Monica BALCAU, Andrei KIRALY, Liviu SCURTU, Calin PRODAN

THEORETICAL CONSIDERATIONS REGARDING THE CONSTRUCTION OF TETRAHEDRON

Abstract: *The paper presents an analytical study on different theoretical aspects regarding the construction of the tetrahedron. It covers some of the particular construction cases which use descriptive geometry methods. Starting from these cases, one can also construct other particular cases. The matter seems simple enough, yet some of the cases require more complex solutions.*

Key words: *tetrahedron, particular case, regular tetrahedron.*

1. INTRODUCTION

The present paper aims to deal with a few cases relating to the construction of a tetrahedron using Descriptive Geometry. The matter seems simple enough, however, we wish to study some particular cases.

The tetrahedron is a regular polyhedron with four equal faces, shaped as equilateral triangles. The vertex of tetrahedron is projected in the orthocentre of the base.

In the first case, we construct a tetrahedron with the base on a projection plane, namely the horizontal one.

The next case we construct a tetrahedron with a vertical edge.

Another particular case is the tetrahedron with two horizontal edges.

Following, we construct a tetrahedron with a known horizontal line and the horizontal projection of another one.

Next case studied here is the construction of the tetrahedron when we know the projection of one of the heights on a symmetry plane of the tetrahedron.

The last case studied the regular tetrahedron with the base in a different plane than the projection planes.

The paper ends with discussions regarding result analysis.

2. PARTICULAR CASES WHEN CONSTRUCTING A TETRAHEDRON

We take the regular tetrahedron with the base situated in the horizontal projection plane (fig. 1). The side of the tetrahedron is marked with *l*.

In the case of the SABC tetrahedron with the base in the horizontal projection plane, the ABC base is displayed in its real size. The *abc* horizontal projection plane being an equilateral triangle, the vertex of the tetrahedron is projected in the center of gravity of the triangle. The foot of the height of the tetrahedron is also projected in the same point. In vertical projection, *a'*, *b'* and *c'* are projected on the ground line since the value of z is zero (the tetrahedron has the base on the horizontal projection base).

The problem is to determine the height of a regular tetrahedron that has side *l*. We notice with $\Omega(\omega, \omega')$ the center of gravity. In this respect, the horizontal projection builds the right angle triangle $b\omega s_o$ that has a cathetus the ωb projection and the hypotenuse the length a of the edges. In this triangle, the cathetus is ωs_o the height of the tetrahedron and respectively the value of z for the vertex S. The foot of the height of the tetrahedron is a point situated in the horizontal projection plane. Its value is zero [4].

We measure the value of z for point S that gives the vertex of the tetrahedron. By joining the projection *s'* with projections *a'*, *b'* and *c'*, we get the vertical projection of the tetrahedron (fig.1).

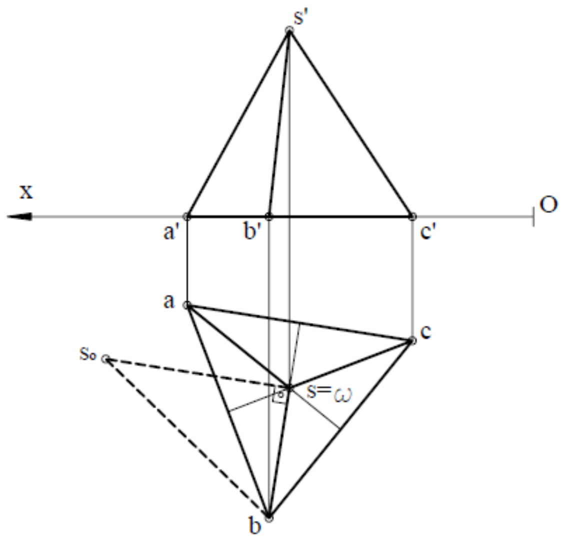

Fig. 1 Constructing a tetrahedron

We are taking into consideration the case in which the tetrahedron has one of the vertical lines of lenght *1*, and the vertex that contains the vertical line is situated in the horizontal projection plane (fig.2).

Take the horizontal line AC(ac,a'c') with given length *l*, and beacause the point S(s,s') situated in the horizontal projection plane, the point S has got the zero value of z, and represents the vertex of the tetrahedron.

Line SC(sc,s'c') makes a 30° angle with the vertical projection plane, also makes a 60° with edge SA, respectively 30° with the horizontal projection plane.

We construct the line D(d,d') which passes through the vertex of the tetrahedron and makes a 30° with both projection planes. Next, we construct the horizontal projection *si* that forms a 30° angle with both projection planes [1].

On this *si* projection we construct the segment $sc_o=s'b'=l$.

The point C will be restored and we construct the line D(d,d') which contains the horizontal projection *c*. This leads to the vertical projection c'.

We construct the isosceles triangle *sca* in which $ca= s'c'=l$. We notice that the line CA(ca,c'a') is a perpendicular line to [V].

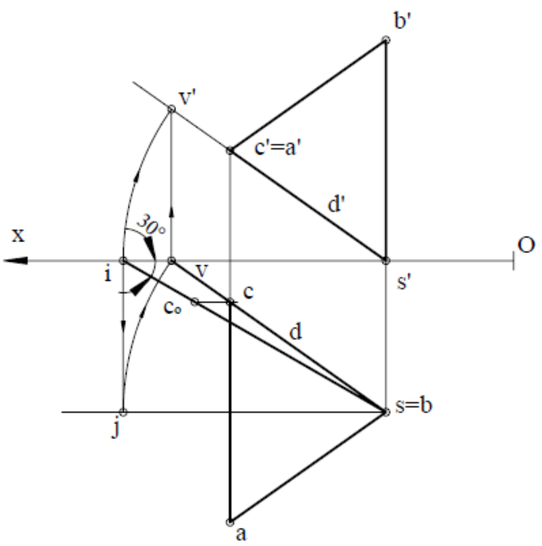

Fig. 2 Constructing the tetrahedron with an vertical line and the vertex that contains the vertical line is situated in the horizontal projection plane

The next case will construct a tetrahedron with an edge of length *l*, with two opposing horizontal lines.

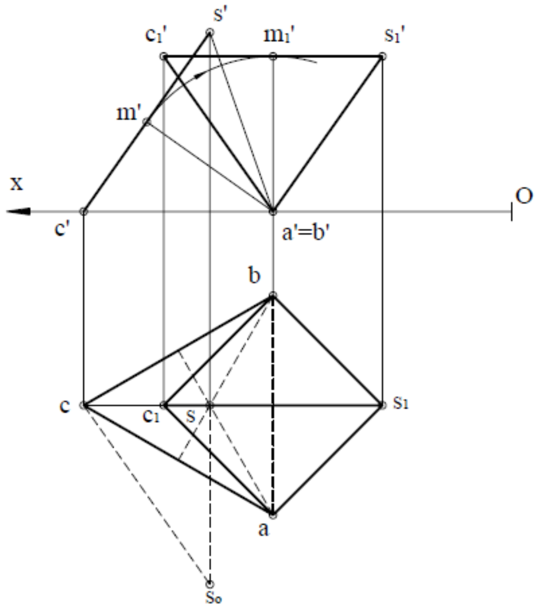

Fig. 3 Constructing the tetrahedron with two opposing horizontal lines

In order to simplify the graphic construction, we construct the horizontal line of the tetrahedron AB(ab,a'b') which is chosen as perpendicular line to [V] situated in the horizontal projection plane.

The horizontal projection *ab* is of lenght *l*. We construct the horizontal projection *abc* which is the equilateral triangle. The vertex of the tetrahedron is projected in the centre of gravity of this triangle.

The base of the height of the tetrahedron is projected in the same point. In order to determine the height of the tetrahedron, we construct the right triangle css_o with one cathetus the projection *sc* and the hypotenuse is length of the edges *l*. In the constructed triangle ss_o is the height of the tetrahedron, respectively S is the vertex of the tetrahedron.

We rotate the SABC tetrahedron so that the frontal line SC(sc,s'c') becomes frontal horizontal line.

In the vertical projection *a's'c'* we construct the height a'm'; the edge SC is a frontal line so the vertical projection *s'c'* appears in its real size. We rotate the projection *s'c'* using vertical projection *m'* of *M* point; result the frontal horizontal S_1C_1.

Because $ab=c_1s_1=l$ the horizontal projection of the tetrahedron is the apparent contour of the horizontal projection s_1ac_1b. The tetrahedron we are looking for is S_1AC_1B (fig.3).

Next we construct the projections of a tetrahedron with the edge AB with length *l* (fig. 4). We construct the horizontal line AB of length *l* projection $ab=l$.

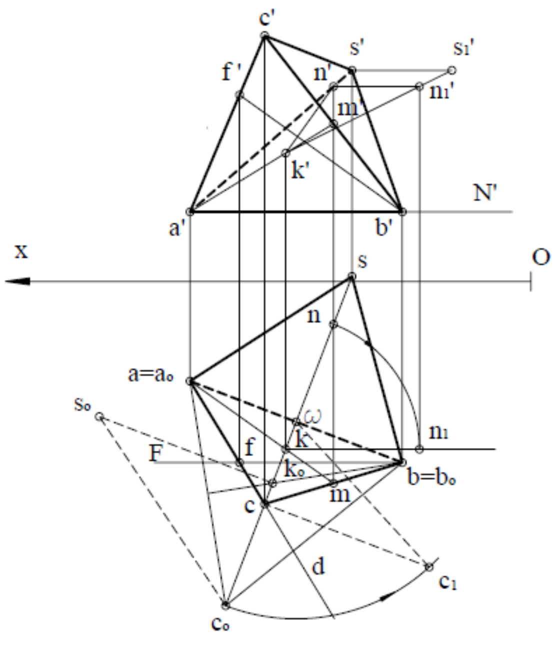

Fig. 4 Construction of the tetrahedron with a horizontal line

Further we build the side abc_o by making a concidence to a horizontal plane N' and restore from coincident c_o in (c,c'). In order to find point C(c,c') we construct the horizontal projection *d* of the edge AC.

In the horizontal projection abc_o we construct the centre of gravity k_o. Next we need to determine the height of the tetrahedron.

This is determined as it's been explained in the previous paragraphs, with the help of the right triangle $s_ok_oc_o$. The cathetus s_ok_o represents the height of the tetrahedron. In order to find the projection *k'* we construct point M(m,m') in the middle of the edge AC.

The horizontal projection *am* meets $c\omega$ in horizontal projection *k*. We draw the perpedincular line KN(kn,k'n') in point K(k,k') on the plane of the side ABC. For this we need to construct the frontal line of plane of the face ABC, namely (bf,b'f'). The projection *k'n'* is perpendicular on the vertical projection *b'f'* of the frontal line of this plane [7].

We rotate this perpendicular line changing it into a frontal one on the vertical projection $k'n_1'$, we measure the height of the tetrahedron, the result is s_1', and we turn it from rotation in S(s,s') [2].

Another case of constructing the tetrahedron is the one in which we know the projection of one of the heights on a symmetrical plane of the tetrahedron. We know the horizontal projection of the height of the tetrahedron on a symmetrical plane.

The tetrahedron SABC has the height projection SI(si,s'i'). To construct the real size of the triangle SAI we take as the new vertical projection plane the symmetry plan of the tetrahedron [3] [8].

We find the projection $s_1'i_1'$ that represents the height of the tetrahedron. Since the edge BC is perpendicular on this symmetry plan, this means that this edge is horizontal line and its new vertical projection is $b_1'=c_1'$.

In the new projection plane, the line B_1C_1 is a perpendicular line to [V]. The new vertical projection of point *i* is i_1'. The plane of the triangle ABC becomes (a_1',$b_1'=c_1'$) perpendicular on $s_1'i_1'$.

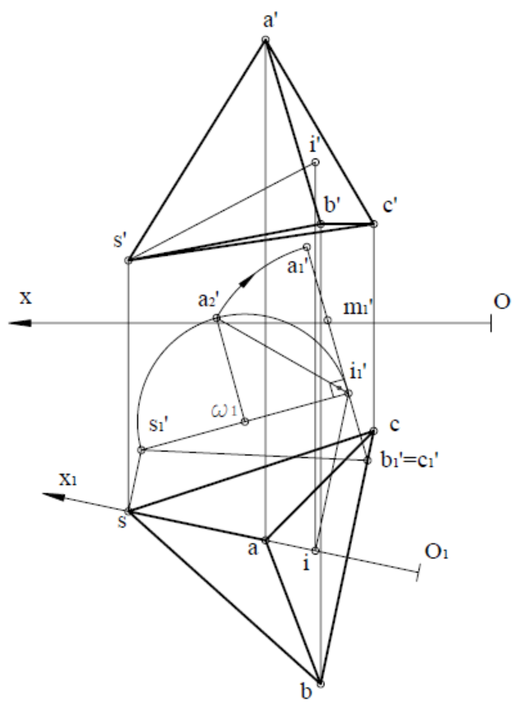

Fig. 5 Constructing the tetrahedron when we know the projection of the height

We notice that AI is the radius of the circle circumscribed to the triangle ABC thus

$$AI=\frac{l}{\sqrt{3}} \qquad (1)$$

and in the triangle SAI results that:

$$AI=\frac{s_1'i_1'}{\sqrt{2}} \qquad (2)$$

From the two relations (1) and (2) results that:

$$l=s_1'i_1'\frac{\sqrt{3}}{\sqrt{2}} \qquad (3)$$

The semicircle constructed on the segment $s_1'i_1'$ with the center in ω_1' is met in a_2' by the perpendicular raised in ω_1', and the arc of the radius $i_1'a_2'$ meets a_1' at the intersection with $b_1'i_1'$. Point A (a,a') is determined at the same time with the length of the edge SA of the tetrahedron.

Because

$$AI=\frac{2}{3}AD \qquad (4)$$

where AD is the apotema of the lateral side of the tetrahedron, we can take

$$i_1'b_1'= i_1'a'/2= i_1'm_1'= m_1'a_1' \qquad (5)$$

so that

$$b_1'= c_1' \text{ and } BC=SA \qquad (6)$$

And with these, we determine the projections of the tetrahedron (fig.5).

In the next case will consider the regular tetrahedron with the base in an oblique plane (fig. 6) [6] [9].

Let P and P' the traces of the plane and A, B, C the points of the regular tetrahedron with edge *l*.

The *abc* and a'b'c'projections are build by restoring from coincidence $a_ob_oc_o$. Plane P was brought into coincidence to the horizontal projection plane with the help of the vertical line. The projections *v* and *v*' are also restored. In this fold, we represent the arc of circle with the radius P_xv_o.

This triangle was restored from coincident, results the horizontal and vertical projections of the base triangle.

Next we need to determine the height of the tetrahedron [5]. This is determined as it's been explained in the previous paragraphs, with the help of the right triangle $s_1s_2b_o$. In this respect, the horizontal projection builds the right triangle $s_1s_2b_o$ that has a cathetus the s_1b_o projection and the hypotenuse the length a of the edges. In this triangle, the cathetus is s_1s_2 the height of the tetrahedron and respectively the value of z for the vertex S. The foot of the height of the tetrahedron i_o is restored from coincident in point *i*, respectively *i'* point.

Next we construct the vertical plane Q through the perpendicular through point I(i,i'). The plane Q intersects with the plane P after an edge of which the coincidence is on the horizontal plane hi_1. We measure the height s_1s_2 on the perpendicular in i_1 on hi_1, and we built s_1 that restored in S(s,s').

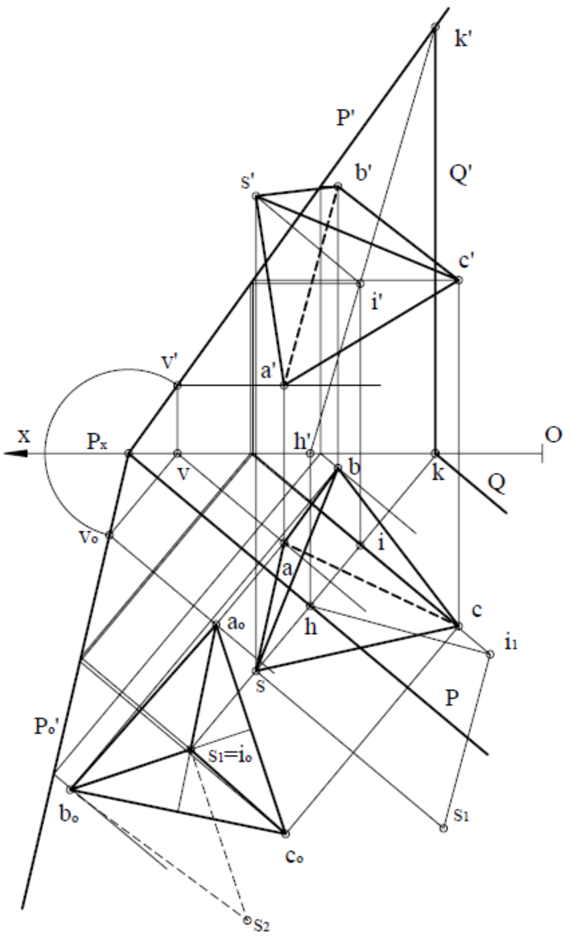

Fig.6 Constructing the tetrahedron with the base in an oblique plane

3. CONCLUSION

The present paper deals with a few particular aspects regarding the construction of a tetrahedron for the purpose of applying in education. Not all particular cases studied there are rapid constructions. The aim is to obtain results by intuitive constructions and not by complex and complicated constructions. Some of the graphic constructions are quick, but others require deep knowledge of Descriptive Geometry. The simplest and the intuitive particular cases regarding the construction of tetrahedron are the first two cases. These can be applied in classical teaching method of Descriptive geometry. More difficult constructions results in the case of the regular tetrahedron situated in an oblique plane and in the case where a tetrahedron edge is a horizontal line and we know the orizontal projection to another one. Of course, when using the CAD applications, all these constructions can be made very quickly.

Today we cannot imagine engineering and technology without the use of computers and computer generated techniques. However, the human factor remains the main element in the areas of development and education. In order for people to develop a better special vision, one needs to understand and use traditional or conventional methods combined with more modern approaches.

REFERENCES

[1] A., Matei, V., Gaba, T., Tacu, *Geometrie Descriptiva*, Editura Tehnica Bucuresti, 1982.

[2] M., Orban, *Proiectii si metode de transformare a proiectiilor*, Editura Casa Cartii de Stiinta 2008, ISBN 978-973-133-393-9.

[3] A., Tanasescu, *Probleme de Geometrie Descriptiva*, Editura Didactica si Pedagogica, Bucuresti 1962.

[4] M,. St., Botez, *Geometrie Descriptiva*, Editura Didactica si Pedagogica, Bucuresti 1965.

[5] A., Tanasescu, *Probleme de Geometrie Descriptiva*, Editura Didactica si Pedagogica, Bucuresti 1967.

[6] S., Bodea, *Geometrie Descriptiva*, Editura RISOPRINT, ISBN 976-656-989-6, Cluj-Napoca 2006.

[7] A., Kiraly, *Grafica Inginereasca*, Editura U.T.PRESS, ISBN 973-8335-35-3, Cluj-Napoca 2002.

[8] V., Iancau, s.a., *Reprezentari geometrice si Desen Tehnic*, E.D.P. 1982.

[9] D., Dragan, s.a., *Descriptive geometry problems*, Editura U.T.PRESS, ISBN 978-973-662-610-4, Cluj-Napoca 2011.

[10] S., Bodea, *Reprezentari grafice ingineresti*, Editura RISOPRINT, ISBN 978-973-53-0144-6, Cluj-Napoca 2010.

[11] M., Orban, s.a., *Geometrie Descriptiva, Suprafete si Corpuri cu Aplicatii in Tehnica*, Editura U.T.PRES, ISBN 97-8335-71-X, Cluj-Napoca 2002.

Authors:

Lecturer PhD. Eng. Monica BALCAU, Technical University of Cluj-Napoca, Faculty of Mechanical Engineering, Department of Automotive Engineering and Transports, E-mail: monica.balcau@auto.utcluj.ro

Associate Professor PhD. Eng. Andrei KIRALY, Technical University of Cluj-Napoca, Faculty of Mechanical Engineering, Department of Automotive Engineering and Transports, E-mail: andrei.kiraly@auto.utcluj.ro

Lecturer PhD. Eng. Liviu SCURTU, Technical University of Cluj-Napoca, Faculty of Mechanical Engineering, Department of Automotive Engineering and Transports, E-mail: liviu.scurtu@auto.utcluj.ro

Teaching Assistant PhD. Eng. Calin PRODAN, Technical University of Cluj-Napoca, Faculty of Mechanical Engineering, Department of Automotive Engineering and Transports, E-mail: vasile.prodan@auto.utcluj.ro