is drawn by the point M, $[T_1]$, which then intersects the plane [P], determined by the straight line D(d,d') and the point M(m,m'). The resulting straight line is exactly the straight line $\Delta(\delta,\delta')$, we were looking for:

$$[T_1] \cap [P] = \Delta(\delta,\delta')$$

Drawing also the second possible tangent plane, $[T_2]$, a second solution would be obtained for the straight line tangent to the cone.

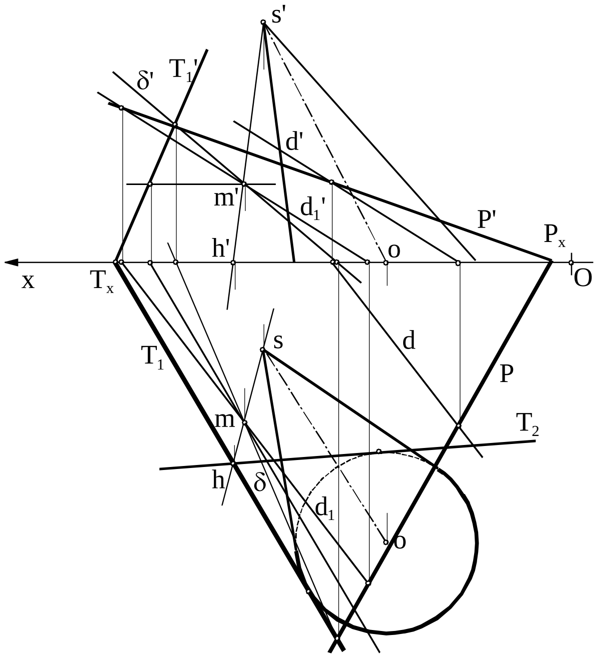

Fig. 9 Straight line tangent to the cone.

Like the common tangent plane to two conical surfaces, the common tangent straight line can also be drawn. In order to determine such a common tangent to two conical surfaces having the bases circles included in the horizontal plane, with the center in points O_1, O_2 and the apexes S_1, S_2, in Figure 10 were drawn the tangent planes $[T_1]$ and $[T_2]$, through the point M(m,m'). The common tangent straight line $\Delta(\delta,\delta')$, looked for is given by the intersection of the two tangent planes:

$$[T_1] \cap [T_2] = \Delta(\delta,\delta').$$

Since for each plane, two tangent planes could be drawn through the point M(m,m'), there are four solutions for the common tangent straight line to the two surfaces, resulting from the intersection of the four planes.

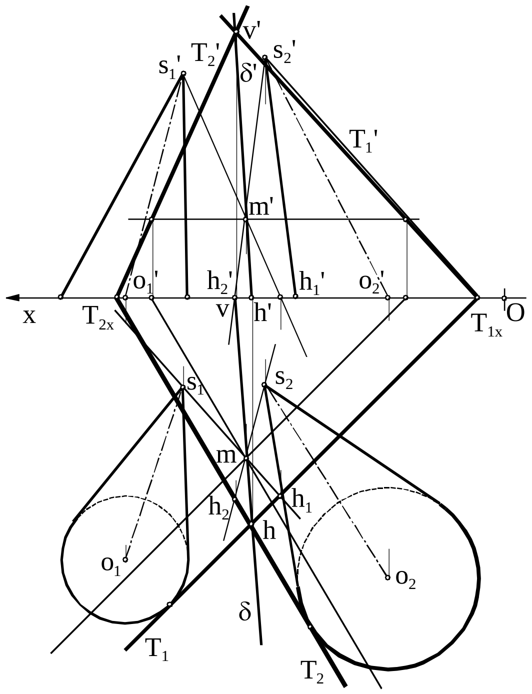

Fig. 10 Straight line tangent to two cones.

6. CONCLUSION

The paper summarizes the possibilities for determining the planes and straight lines tangent to conical surfaces, observing certain imposed conditions.

The possible cases of drawing the planes tangent to conical surfaces based on the angle they make with the horizontal plane of projection are analyzed.

The determination of the straight line tangent to conical surfaces is conditioned by drawing the planes tangent to those surfaces, that contain them.

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